ON SIGNAL DENOISING BY EMD IN THE FREQUENCY DOMAIN

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ABSTRACT

In this work a new denoising scheme based on the empirical mode decomposition associated with a frequency analysis is introduced. Compared to classical approaches where the extracted modes are thresholded in time domain, in the proposed strategy the thresholding is done in the frequency domain. Each mode is divided into blocks of equal length where the frequency content of each one is analyzed. Relevant modes are identified using an energy and a frequency thresholds obtained by training. The denoised signal is obtained by the superposition of the thresholded modes. The effectiveness of the proposed scheme is illustrated on synthetic and real signals and the results compared to those of methods reported recently.

Index Terms— Empirical Mode Decomposition, Intrinsic Mode Function, Denoising,

1 Introduction

The Empirical Mode Decomposition (EMD) is a powerful signal processing tool for extracting signals from a noisy and non-stationary data. The EMD decomposes any data into multi AM-FM components called Intrinsic Mode Functions (IMFs) by the means of an empirical process. When EMD is applied to a noisy data, a physical interpretation of the resulted modes is necessary to determine which IMFs are pure noise, pure signal or contains both [1]. Wu and Huang [2], [3] revealed a statistical significance of the IMFs by studying the statistical characteristics of the uniformly distributed white noise. Peng et al. [4] and Ayenu-Prah and Attoh-Okine [5] proposed a correlation-based threshold to discriminate between relevant and irrelevant IMFs. For very noisy signals, both of these methods have met with limited success, due mainly to the strong correlation between the signal and the first modes. However, a Consecutive Mean-Squared-Error (CMSE) algorithm has been proposed by Boudraa and Cexus [6], in which the signal is reconstructed from the mode for which the CMSE criterion is minimal.

Even if interesting results on synthetic and real data, are obtained using the filtering method defined in the time domain, it is still facing difficulty while dealing with the mode mixing problem, which means that different AM-FM oscillations coexist in a single IMF [7]. Regarding this description, there is no efficient "filtering" method that could overcome this problem due to the fact that an IMF is considered as either signal or noise, thus, it is retained (signal) or rejected (noise). However, to deal with such problem, Wu and Huang proposed EEMD, which proved to be efficient in removing mode-mixing, though, its very motivation becomes under questioning when dealing with non-white noise [?]. We show in the following that the mode mixing can be reduced by combining the standard EMD with a signal block strategy.

One of the first denoising methods introduced in the literature is the EMD-based signal noise reduction approach [8] where the authors dropped the assumption that the noise is spread over only the first IMF. Therefore, all IMFs are preserved and the signal is fully reconstructed using all the "preprocessed" IMFs. This denoising has further been improved recently by Kopsinis and McLaughlin by introducing the Interval Thresholding techniques termed as EMD-IT and EMD-CIIT which stands for EMD-Clear Iterative Interval-Thresholding [9]. The very basic idea is to treat every zero-crossing interval and estimate if it is noise-dominant or signal-dominant based on the single extrema on this interval. If the extrema amplitude exceeds a fixed threshold, the interval is considered as "useful", otherwise, it is considered as noise.

All aforementioned methods use time-domain energy criterion to discriminate between relevant and irrelevant modes. However, sometimes a decision based on only the time-domain energy is not always significant like the case in Figure 1 where we can clearly see that the time-domain information is not sufficient to distinguish between a noise-only signal and a useful one, due mainly to the high power of noise. Whereas by just looking at the frequency domain, the decision can be made so obviously. Hence, we understand the need to treat the IMFs using the frequency domain (Fourier spectrum) instead of their time-domain information. Another limitation concerning all previously cited methods is that they all

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use an energy-based threshold, either in the form of a correlation function, amplitude,.... Although being a sufficient criterion in a lot of applications, it ignores the fact that in some particular cases, the energy is not the only criterion distinguishing useful signals from noise-only ones. It turned out that in the case of a Doppler signal like in Figure 1, a very interesting part of the signal (highlighted with a red rectangle) has very low amplitude, which means very low energy. This part of the signal can be clearly seen in the EMD decomposition in Figure **??**, particularly in the 2^{nd}, 3^{rd} and 4^{th} modes. The horizontal lines represent the thresholds used in [9], one can notice that theses thresholds remove a very important part of the signal, because it does not have a sufficient energy to exceed the thresholds in any of the three modes. Hence the need to add another criterion that can eventually lead to a better thresholding strategy. It turned out that adding a bandwidth-based threshold can be useful in cases where signal energy is small. It should be noted that neither the EMD or the EEMD did not succeed in finding the regions of interest in the signal (see Figures **??** and 2). None of the aforementioned methods succeeds in dealing with such cases. Hence the necessity to develop a denoising algorithm that can extract useful information from low energy signal intervals.

A new EMD-based denoising strategy for detecting the IMF's signal-dominant intervals is presented. In classic situations, when the EMD succeeds in separating noisy modes from signal ones, one can use a hard-thresholding method [6], [10]. However, when signal modes are mixed with noise ones, while the signal ones has significant energy level, any of theses methods [8], [9] will do. But in more complicated cases, when the original signal has low energy level, we proved that the new thresholding strategy outperforms the other methods in terms of SNR especially for low energy parts of the signal.

1.1 EMD-FT denoising

The EMD-based methods studied in [6], [10] are very fast and robust when the modes are not mixed, otherwise, we will certainly lose relevant information or add unwanted noise because of the hard thresholding decision applied (i.e., every mode is either selected or rejected). To overcome the mode mixing problem, one should look inside every IMF and make a decision on every interval. In order to do that, we must understand how the noise does effect the EMD. The white noise has a constant power spectral density, i.e., it has equal power within a fixed large enough bandwidth. On the other hand, for many categories of signals (e.g., Doppler), one interval

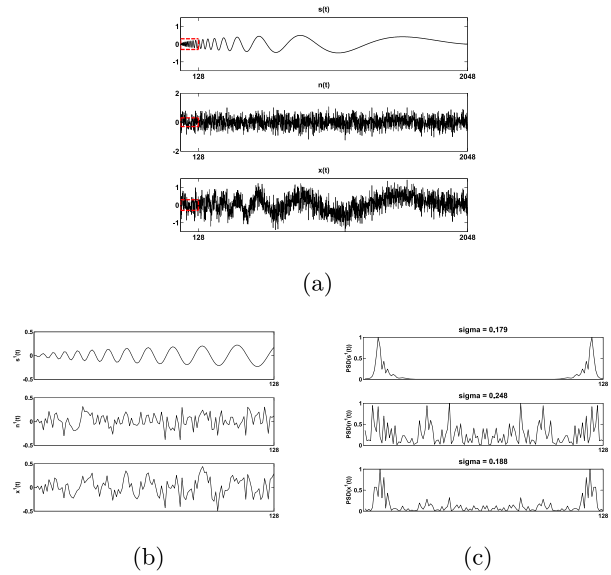


Fig. 1. (a) from top to bottom: original Doppler signal, -2dB noise and noisy Doppler signal, the red square represents a sliding window of 128 samples (b) from top to bottom: the windowed Doppler, the windowed noise and the windowed noisy Doppler (c) from top to bottom: power spectral density of the original Doppler, PSD of the noise and the PSD of the noisy Doppler

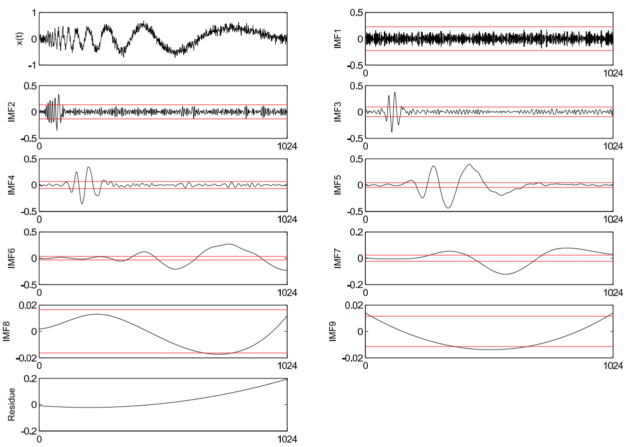


Fig. 2. EMD applied to a Doppler signal. The lines in red represents the thresholds used for CIIT method.

of a certain IMF of an almost noiseless signal is likely to have narrow bandwidth. Thus, one way to discriminate between very noisy intervals and almost noiseless intervals, is to study the frequency domain of each interval of each IMF and decide basing on two criteria: bandwidth and energy. In order to do that, one can start by decomposing each IMF into a number of 50% overlapping blocks and then multiplying each block by a certain analysis window (e.g., a sine window) and then applying the Fourier Transform (FT) on each one of the windowed blocks. Then, we choose two thresholds (one for bandwidth and the other for energy) and we compare the bandwidth and the energy of the block with the chosen thresholds. Based on this comparison, we can estimate the nature of the block (useful signal or noise). For example, we will compare the power spectral densities of two different IMF intervals $|P_k^{j_1}(f)|$ and $|P_k^{j_2}(f)|$, the one that has a narrow-band frequency domain will be considered as signal, while the one with a wide-band frequency domain will be considered as noise.

Let $s(t)$ be a noise free signal contaminated by a white Gaussian noise $b(t)$: $x(t) = s(t)+b(t)$. The algorithm described above can be summarized in the following steps:

1. Decompose the noised signal $x(t)$ using EMD.
2. Decompose each mode $p_i(t)$ into k blocks: $p_i(t) = \{p_i^1(t), \ldots, p_i^j(t), \ldots, p_i^k(t)\}$, $i \in \{1, 2, \ldots, N\}$, where N is the number of modes, with $p_i^j(t) = p_i(t).w_a\left(\frac{t-\tau}{\lambda}\right)$ where $w_a\left(\frac{t-\tau}{\lambda}\right)$ is a sliding window, λ is its width and τ is its delay. Each block has 50% overlapping with the previous one.
3. Compute the discrete FT on each one of the blocks: $\mathcal{F}(p_i^j(t)) = P_i^j(f)$.
4. Compute the energy E_i^j and the bandwidth σ_i^j of each $P_i^j(f)$ [11].
5. Find by learning (Figure 3) the frequency threshold $\overline{\sigma}$ and the energy threshold $\overline{E}$.
6. Select only the intervals having their energy above $\overline{E}$ and their bandwidth below $\overline{\sigma}$. The estimated blocks are expressed as:

$$\tilde{P}_i^j(f) = P_i^j(f)\mathcal{H}(E_i^j - \overline{E})\mathcal{H}(\overline{\sigma} - \sigma_i^j) \tag{1}$$

   where $\mathcal{H}$ denotes the function:

$$\mathcal{H}(E_i^j - \overline{E}) = \begin{cases} 1 & \text{si} \quad E_i^j \geq \overline{E} \\ 0 & \text{si} \quad E_i^j < \overline{E} \end{cases}$$

   and

$$\mathcal{H}(\overline{\sigma} - \sigma_i^j) = \begin{cases} 1 & \text{si} \quad \sigma_i^j \leq \overline{\sigma} \\ 0 & \text{si} \quad \sigma_i^j > \overline{\sigma} \end{cases}$$

7. Compute the inverse FT of each block, then multiply it by the synthesis window w_s:

$$\tilde{p}_i^j(t) = w_s\left(\frac{t-\tau}{\lambda}\right) \Re\{\mathcal{F}^{-1}(\tilde{P}_i^j(f))\} \tag{2}$$

   where $\Re$ is the real part. The global window $w = w_a * w_s$ must satisfy the following condition: $w(\frac{\lambda}{2} + n) + w(n) = 1 \; \forall n \; 1 \leq n \leq \frac{\lambda}{2}$ (because of the 50% overlapping). We may use a synthesis window (e.g., a sine window) for reconstructing the overlapping blocks. i.e., we apply the inverse FT after having chosen the good blocks.
8. Perform the partial reconstruction using only the selected intervals.

$$\tilde{s}(t) = \sum_{i=1}^{N}\sum_{j=1}^{k} \tilde{p}_i^j(t) + r(t) \tag{3}$$

It turns out that the optimal thresholds depend on the signal. More precisely, they depend on the nature of the signal. But this is not really a problem, since in almost all practical cases, we already know what is the nature of the signal. For example, we can predetermine optimal thresholds for a Doppler signal, and these thresholds would also be good for any other signal having the same nature as the Doppler signal, such as FM signals or any modulated signals having a continuous phase (e.g., CPFSK or MSK).

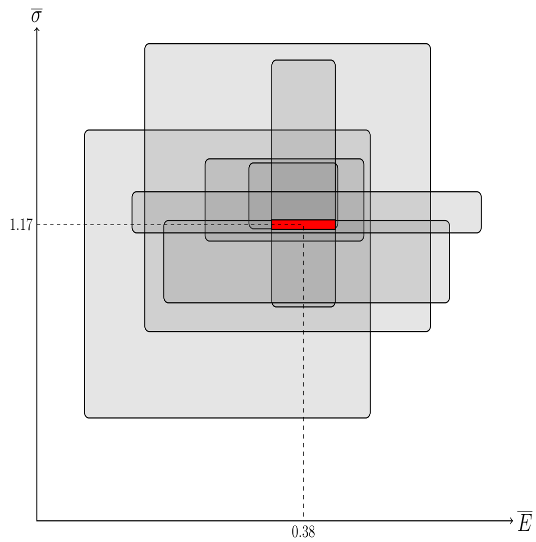


Fig. 3. Learning process: each of the seven grey rectangles corresponds to the maximum SNR region. The red area corresponds to the intersection between the grey regions. The thresholds in this region are considered as optimal thresholds.

1.2 Results and discussions

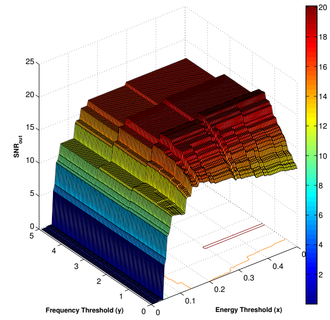
The denoising method is tested using a synthetic Doppler signal and a real-world signal (ECG). The obtained results are compared with wavelet approach [12], [13], [?] and EMD-based denoising methods [9]. First we proceed by estimating the threshold values $\overline{\sigma}$ and $\overline{E}$ by learning. Figures **??** and 4 show the variations of SNR$_{out}$ of the reconstructed Doppler signal versus a range of possible thresholds values. The frequency threshold range is chosen so as $\overline{\sigma} \in [0, f_{m_{(N-1)}}]$ with $f_{m_{(N-1)}}$ denotes the mean frequency of the $(N-1)^{th}$ IMF, which is considered as narrow-band. The energy threshold $\overline{E} \in [0, S]$, where S is the maximum energy threshold defined as $S = C\sqrt{E_{1_{wa}} 2 \ln \lambda}$, where $E_{1_{wa}}$ denotes the energy of a window from the first mode of length $'\lambda'$ (it's worth recalling that the first mode is considered as noise)and C is a constant. Thus, the value of SNR$_{out}$ is calculated for all couples $(\overline{\sigma}, \overline{E})$. It should be noted that for some particular combinations, the energy and bandwidth criterion are not suitable, which leads to a low SNR$_{out}$. In contrast, there are other combinations $(\overline{\sigma}, \overline{E})$ resulting in an optimal SNR$_{out}$, which results in a region of optimal combinations. The retained couple is chosen as the barycenter of this region ($\overline{\sigma} = 1.8$,$\overline{E} = 0.35$).

As for the mode reconstruction, $\tilde{p}_i^j(t)$, (Eq. 2), we chose $w_a = w_s = \sin\left(\pi \frac{t}{\lambda} - \frac{j-1}{2}\right)$ which is a smooth window. The resulting block is thus guaranteed to be circular smooth. Hence, no irregularities will affect the FT. All simulations are performed using sinusoidal window.

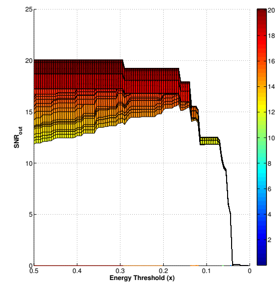
The robustness of the method is proved by calculating the SNR$_{out}$ for the Doppler signal (Figure 5), for a range of SNR$_{in}$ values, using 1024 and 4096 samples. EMD-FT and EMD-CIIT [9] have almost similar performance for small SNR$_{in}$ with a small advantage for the EMD-FT. These two methods offer better performance than the Bayesien and the wavelet methods [13], [12]. However, for higher values of SNR$_{in}$, the EMD-FT gives the best performances in terms of SNR$_{out}$. It is worth noting that for all aforementioned EMD-based methods, the more data points we have (number of samples), the better performance we get.

We have estimated the processing time of these five methods (Table 1). Simulations are conducted on a Intel Core i3 (3^{rd} generation) 3120M/2.1 GHz, with 6 GB of RAM. The results show that our denoising method is faster than the others, which make it more useful for real-time applications.

The proposed method is also tested for a real-world ECG signal. Three signals are presented in Figure 6, the original ECG signal $s(t)$, its noised version $x(t)$ and its denoised version $\tilde{s}(t)$. The results show that the main



(a)



(b) (c)

Fig. 4. SNR$_{out}$ of the Doppler signal reconstructed in function of (a) both the frequency threshold $\overline{\sigma}$ and the energy threshold $\overline{E}$. (b) in function of the energy threshold $\overline{E}$ (c) in function of frequency threshold $\overline{\sigma}$

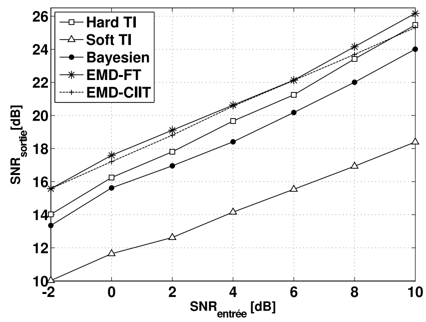


Fig. 5. SNR$_{out}$ versus SNR$_{in}$ for the reconstructed Doppler signal (a) 1024 samples (b) 4096 samples.

Table 1. Processing time for the denoising methods.

Processing time [s]		
	Number of samples	
Methods	**1024**	**4096**
Hard-TI [13]	0.0324	0.0672
Soft-TI [14]	0.0291	0.0689
Bayesien [12]	0.0092	0.0248
EMD-CIIT [9]	1.5817	3.5335
EMD-FT	**0.0067**	**0.0228**

characteristics of the signal are well preserved, while reducing enormously the noise.

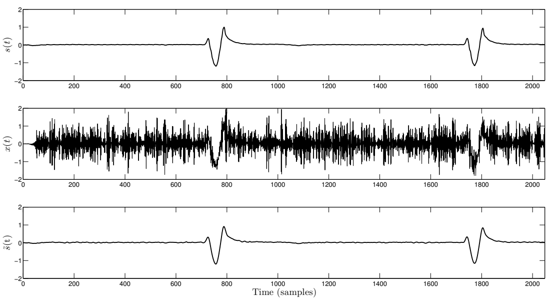


Fig. 6. ECG signal (original, noisy and denoised).

1.3 Conclusion

This paper presented a new EMD-based method associated with a frequency analysis for denoising. The first results are very promising and show the importance of frequency thresholding. Indeed, the method leads to a SNR_{out} of the order of 20 dB for very low SNR_{in}. The reduced processing time should be emphasized, which permits the use of this method for real-time applications. This method should be validated using a larger class of real-world signals contaminated with different types of noise, while varying the window length, so we can associate an optimal window length to each case of study. Also, a special attention should be paid to mathematical formulation of this method, especially the learning process.

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