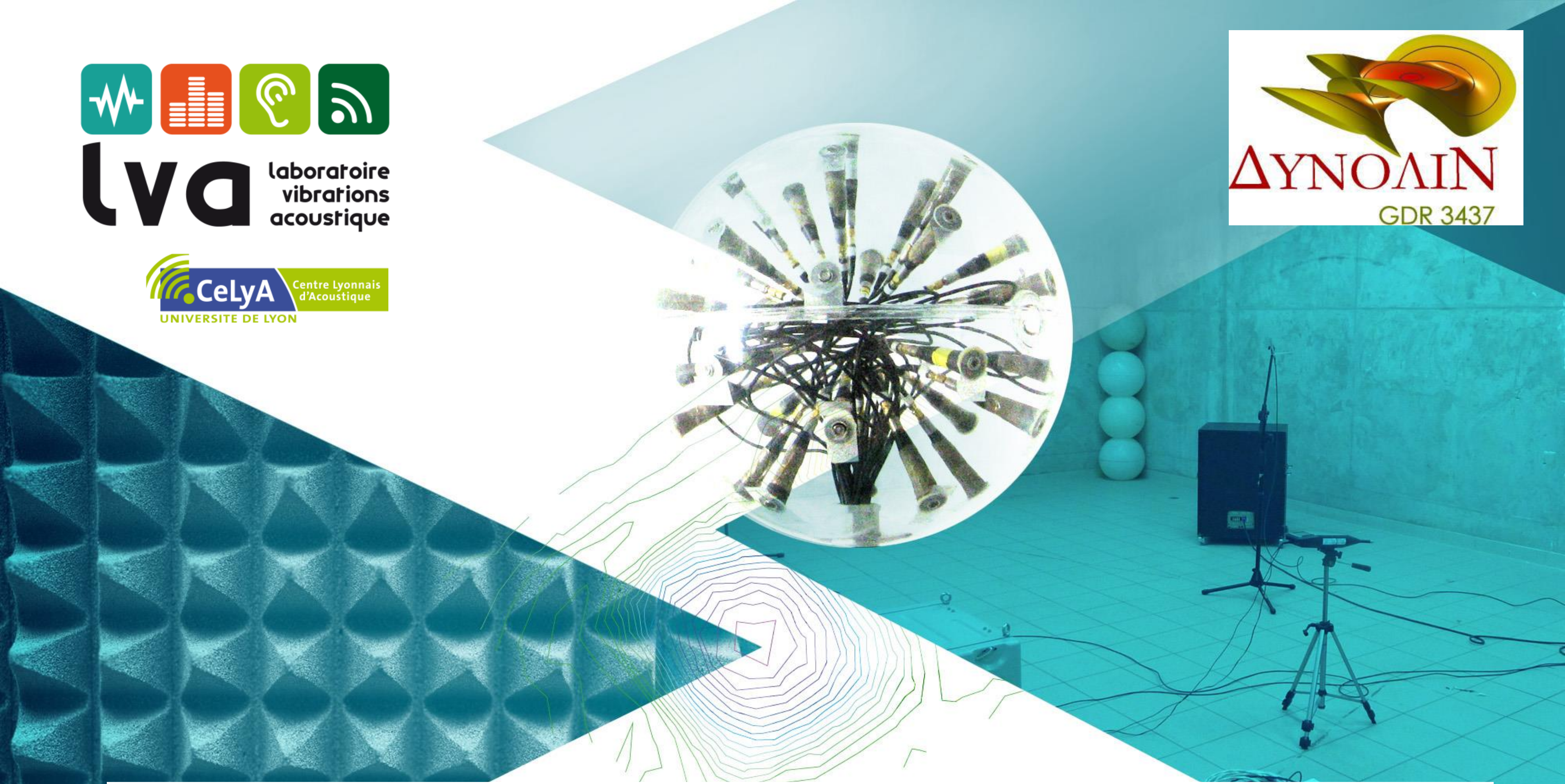


Exponential sine sweeps for the autonomous estimation of nonlinearities and errors assessment by bootstrap
Application to thin vibrating structures

Marc Rébillat[1], Kerem Ege[2], Maxime Gallo[2], Jérôme Antoni[2]

1. 2.

A linear behavior in vibroacoustics ?

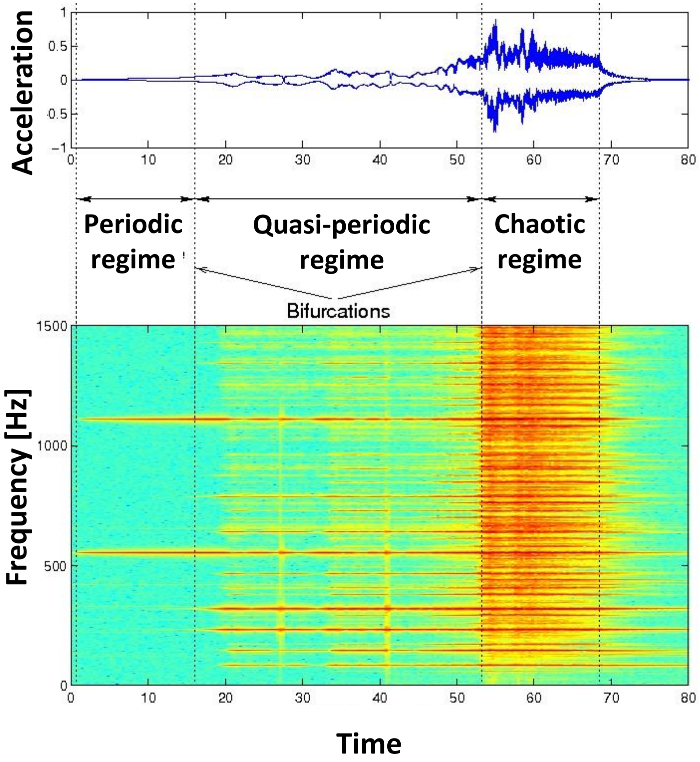
- Geometrically nonlinear vibrations of plates, shells:

when the amplitude of the transverse displacement w
exceeds the plate/shell thickness h

$\longrightarrow$ *jump phenomenon, hysteresis, internal resonance*

Example : Gong

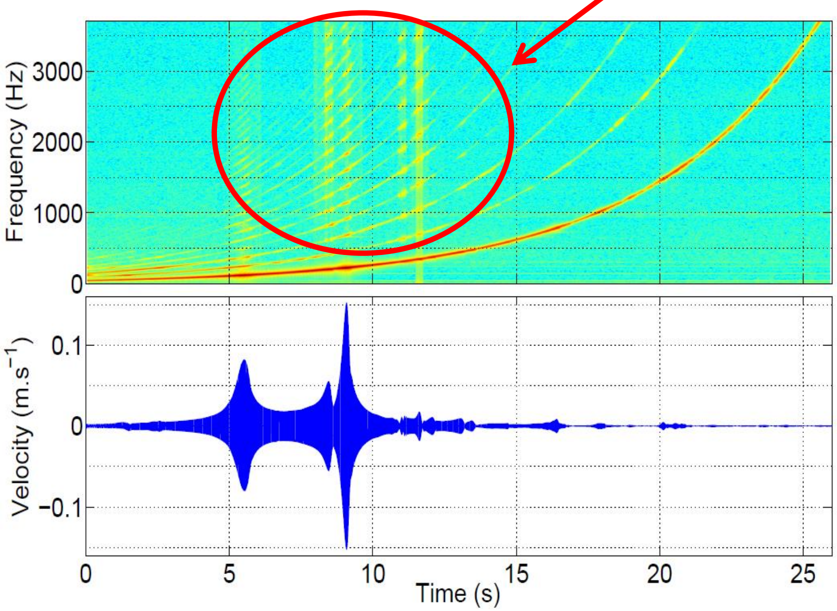
Cyril Touzé, ***Olivier Thomas***



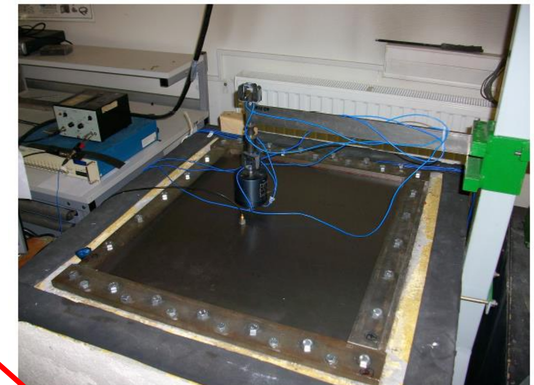
- Boundary conditions, joints, contacts...

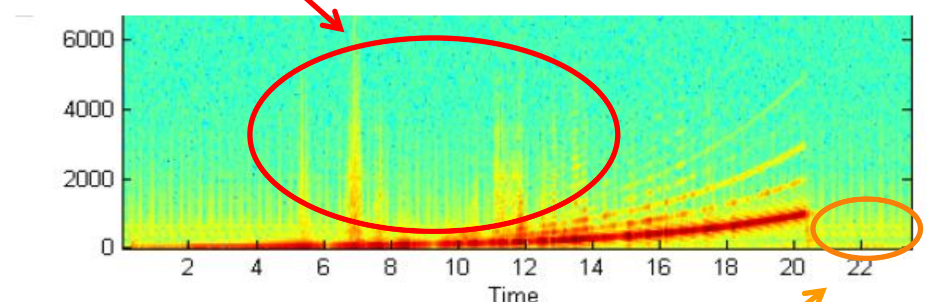
A linear vibroacoustic measurement ?

Acoustical excitation



Vibrational excitation





How to extract the linear/nonlinear parts of a typical vibroacoustic measurement ?

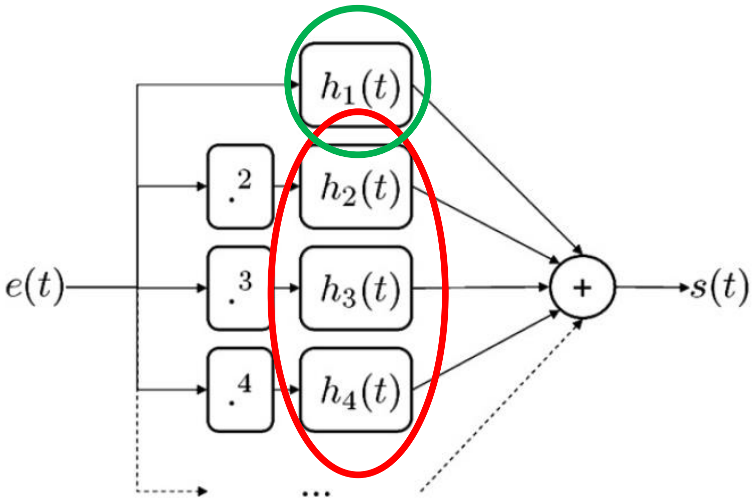
How to estimate the effects of experimental noise ?

Outline

- Estimation of Parallel Hammerstein models
  - Theory
  - Application to a vibrating plate
  - Separation of the intrinsic nonlinear contributions
- Improvements of the method (sweep repetition)
  - Theory
  - Noise estimation by synchronous averaging
  - Uncertainty estimation by bootstrap
  - Autonomous kernel order estimation
- Conclusion and perspectives

Parallel Hammerstein models estimation

⇒ Slightly nonlinear system modeled as **parallel Hammerstein models**

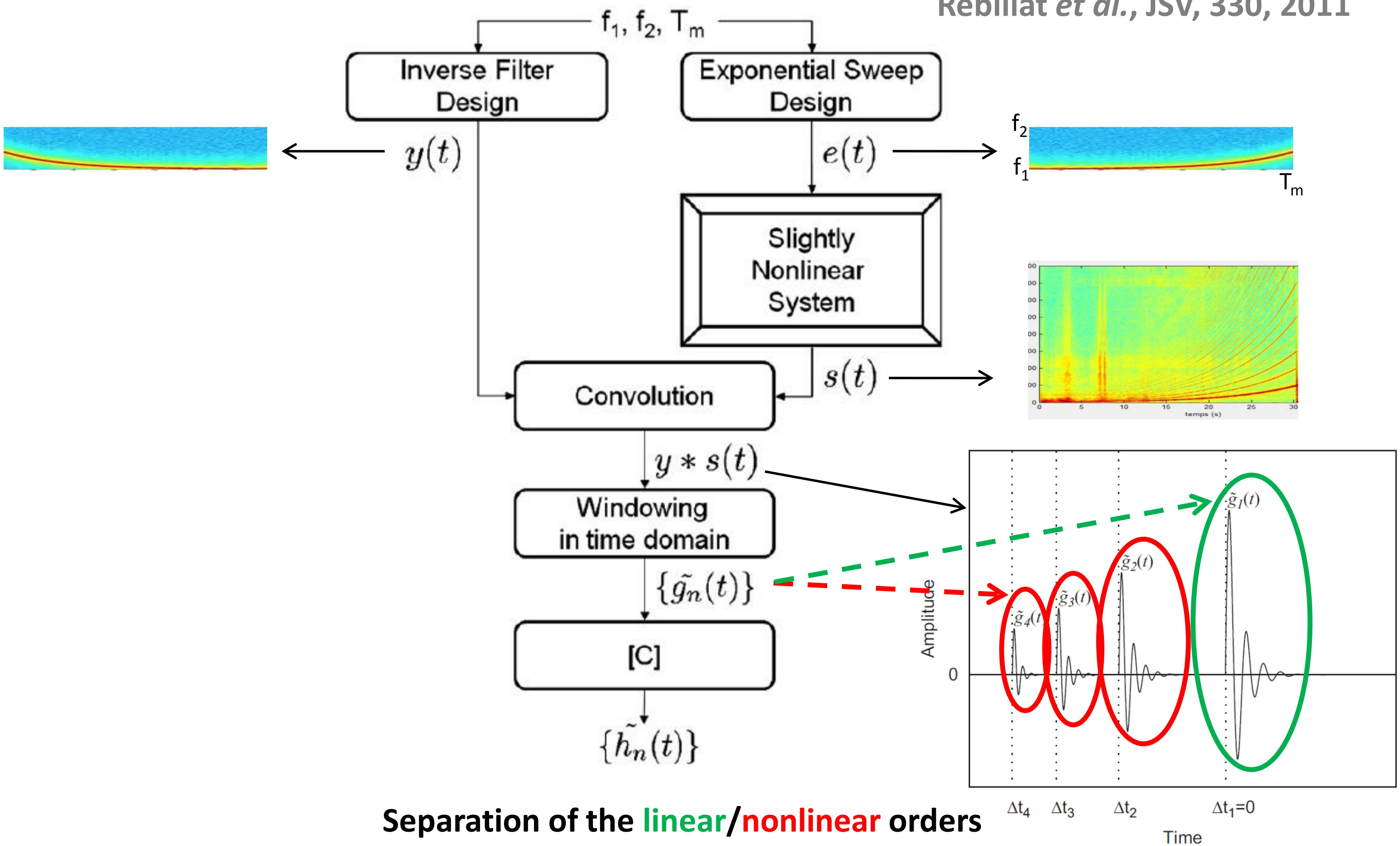


⇒ Kernels easily estimated using **exponential sine sweeps**
(see [Farina, AES, 2000], [Rébillat, JSV, 2011] or [Novak, IEEE Instrumentation, 2011] for example)

⟶ Separation of the **linear**/**nonlinear** orders

Parallel Hammerstein models estimation

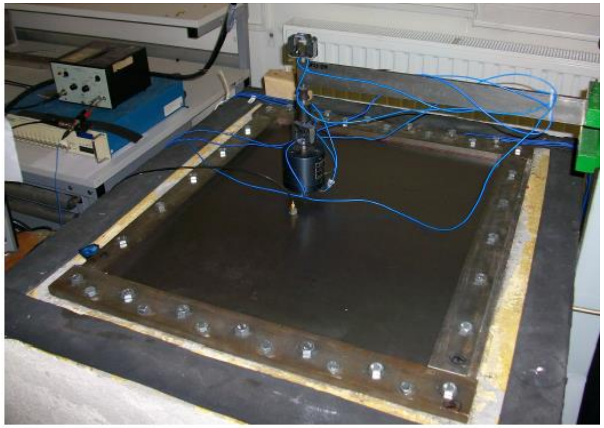
Rébillat *et al.*, JSV, 330, 2011



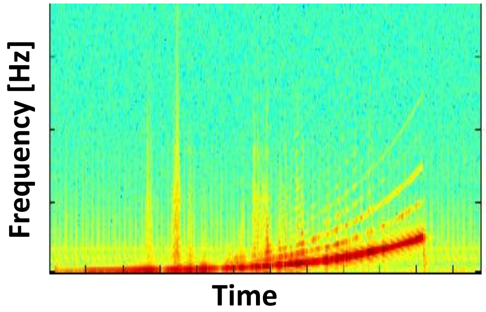
Separation of the linear/nonlinear orders
<u>in time domain</u>

Application to a vibrating plate

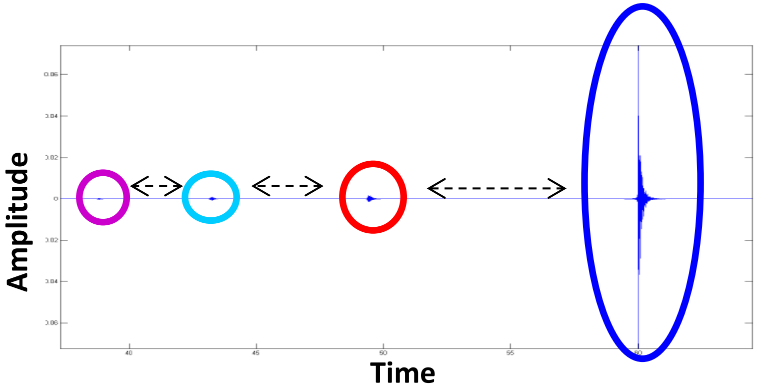
- Clamped Steel plate (1mm)
- Shaker + Accelerometer



One sweep of **20 seconds** [20Hz-1kHz]

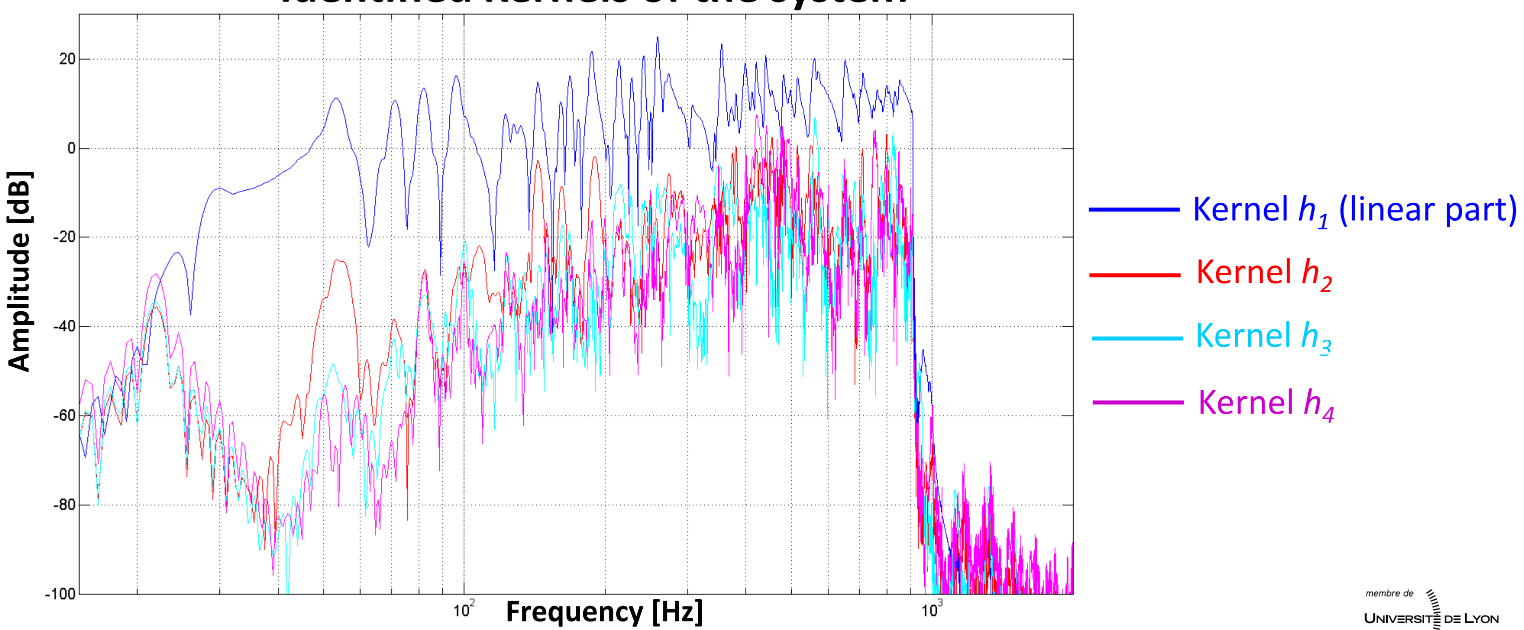


After deconvolution:



Importance of the **length of the sweep** for time domain separation

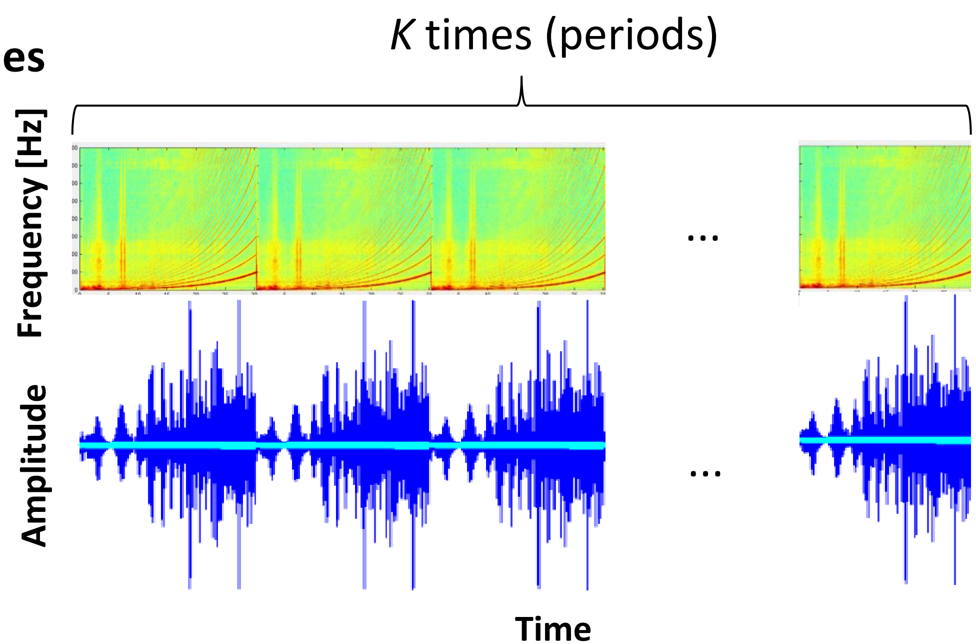
Identified Kernels of the system



Improvement of the method: Sweep repetition & Noise estimation

- Measured signal = system response + experimental noise $x(t) = s(t) + n(t)$

- **Repetition of the same sweep *K* times**



- **Estimation of the noise** $\bar{n}(t) = x(t) - \bar{s}(t)$

  using the **time synchronous averaging** $\bar{s}(t)$ of the system response $x(t)$

$$\bar{s}(t) = \frac{1}{K} \sum_{k=0}^{K-1} [x(t - kT)]$$

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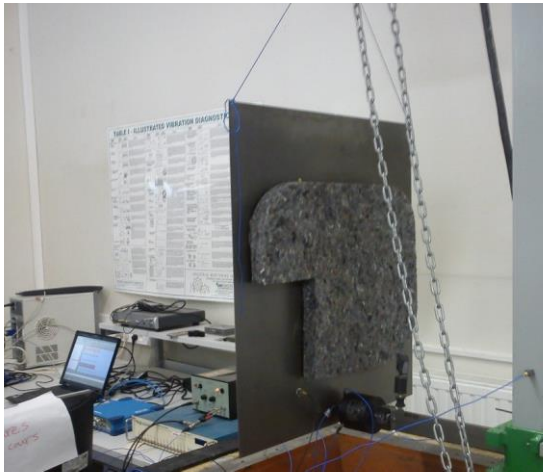
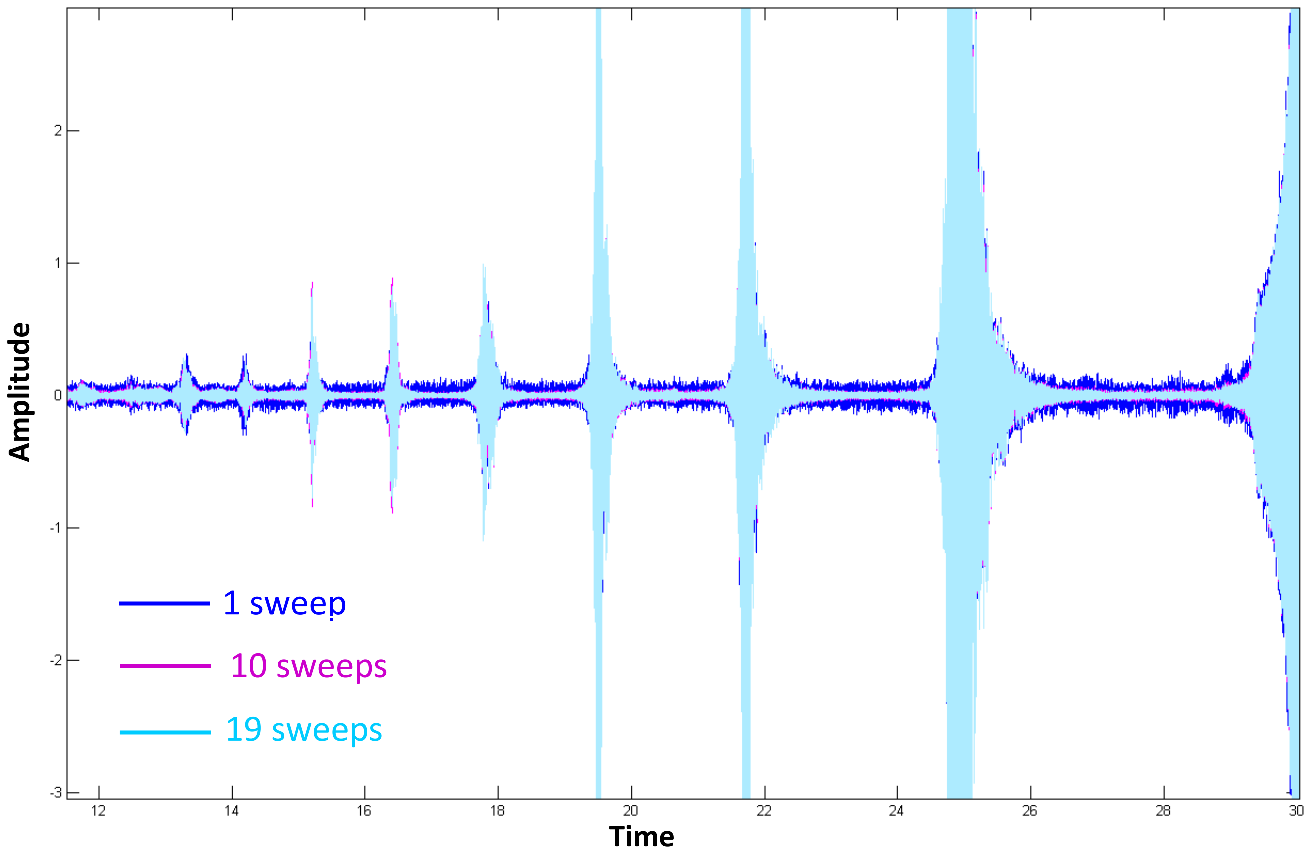
membre de UNIVERSITÉ DE LYON

Application to a vibrating plate – Noise estimation

- Free-free damped steel plate (1mm)
- Shaker + Accelerometer



Influence of sweep repetitions (number of periods K)



Increase of *K*

↓

Synchronous averaging on **more periods**

$$\bar{s}(t) = \frac{1}{K}\sum_{k=0}^{K-1}[x(t-kT)]$$

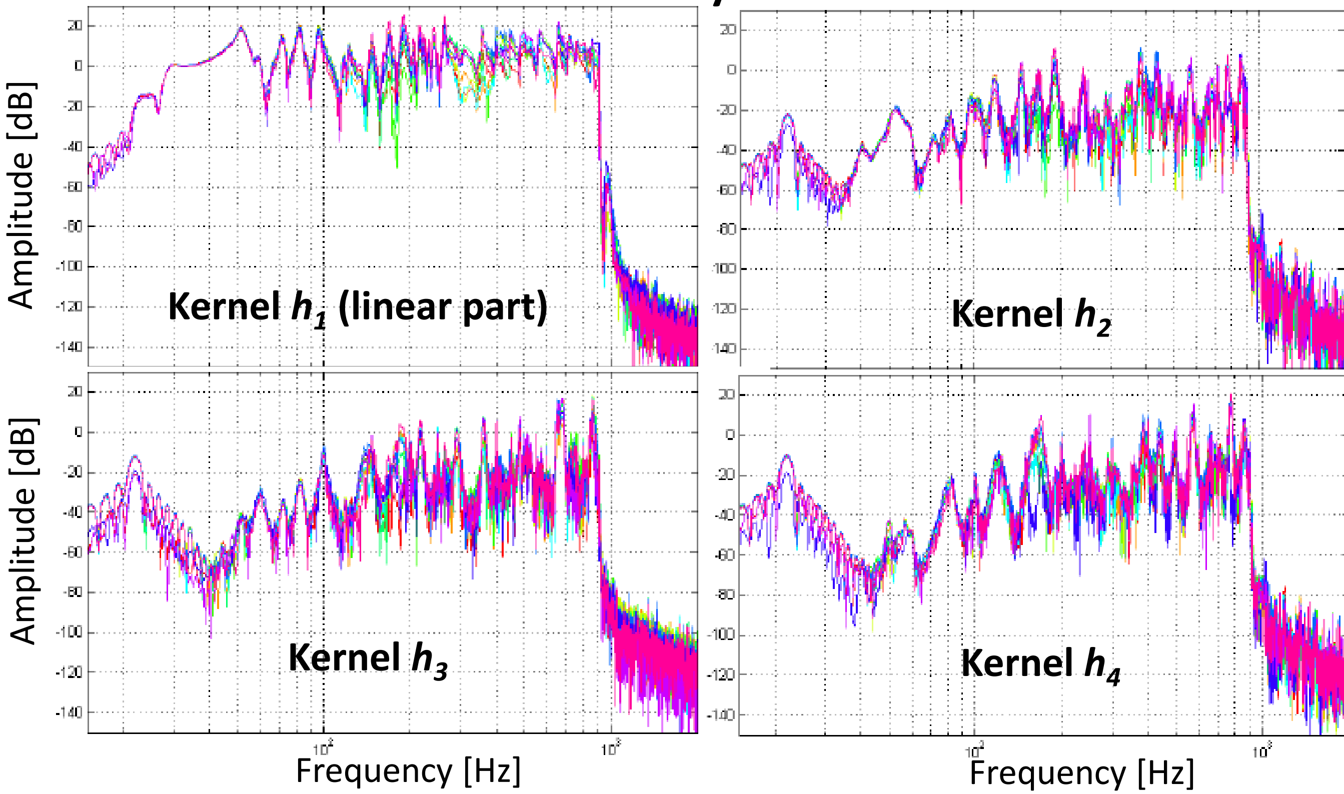
→ Better extraction of the noise → **More precise estimations of the kernels of high orders**

Uncertainty of measurement using Bootstrap

How to study the **variability of the measurements ?** → **Bootstrap method**

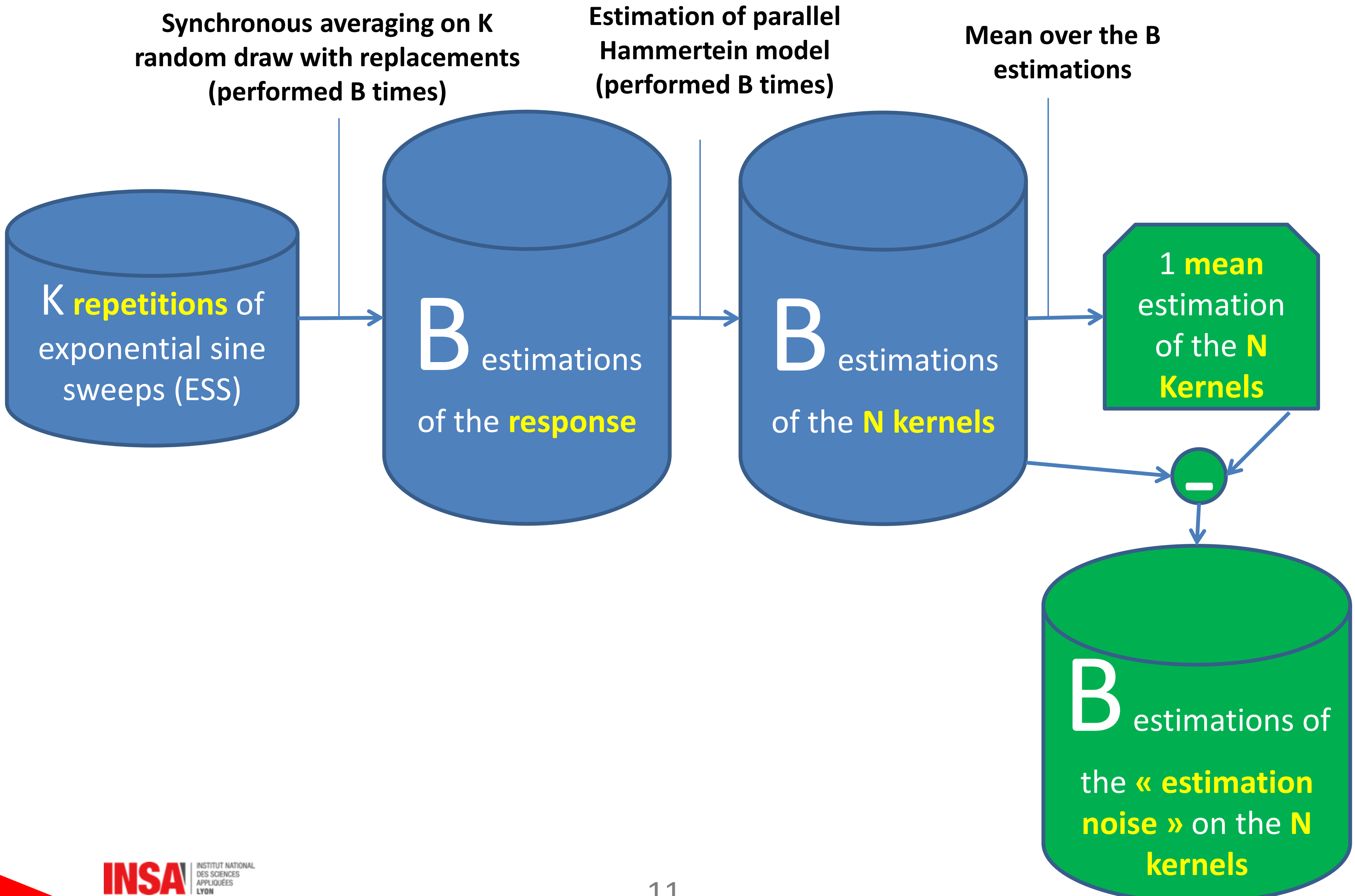
B random sample with replacement (of K sweeps each time)
Synchronous averaging on K repetitions → B different kernel estimations

Identified Kernels of the system for B=10 and K=19

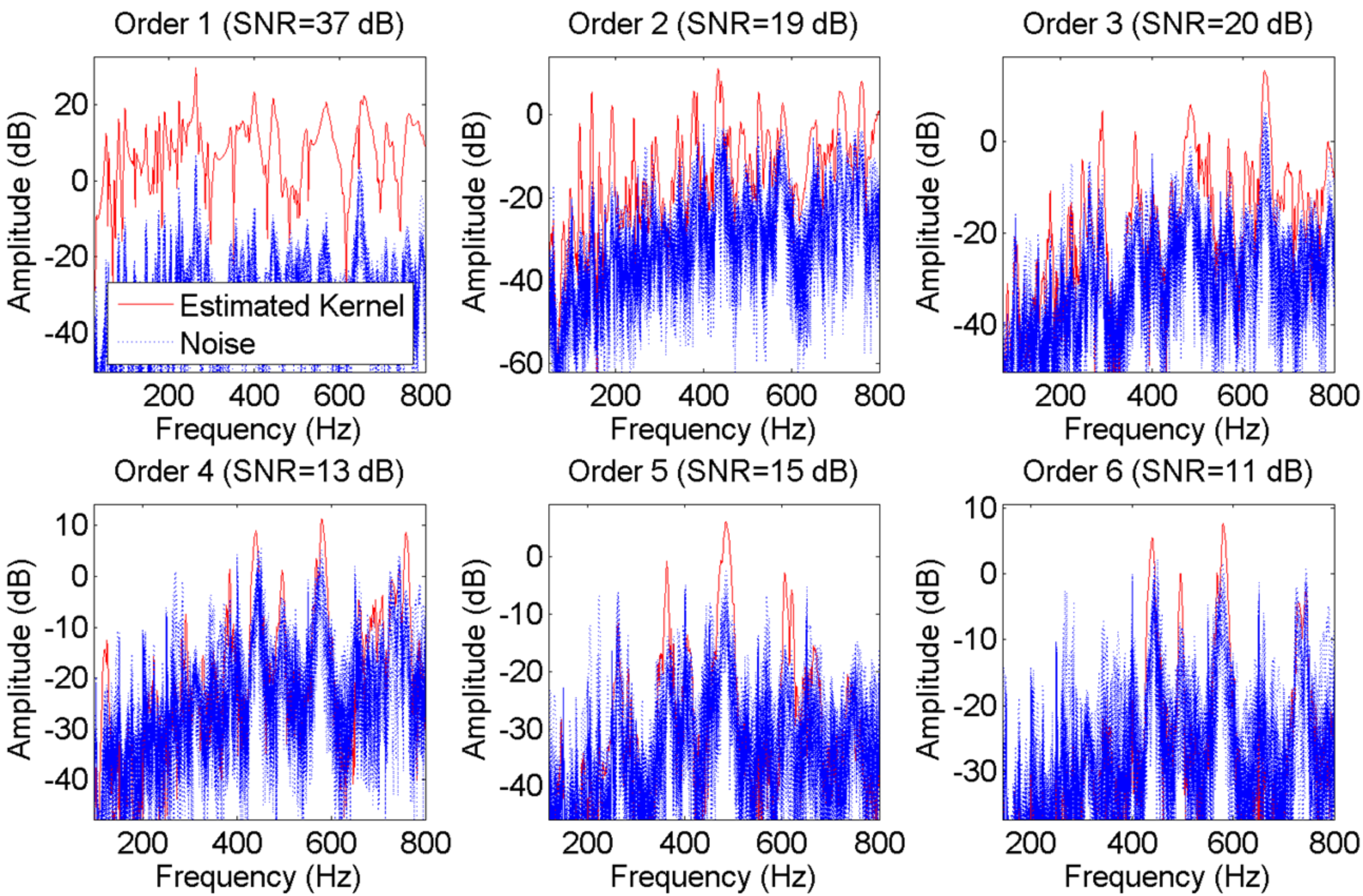


Way to identify and quantify variability (uncertainty) of plate nonlinearities estimations

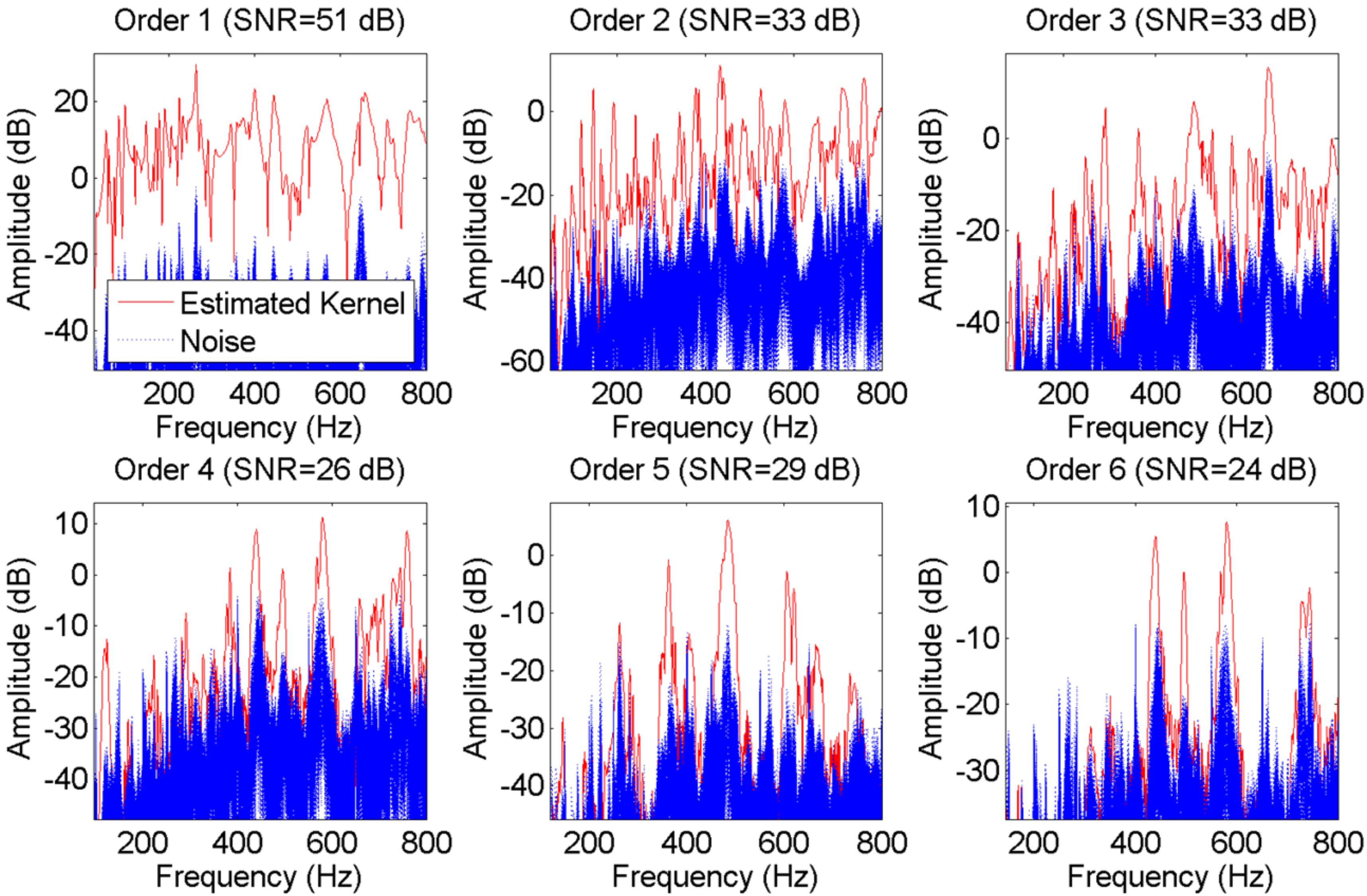
Uncertainty of quantification



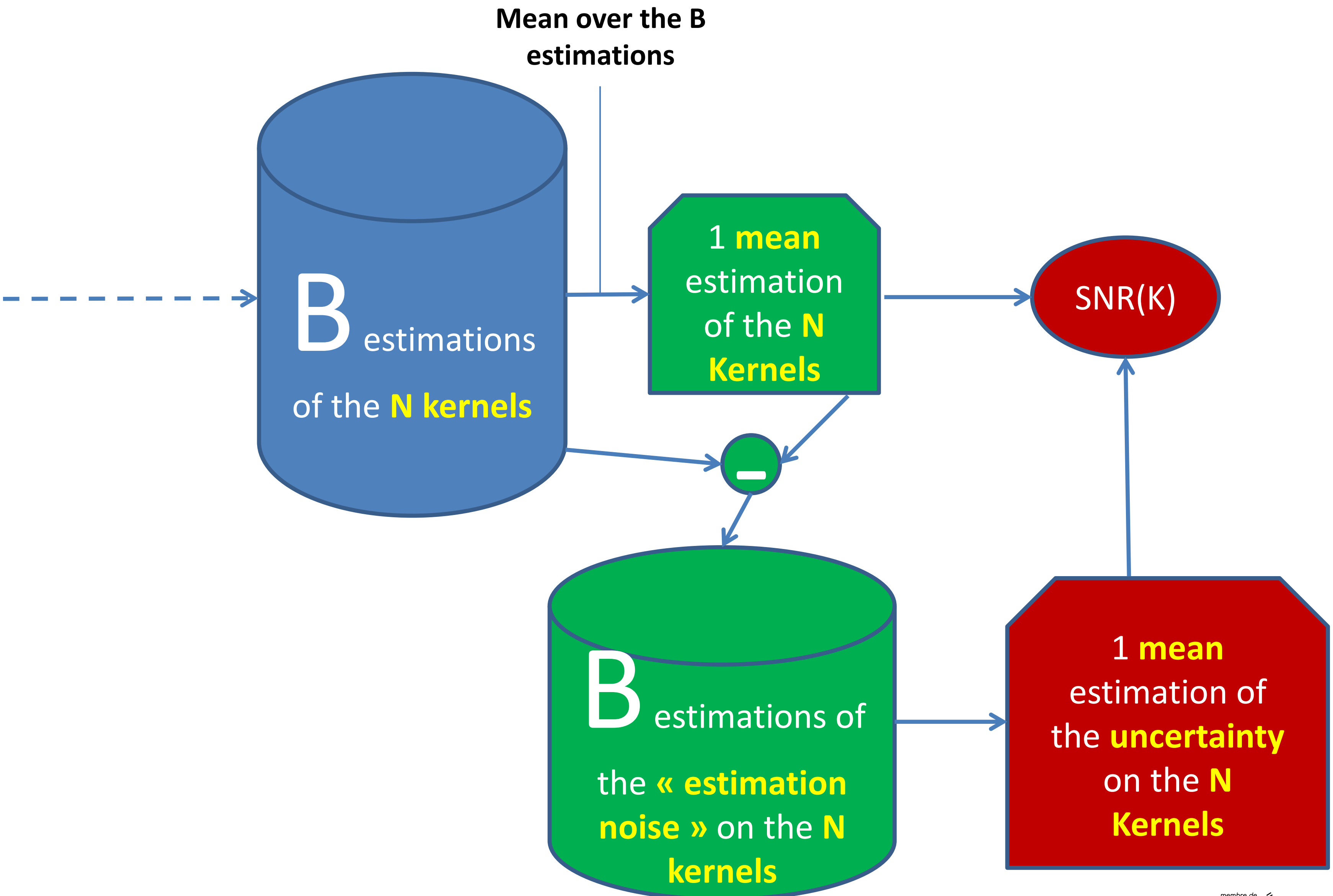
No averaging (K=1, B=150)



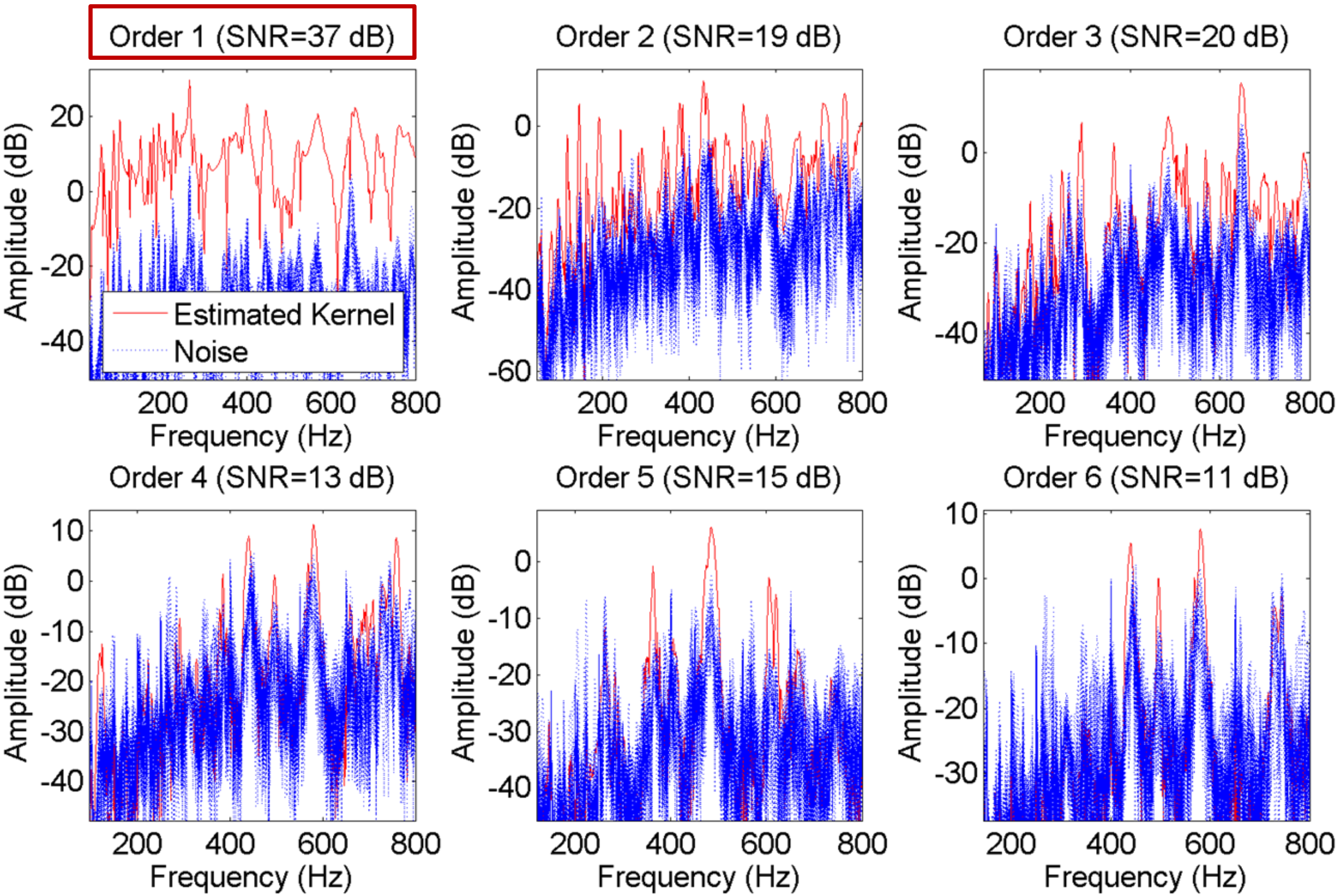
Averaging (K=19, B=150)



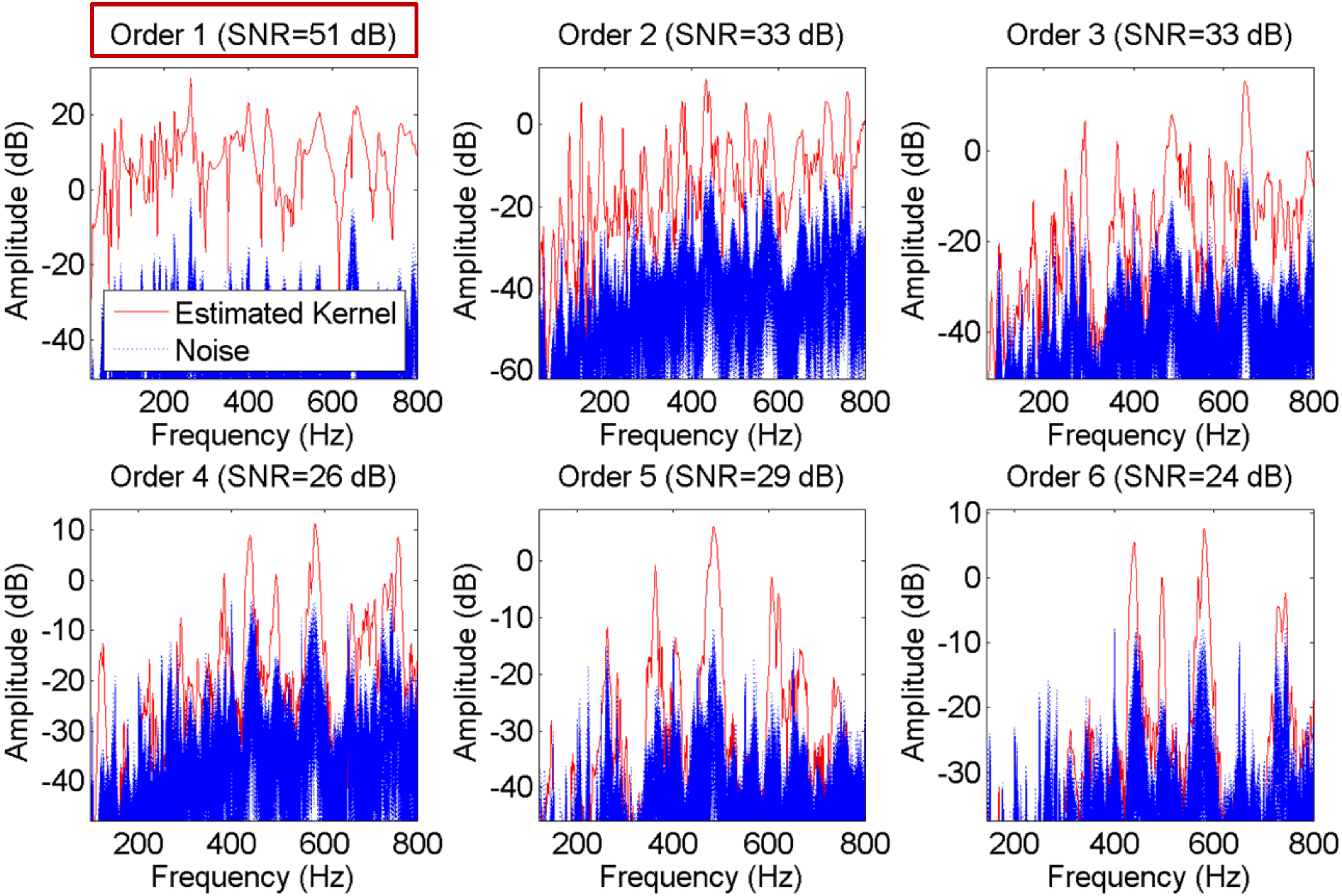
Uncertainty of quantification



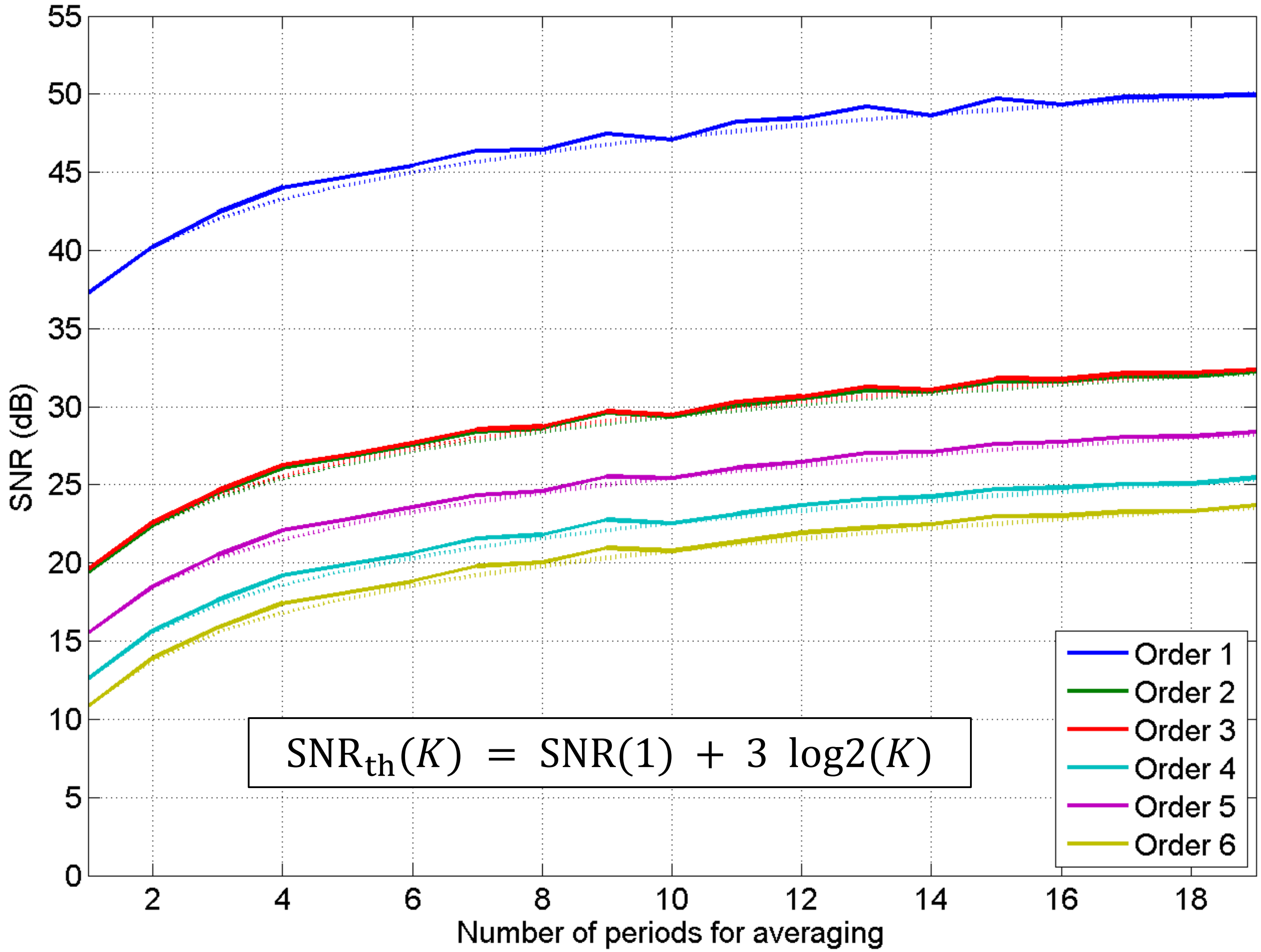
No averaging (K=1, B=150)



Averaging (K=19, B=150)

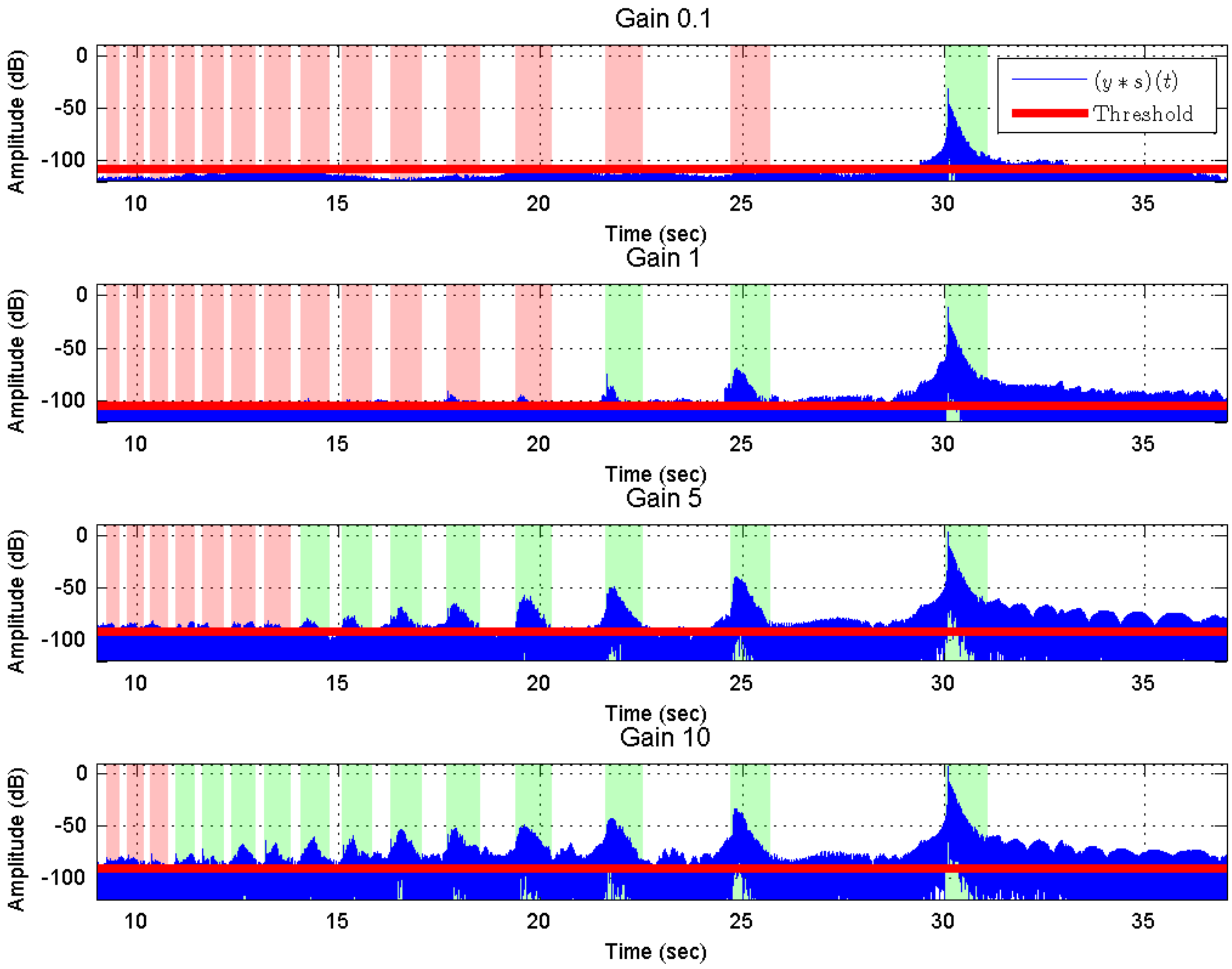


Effect of repetition number K on SNRs



Autonomous kernel order estimation

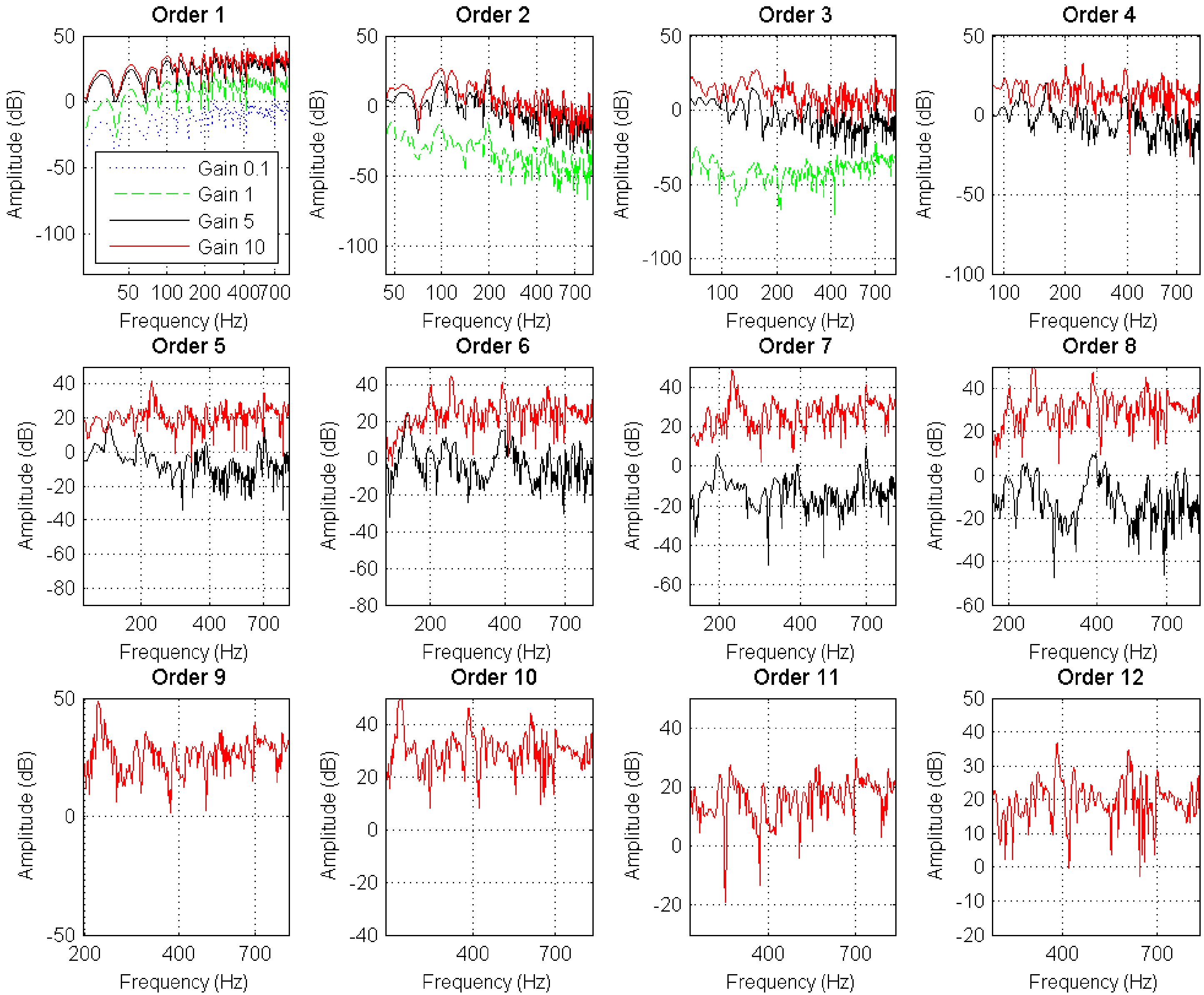
→ **Example on the suspended damped plate (5 different gains)**



Estimation of the proper kernel order N to identify

Threshold defined following a statistical F-test (Fisher)

Example of autonomous Kernel estimation



Conclusion & Perspectives

- Original method to **estimate the nonlinearities** of a vibro-acoustical structure
  - Slightly nonlinear systems modelled as **parallel Hammerstein models**
  - Kernels easily estimated using **exponential sine sweeps**

- **Improvements** of the sine sweep method
  - **Repetition of the excitation signal** – *K* sweeps
  - **Extraction of the noise** through time synchronous averaging
  - **Uncertainty estimation** by bootstrap
  - **Autonomous kernel order estimation**

- **Perspectives**
  - **Comparisons** with other methods (spectral domains): **periodic multisines**...