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Locking-free formulation for the stabilized enhanced strain solid-shell element (SHB8PS): geometrically non-linear applications

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Abstract

In this work, a new locking-free and physically stabilized formulation of the SHB8PS solid-shell element is presented. The resulting finite element consists of a continuum mechanics shell element based on a purely three-dimensional approach. This eight-node hexahedron is integrated with a set of five Gauss points, all distributed along the "thickness" direction. Consequently, it can be used for the modeling of thin structures, while providing an accurate description of various through-thickness phenomena. The reduced integration has been used in order to prevent some locking phenomena and to increase its computational efficiency. The spurious zero-energy deformation modes due to the reduced integration are efficiently stabilized, whereas the strain components corresponding to locking modes are eliminated with a projection technique following the Enhanced Assumed Strain (EAS) method.

1. Introduction

Over the last decade, considerable progress has been made in the development of three-dimensional finite elements capable of modeling thin structures [1], [3-5], [7]. The coupling between solid and shell formulations has proven to be an interesting way to provide continuum finite element models that can be efficiently used for structural applications. These solid-shell elements have numerous advantages for the analysis of various complex structural geometries that are common in many industrial applications. Their main advantage is to allow such complex structural shapes to be meshed without classical problems of connecting zones meshed with different element types (continuum and structural elements for instance). Another important benefit of the solid-shell concept is the avoidance of tedious and complex pure-shell element formulations needed for the complex treatment of large rotations.

In this work, a new locking-free formulation for the SHB8PS solid-shell element is performed. Note that the SHB8PS element was first implemented into the dynamic explicit finite element code (Europlexus) for impact problem simulations (Abed-Meraim and Combescure [1]). Later, an implicit version was implemented into the quasi-static implicit code (Inca) for stability analysis of shells (Legay and Combescure [4]). This element is an eight-node, three-dimensional hexahedron with a preferential direction called "the thickness". Therefore, it can be used to represent thin structures while providing an accurate description of various through-thickness phenomena thanks to the use of a numerical integration with five Gauss points in that direction. As a result, the element is under-integrated and requires a stabilization procedure to control the associated hourglass modes. The stabilization technique used is based on the Assumed Strain Method (Belytschko and Bindeman [2]).

More specifically, this work focuses on the elimination of the residual membrane and shear locking persisting in the previous formulations. By using orthogonal projections of the discrete gradient operator, these severe shear and membrane locking modes are removed. Several numerical experiments on popular linear and non-linear benchmark problems show that this new formulation of the SHB8PS element is effective under non-linear conditions and demonstrates good convergence without locking phenomena.

2. Formulation of the SHB8PS element

The element coordinates x_i and displacements u_i $(i = 1,...,3)$ are interpolated using the isoparametric trilinear shape functions $N_I(\xi,\eta,\zeta)$ $(I = 1,...,8)$. By introducing the *Hallquist* vectors ($\underline{b}_i$, i = 1,..., 3), defined as:

$$\underline{b}_i^T = \underline{N}_{,i}(0,0,0) \qquad i = 1,2,3 \qquad Hallquist's\ Form \tag{1}$$

where $\underline{N}_{,i} = \partial \underline{N} / \partial x_i$, one can show that the discrete gradient operator, which relates the linear deformations to the nodal displacements (i.e., $\underline{\underline{\nabla}}_S(\underline{u}) = \underline{\underline{B}} \cdot \underline{d}$), is given by equation (3) below. This $\underline{\underline{B}}$ -matrix makes use of the following variables:

$$\begin{cases} \underline{\gamma}_\alpha = \frac{1}{8}\left[\underline{h}_\alpha - \sum_{j=1,3}(\underline{h}_\alpha^T \cdot \underline{x}_j)\underline{b}_j\right] \\ h_1 = \eta\zeta,\ h_2 = \zeta\xi,\ h_3 = \xi\eta,\ h_4 = \xi\eta\zeta \end{cases} ; \quad \begin{cases} \underline{d}_i^T = (u_{i1}, u_{i2}, u_{i3,}, ..., u_{i8}), & \underline{x}_i^T = (x_{i1}, x_{i2}, x_{i3}, ..., x_{i8}) \\ \underline{h}_1^T = (1,1,-1,-1,-1,-1,1,1), & \underline{h}_2^T = (1,-1,-1,1,-1,1,1,-1) \\ \underline{h}_3^T = (1,-1,1,-1,1,-1,1,-1), & \underline{h}_4^T = (-1,1,-1,1,1,-1,1,-1) \end{cases} \tag{2}$$

$$\underline{\underline{B}} = \begin{bmatrix} \underline{b}_x^T + h_{\alpha,x}\underline{\gamma}_\alpha^T & 0 & 0 \\ 0 & \underline{b}_y^T + h_{\alpha,y}\underline{\gamma}_\alpha^T & 0 \\ 0 & 0 & \underline{b}_z^T + h_{\alpha,z}\underline{\gamma}_\alpha^T \\ \underline{b}_y^T + h_{\alpha,y}\underline{\gamma}_\alpha^T & \underline{b}_x^T + h_{\alpha,x}\underline{\gamma}_\alpha^T & 0 \\ 0 & \underline{b}_z^T + h_{\alpha,z}\underline{\gamma}_\alpha^T & \underline{b}_y^T + h_{\alpha,y}\underline{\gamma}_\alpha^T \\ \underline{b}_z^T + h_{\alpha,z}\underline{\gamma}_\alpha^T & 0 & \underline{b}_x^T + h_{\alpha,x}\underline{\gamma}_\alpha^T \end{bmatrix} \left(\begin{array}{c}\text{The convention of implied summation} \\ \text{of repeated subscripts } \alpha \text{ is adopted}\end{array}\right) \tag{3}$$

Despite the geometry of the element (eight-node hexahedron), several modifications are introduced in order to provide it with shell features. Among them, a shell-like behavior is intended for the element, by modifying the three-dimensional elastic constitutive law so that the plane-stress conditions are approached and by aligning all the integration points along a privileged direction, called the thickness. The stiffness matrix is then obtained by Gauss integration:

$$\underline{\underline{K}}_e = \int_{\Omega_e} \underline{\underline{B}}^T \cdot \underline{\underline{C}} \cdot \underline{\underline{B}}\, dv = \sum_{I=1}^{5} \omega(\zeta_I) J(\zeta_I) \underline{\underline{B}}^T(\zeta_I) \cdot \underline{\underline{C}} \cdot \underline{\underline{B}}(\zeta_I) \tag{4}$$

Because the $h_{\alpha,i}$ functions $(\alpha = 3,4;\ i = 1,2,3)$ vanish at the five Gauss points, of coordinates $\xi_I = \eta_I = 0,\ \zeta_I \neq 0,$ the $\underline{\underline{B}}$-matrix in equation (3) reduces to its $\underline{\underline{B}}_{12}$ part, with only the $h_{\alpha,i}$ terms $(\alpha = 1,2;\ i = 1,2,3)$. This leads to six hourglass modes generated by $\underline{h}_3$ and $\underline{h}_4$. These spurious modes are stabilized following the approach given in Belytschko and Bindeman [2]. Moreover, we apply an assumed strain method in order to eliminate locking. The $\underline{\underline{B}}$ -matrix is thus projected onto $\overline{\underline{\underline{B}}}$ as:

$$\overline{\underline{\underline{B}}} = \underline{\underline{B}}_{12} + \overline{\underline{\underline{B}}}_{34} \tag{5}$$

Consequently, the stiffness matrix, equation (4), can be rewritten as:

$$\underline{\underline{K}}_e = \underline{\underline{K}}_{12} + \underline{\underline{K}}_{STAB} \tag{6}$$

The first term $\underline{\underline{K}}_{12}$ is obtained by Gauss integration, equation (4). The second term $\underline{\underline{K}}_{STAB}$ represents the stabilization stiffness:

$$\underline{\underline{K}}_{STAB} = \int_{\Omega_e} \underline{\underline{B}}_{12}^T \cdot \underline{\underline{C}} \cdot \overline{\underline{\underline{B}}}_{34} dv + \int_{\Omega_e} \overline{\underline{\underline{B}}}_{34}^T \cdot \underline{\underline{C}} \cdot \underline{\underline{B}}_{12} dv + \int_{\Omega_e} \overline{\underline{\underline{B}}}_{34}^T \cdot \underline{\underline{C}} \cdot \overline{\underline{\underline{B}}}_{34} dv \tag{7}$$

3. Numerical results and discussions

The performance of this new formulation has been tested through a variety of linear and non linear mechanical problems. In all of these tests, the new version showed better performance than the previous formulation. In particular, the improvement is significant in the pinched hemispherical shell test.

3.1 Pinched hemispherical shell problem

This test has become very popular and it is used by many authors. It is severe since the shear and membrane locking phenomena are very important and emphasized by the problem geometry (distorted, skewed elements). As reported by many authors, in this doubly-curved shell problem, the membrane locking is much more severe than shear locking. Figure 1 shows the geometry, loading and boundary conditions for this problem.

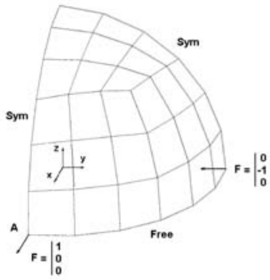

Figure 1: Geometry and loading of the pinched hemispherical shell problem

The radius is R=10, the thickness t=0.04, Young's modulus E=6.825×10^7 and Poisson's ratio ν =0.3. Using the symmetry, only a quarter of the hemisphere is meshed using a single element through the thickness and with two unit loads along directions Ox and Oy. The reference solution for the radial displacement at the load point is 0.0924. The convergence results are reported in Table 1 in terms of the normalized displacement at the load point. The new version of the SHB8PS element is compared with the former one and with the three elements HEX8, HEXDS and H8-ct-cp. The HEX8 element is the standard, eight-node, full integration solid element (eight Gauss points). The HEXDS element is an eight-node, four-point quadrature solid element (Liu *et al.* [6]). The H8-ct-cp element was developed by Lemosse [5]. Table 1 shows that the new version of the SHB8PS element provides an excellent convergence rate and shows no locking.

Number of	SHB8PS previous version	HEX8	HEXDS	H8-ct-cp	SHB8PS new version
	Ux/Uref	Ux/Uref	Ux/Uref	Ux/Uref	Ux/Uref
12	0.0629	0.0005		0.05	0.8645
27	0.0474	0.0011			1.0155
48	0.1660	0.0023	0.408	0.35	1.0098
75	0.2252	0.0030	0.512	0.58	1.0096
192	0.6332	0.0076	0.701	0.95	1.0008
363	0.8592	0.0140	0.800		1.0006
768	0.9651	0.0287			1.0006
1462	0.9910	0.0520			1.0009

Table 1: Normalized displacement at the load point of the pinched hemispherical shell

3.2 Pinched cylinder with end diaphragms

A cylindrical shell loaded by a pair of concentrated vertical forces at its middle section is considered here. Both ends of the cylinder are covered with rigid diaphragms that allow displacement only in the axial direction (Figure 2). This test is considered as a selective test problem since it has been shown that shear locking is more severe than membrane locking. The length is L=600, the radius R=300, the thickness t=3, Young's modulus

$E=3\times10^6$, Poisson's ratio $\nu=0.3$ and the applied load P=1. Owing to symmetry, only one eighth of the cylinder is modeled using a single element through the thickness and 2×2, 4×4, 8×8, 16×16 and 32×32 meshes: Figure 2.

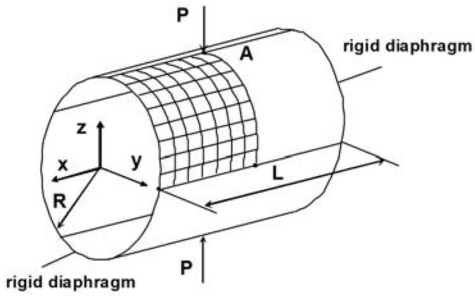

Figure 2: Geometry, boundary conditions and loading for the pinched cylinder

The displacement at the loaded point in the loading direction is normalized with respect to the reference solution of 0.18248×10^{-4} and reported in Table 2 below.

Mesh layout	SHB8PS previous version Uz/Uref	SHB8PS new version Uz/Uref
2x2	0.043	0.101
4x4	0.223	0.387
8x8	0.708	0.754
16x16	0.937	0.940
32x32	0.996	0.997

Table 2: Normalized displacements at the loaded point of the pinched cylinder

As we can see in Table 2, the new version of the SHB8PS element performs better than the former one, especially for coarse meshes. This new version has also been tested on a variety of non linear, elastic and elastic-plastic problems. We demonstrate that the projection adopted in this formulation better eliminates the locking phenomena. As shown particularly in the pinched hemisphere test problem, Table 1, this element also exhibits excellent efficiency and convergence through numerous other tests.

References

[1] Abed-Meraim F and Combescure A. SHB8PS a new adaptive, assumed-strain continuum mechanics shell element for impact analysis. *Computers & Structures* 2002; **80**:791-803.

[2] Belytschko T and Bindeman LP. Assumed strain stabilization of the eight node hexahedral element. *Computer Methods in Applied Mechanics and Engineering* 1993; **105**:225-260.

[3] Hauptmann R and Schweizerhof K. A systematic development of solid-shell element formulations for linear and non-linear analyses employing only displacement degrees of freedom. *International Journal for Numerical Methods and Engineering* 1998; **42**:49-69.

[4] Legay A and Combescure A. Elastoplastic stability analysis of shells using the physically stabilized finite element SHB8PS. *International Journal for Numerical Methods and Engineering* 2003; **57**:1299-1322.

[5] Lemosse D. Eléments finis iso-paramétriques tridimensionnels pour l'étude des structures minces. PhD Thesis, Ecole Doctorale SPMI/INSA-Rouen, 2000.

[6] Liu WK, Guo Y, Tang S and Belytschko T. A multiple-quadrature eight-node hexahedral finite element for large deformation elastoplastic analysis. *Comp. Meth. in Applied Mech. and Engng.* 1998; **154**:69-132.

[7] Wall WA, Bischoff M and Ramm E. A deformation dependent stabilization technique, exemplified by EAS elements at large strains. *Computer Methods in Applied Mechanics and Engng.* 2000; **188**:859-871.