Numerical approximations of hyperbolic systems with source terms and applications
Cortona, Italy, June 14-20, 2015

Eulerian Scheme for multimaterials with plasticity

Alexia de Brauer[1], Angelo Iollo[1] & Thomas Milcent[2]
[1]Institut de Mathématiques de Bordeaux, UMR CNRS 5251, Université de Bordeaux
Inria Bordeaux - Sud Ouest, 33405 Talence, France
[2]Institut de Mécanique et d'Ingénierie, UMR CNRS 5295, Université de Bordeaux
Arts et Métiers Paritech, 33405 Talence, France
Corresponding author: alexia.de-brauer@math.u-bordeaux1.fr

We propose a simple Cartesian method to simulate the interaction of compressible materials separated by a sharp interface [2]. The model describes fluids, hyper-elastic and perfectly plastic solids in a fully Eulerian frame involving a unique scheme for all materials. The scheme can handle large deformations as well as elastic rebounds.

1. The Eulerian Model

The conservative form of fluid or elastic media equations in the Eulerian framework includes the Euler equations on density, momentum and energy and the gradient of a transport equation on the backward characteristics $Y(x,t)$:

$$\partial_t(\nabla_x Y) + \nabla_x(u \cdot \nabla_x Y) = 0 \tag{1}$$

where u is the velocity field. These characteristics describe the evolution of a solid material in the Eulerian framework. To close the system a general constitutive law is chosen so that it models either gas, liquids or neo-hookean elastic solids. This system is solved following the method developed in [2]. Explicitly, the numerical fluxes are calculated using an HLLC approximate Riemann solver [3] adapted to multimaterial flows.

2. The plastic behaviour

According to [1], the deformation gradient, denoted $\mathrm{F} = \nabla_x Y^{-1}$, is multiplicatively decomposed into an elastic part and an irreversible inelastic part : $\mathrm{F} = \mathrm{F}^e\mathrm{F}^i$. In facts, during unloading the solid reaches a state where $\mathrm{F}^e = \mathrm{Id}$ and, thus, the left deformations are plastic. The energy contribution due to elastic strains dominates in comparison to the contribution due to plasticity. Hence, the energy and, as can be demonstrated, the stress tensor are only functions of the elastic deformation. Applying this property to equation (1) along with the decomposition of F gives:

$$\partial_t(\nabla_x Y) + \nabla_x(u \cdot \nabla_x Y) = \mathrm{L}^i\ \nabla_x Y \qquad \text{with, following [1],} \quad \mathrm{L}^i = \frac{1}{\tau}\nabla_x Y \mathrm{dev}(\sigma)\nabla_x Y^{-1} \tag{2}$$

where τ is a relaxation time and $\mathrm{dev}(\sigma) = \sigma - \frac{1}{3}\mathrm{Tr}(\sigma)$ is the deviatoric part of the Cauchy stress tensor σ. The plastic behaviour only occurs when the total stress overtakes the elastic yield stress σ_{y}. Following the von Mises criteria plasticity starts when $\|\mathrm{dev}(\sigma)\|^2 > \frac{2}{3}\sigma_{\mathrm{y}}^2$. Then, the ODE

$$\partial_t(\nabla_x Y) = \frac{1}{\tau}\nabla_x Y \mathrm{dev}(\sigma) \tag{3}$$

is treated with $\nabla_x Y$ implicit and $\mathrm{dev}(\sigma)$ explicit. Equation (3) penalizes the right-hand side so that when $\tau \ll 1$ the deviator tends to zero.

3. Results

We present an impact simulation of a 800m.s^{-1} copper projectile on a copper plate in air (see Fig. 1). The plastic behaviour appears at the beginning of the computation.

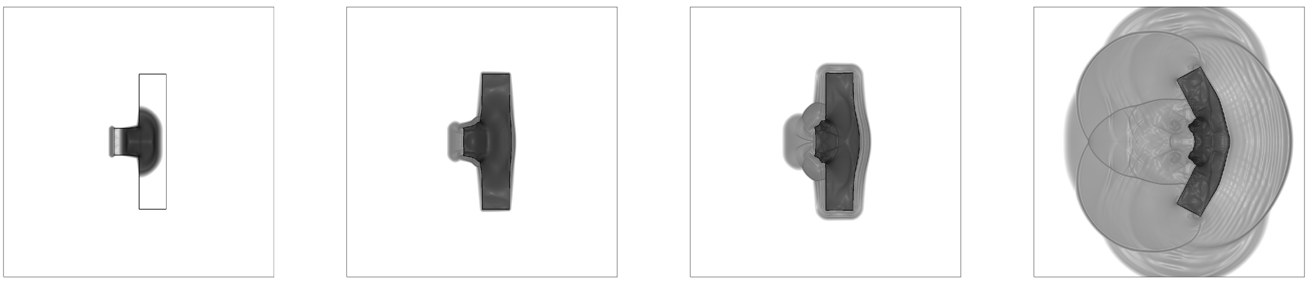

Figure 1: *Impact of a 800m.s^{-1} projectile on a plate. Schlieren at time $t = 12\mu s$, $t = 47\mu s$, $t = 118\mu s$, $t = 710\mu s$.*

References

[1] P.T. Barton, R. Deiterding, D. Meiron & D. Pullin. Eulerian adaptive finite-difference method for high-velocity impact and penetration problems. *Journal of Computational Physics*. Vol 240, p76-99, 2013.
[2] Y. Gorsse, A. Iollo, T. Milcent & H. Telib. A simple Cartesian scheme for compressible multimaterial. *Journal of Computational Physics*. Vol 272, p772-798, 2014.
[3] E.F Toro, M. Spruce & W. Speares. Restoration of the contact surface in the HLL-Riemann solver. *Shock Waves*. Vol 4, p25-34, 1994.