Parallel Hammerstein Models Identification using Sine Sweeps and the Welch Method

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Abstract: Linearity is a common assumption for many real life systems. But in many cases, the nonlinear behavior of systems cannot be ignored and has to be modeled and estimated. Among the various classes of nonlinear models present in the literature, Parallel Hammertein Models (PHM) are interesting as they are at the same time easy to understand as well as to estimate when using exponential sine sweeps (ESS) based methods. However, the classical EES-based estimation procedure for PHM relies on a very specific input signal (ESS), which limits its use in practice. A method is proposed here based on the Welch method that allows for PHM estimation with arbitrary sine sweeps (ASS) which are a much broader class of input signals than ESS. Results show that for various ASS, the proposed method provides results that are in excellent agreement with the ones obtained with the classical ESS method.

Keywords: Nonlinear system identification; Input and excitation design;

1. INTRODUCTION

Systems are generally assumed to behave linearly and in a noise-free environment. This is in practice not perfectly the case. First, nonlinear dynamic behaviors are very common in real life systems. Second, the presence of noise is a natural phenomenon that is unavoidable for all experimental measurements. In order to perform reliable model estimation of such systems, one should thus keep in mind these two issues and care about them. Indeed, all the noise that is not correctly removed from the measurements could be misinterpreted as nonlinearities, thus polluting measurements. And if nonlinearities are not accurately estimated, they will end up within the noise signal and information about the system under study will be lost.

The first problem addressed here is related to the estimation of nonlinear models of real life systems (Pearson, 1999; Kerschen et al., 2006). As nonlinear behaviors are complex and various and as it is not intended to build a model for each case, it is chosen here to rely on Parallel Hammerstein Models as they can be interpreted easily (see Fig. 1). Such models belong to the class of *"Sandwich models"* (Chen, 1995). Even if the model presented in Fig. 1 involves only monomial nonlinearities, it has been shown to possess a good degree of generality (Pearson, 1999)]. Classical estimation methods for this class of system rely on a least-square formulation of the problem but are computationally costly and prone to conditioning issues (Gallman, 1975). Hopefully, thanks to exponential sine sweeps (ESS), non-parametric versions of such models can be very easily and rapidly estimated (Novak et al., 2010, 2015; Rébillat et al., 2011, 2016). However, one drawback of these ESS based methods is that they rely on a specific class of input signals (ESS) that limits their use in practice.
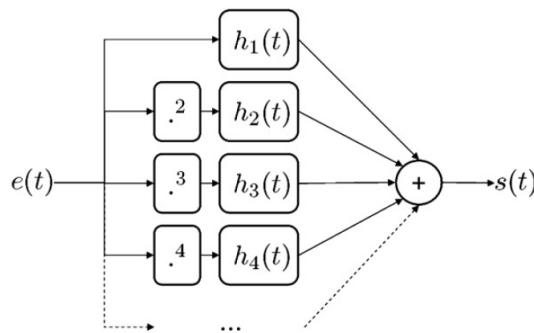

Fig. 1. Representation of parallel Hammerstein models

The second problem addressed here is related to the rejection of uncertainties caused by the presence of noise. One way to address this issue relies on a careful design of the excitation signal: special types of periodic excitations (Pintelon and Schoukens, 2001), non-stationary excitation with an underlying periodic structure (Zhang et al., 2010), or repeated ESS (Rébillat et al., 2016) for example. Another way to remove noise consists, independently of the input signal, to segment the output signal in several slices on which output noise is assumed to be independently distributed, as done in the classical Welch's Method (Welch, 1967). This kind of approach is very attractive in the context of nonlinear system identification as it puts no constraints on the input signal.

The aim of this paper is thus to provide a methodology that allows for the estimation of Parallel Hammerstein

Models based on the Welch method and on arbitrary sine sweeps (ASS). The paper is organized as follows: the classical ESS and Welch Methods are first briefly recalled in Section 2 and Section 3. Then the proposed method is detailed in Section 4. Results comparing the classical ESS method with the proposed method for various ASS are provided in Section 5 before concluding in Section 6.

Note 1 : In this article, $*$ will stand for the convolution product. The convolution product between a function a and a function b will be denoted as $a(t) * b(t)$ instead of $a * b(t)$ in order to simplify some notations.

Note 2 : f will refer to the continuous frequency whereas k will refer to its discrete numeric version. Both will be used depending on the context.

2. EXPONENTIAL SINE SWEEP METHOD

A Parallel Hammerstein Model (PHM) is a relatively simple model of a weakly non-linear system (see Fig. 1). The input-output relation of such a model can be written as follows:

$$y(t) = \sum_{n=1}^{+\infty} h_n(t) * x^n(t) \tag{1}$$

where $x(t)$ is the input of the system, $y(t)$ its output and $h_n(t)$ the kernels of the Hammerstein model.

Let's consider an arbitrary sine sweep (ASS) $x(t)$ as an input of a PHM truncated to the order N. Such a signal is defined as $x(t) = \sin[\phi(t)]$ with $\phi(t)$ a strictly increasing function of time. By using trigonometry formula, one can then write (Novak et al., 2010, 2015; Rébillat et al., 2011, 2016):

$$y(t) = \sum_{n=1}^{N} g_n(t) * \sin[n\phi(t)] \tag{2}$$

With $g_n(t) = \sum_{m=1}^{n} A_{mn} h_m(t)$ a linear combination of the kernels $h_m(t)$ with $m \leq n$. This clearly highlights the harmonics that commonly appear when measuring the frequency response of a nonlinear system.

The identification of a PHM consists in finding the set of kernels $h_n(t)$, by means of the known input $x(t)$ and the measured output $y(t)$. Let's consider the input $x(t)$ as an Exponential Sine Sweep (ESS) of duration T starting from a frequency f_1 and ending with a frequency f_2. Its instantaneous frequency can then be written as follows:

$$2\pi f(t) = \phi'(t) = 2\pi f_1 \exp\left[\frac{t}{T}\ln(f_2/f_1)\right] \tag{3}$$

The integration of this formula then gives:

$$\phi(t) = 2\pi \frac{f_1 T}{\ln(f_2/f_1)} \exp\left[\frac{t}{T}\ln(f_2/f_1)\right] + C$$

with C a constant value. To be able to identify the kernels $h_n(t)$, the following relationship is desired and can be obtained by different means depending on the C value (Novak et al., 2010, 2015; Rébillat et al., 2011, 2016):

$$\sin[k\phi(t)] = sin[\phi(t + \Delta_{t_k})] \tag{4}$$

Then, from Eq. (2) the following relationship is obtained:

$$y(t) = \sum_{n=1}^{N} g_n(t) * \sin[\phi(t + \Delta_{t_n})]$$

The function $h(t)$ can then be computed as follows:

$$h(t) = y(t) * x_{-1}(t) = \sum_{m=1}^{+\infty} g_n(t - \Delta_{t_k}) \tag{5}$$

with $x_{-1}(t)$ defined such that: $x(t) * x_{-1}(t) = \delta_0(t)$, where δ_0 is the Dirac distribution. This deconvolution operation is crucial for the ESS method and can be achieved by different means (Novak et al., 2015; Rébillat et al., 2011). Another way based on the Welch to perform this step is proposed in this article.

Note from Eq. (5) that the linear combination $g_n(t)$ of high order Hammerstein kernels $h_m(t)$, $m < n$ of the system are in advance in time of a given value Δ_{t_n} thanks to Eq. (4). This justifies the usefulness of this property. Then, the different terms $g_n(t)$ can be easily isolated by means of a simple temporal windowing of $h(t)$ and the kernel $h_n(t)$ can be found by inverting the linear combination defined by the matrix $\mathbf{A} = (A_{mn})_{1 \leq m, n \leq N}$. Also note that according to the classical ESS method implementation, the length T_{cut} of the temporal window used to extract $h(t)$ has to be smaller than $T\ln((N+1)/N)/\ln(f_2/f_1)$ where N is the maximal order being considered. Consequently, the power of the noise polluting the estimation is relative to this length T_{cut}. This temporal window is usually build with a Hanning window to avoid side effects.

3. WELCH METHOD

The Welch method was first introduced by Welch (1967) and provides an estimation of the cross power spectral density of two signals through an unbiased estimator that belongs to the modified periodogram class.

3.1 Cross power spectral density estimation

Let's assume the signals $x[n]$ and $y[n]$ to be realizations of length N_T of an ergodic and stationary process. By definition, the Cross Power Spectral Density (CPSD) of these signals is:

$$\Gamma_{xy}(k) = \mathcal{DFT}\left\{\mathbb{E}\left[x[n]\overline{y[n-m]}\right]\right\}(k)$$

where $\mathbb{E}$ denotes the expected value, $\overline{a}$ the conjugate product of a and $\mathcal{DFT}$ the Discrete Fourier Transform operation.

This CPSD can be estimated with a certain bias by means of the classical periodogram $S_{xy}(k)$, defined as follows:

$$S_{xy}(k) = \frac{1}{N_T f_s}\mathcal{DFT}[x](k)\mathcal{DFT}[\overline{y}](k)$$

Taking advantage of ergodicity and stationarity, another estimator of $\Gamma_{xy}(k)$ based on a temporal average process can also be defined. The idea is to split the signals x and y into P overlapping segments of length M containing each D overlapping sample using a window w, and to estimate the CPSD of each segment $\hat{\Gamma}_{xy}^p(k)$ through the

periodogram before averaging it. An unbiased CPSD estimator can thus be defined as (Welch, 1967; Akcay, 2012):

$$\hat{\Gamma}_{xy}(k) = \frac{\alpha}{P}\sum_{p=1}^{P}\hat{\Gamma}_{xy}^{p}(k) = \frac{\alpha}{P}\sum_{p=1}^{P}S_{x_p y_p}(k) \qquad (6)$$

with $x_p(k) = w[k]x[k+pD]$ and $y_p(k) = w[k]y[k+pD]$ and α a normalization factor such that: $\alpha = M/\sum_{m=1}^{M}|w(m)|^2$.

The main advantage of this approach is that when considering deterministic signals polluted by ergodic noise, the error committed will be relative to the noise power considered as a length M instead of length N_T. However, some spectral precision will necessarily be lost by using smaller windows than the single whole signal.

3.2 Linear system identification using CPSD

The Welch method has been largely used to estimate the frequency response of a linear system such as:

$$y(t) = h(t) * x(t) + b(t) \qquad \text{time domain}$$
$$Y(f) = H(f)X(f) + B(f) \qquad \text{frequency domain}$$

where $y(t)$ is the output, $x(t)$ the input and $b(t)$ the noise, and $A(f) = \mathcal{FT}\{a(t)\}(f)$ where $\mathcal{FT}$ denotes the Fourier transform. Note that $b(t)$ is one realization of a ergodic and stationary process with zero mean and then that $y(t)$ has the same statistical properties than this realization.

The simplest way to numerically estimate H is:

$$\hat{H}(f) \approx Y(f)/X(f)$$

and the error committed by doing so is proportional to the power of the noise which last the whole signal length. The idea is now to use the CPSD to define the transfer function and then to use the Welch method in order to reduce errors caused by noise. Using the fact that noise is uncorrelated with the input signal and the deterministic nature of $H(f)$, one obtains:

$$H(f) = \mathrm{E}\left[\frac{Y(f)\overline{X(f)}}{X(f)\overline{X(f)}}\right] - \mathrm{E}\left[\frac{B_\Omega(f)\overline{X(f)}}{X(f)\overline{X(f)}}\right] = \frac{\Gamma_{xy}(f)}{\Gamma_{xx}(f)}$$

Then the error commited is here again proportional to the power of the noise as if it only lasts M samples.

4. WELCH SINE SWEEP OR CPSD METHOD

4.1 General idea of the method

Let's assume a PHM as defined by Eq. (1) and an input signal $x(t)$ defined to be an arbitrary sine sweep (ASS) as done in Eq. (2). That way $x(t) = \sin[\phi(t)]$ with ϕ a strictly increasing function of time. Then, at each time a single frequency is played by the input $x(t)$, involving an output $y(t)$ made up of a discrete set of harmonic frequencies. Indeed $y(t)$ is a linear combination of convolution products between harmonics and $g_n(t)$ functions as described in Eq. (2). This is illustrated in Fig. 2 which shows for a given frequency of the input the frequencies contained in the output of the PHM when considering an ASS as input.

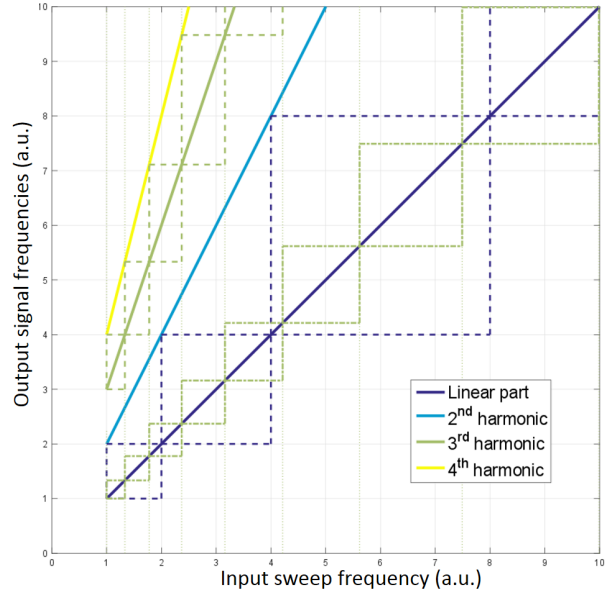The idea behind the Welch Sine Sweep method (also called CPSD method) is to be able to estimate the kernels of

Fig. 2. Input sweep frequency versus output signal frequencies for a 4$^{\text{th}}$ order PHM and windowing choices examples (no overlap): linear part [optimal] (blue dashed lines), third harmonic [optimal] (green dashed lines), linear part [suboptimal] (green dot-dashed line).

the PHM with the same precision than the ESS method but by releasing all the constraints on the sweep itself. As stated in Sec. 2, the ESS method is based on an ESS which leads, when provided as the input of a PHM and after a deconvolution, to time localized linear combination $g_n(t)$ of kernels $k_n(t)$, see Eq. (5). A temporal windowing operation is then performed to extract the impulses responses $g_n(t)$. By making use of the properties of the Welch Method, we directly estimate the impulse responses $g_n(t)$ by performing a temporal windowing operation on slices of input and output signals. The deconvolution operation is avoided and all constraints on the sweep are thus released.

However, in order to still be able to estimate the non-linear model, this temporal windowing cannot be done arbitrarily. It must be done by taking into account the fact that the input signal is an ASS and the fact that the system under study is a PHM. A way to do so is to focus on the different harmonics one by one and to try to get information regarding them, and exclusively them, within a considered time window. The challenge is thus to be able to design temporal windows that ensure that the frequency content of the harmonic under study is contained within a frequency range that is not polluted by the other harmonics. If the harmonic of interest is the nth harmonic, and if the selected windows starts with a frequency f_{start}, then the time at which this temporal window should stop is the time at which the the input frequency is such that $f_{\text{stop}} = (n+1)/n \times f_{\text{start}}$. Thus, the higher the harmonic order is, the smaller the largest window allowed is. This temporal window is called in the following the "optimal" window as it is the longest possible, but any shorter window is acceptable. Note that there is no constraints on the start frequency f_{start} and thus that the windows can start at any arbitrary time, allowing to use overlapped windows. This process is illustrated in Fig. 2 that shows two examples of optimal

windowing choice (without overlapping) when focusing on the linear part (blue dashed lines) and when focusing on the third harmonic (green dashed lines). It is also shown how this "optimal temporal windowing" choice derived when focusing on the third harmonic can be applied to the linear part in green dot-dashed line and thus become suboptimal with respect to the linear part.
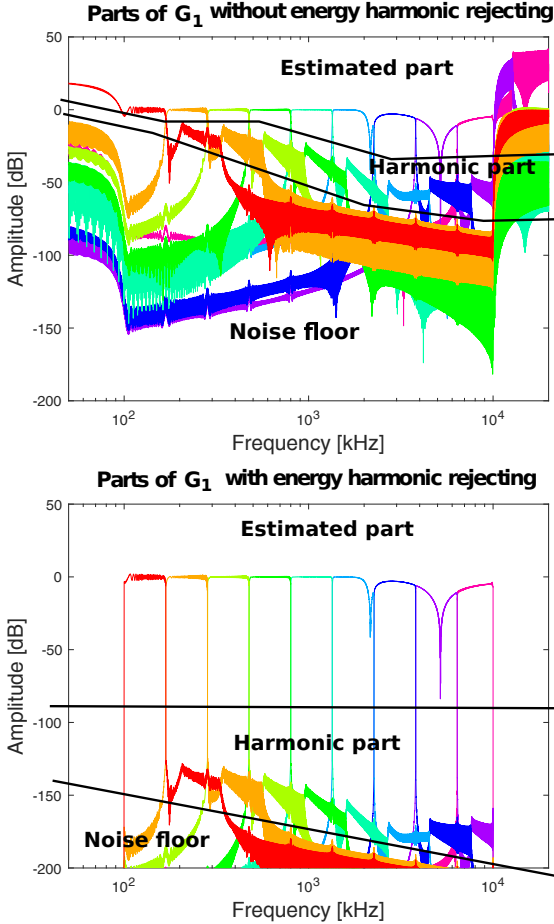

Fig. 3. Estimation of $G_1(f)$ over unity time windows without overlap. *Top:* strict application of Eq. (7). *Bottom:* application of Eq. (7) with regularization (see Sec. 4.3). Time windows are built from Eq. (9) for $n = 1$ with a security factor $3/4$ (see Sec. 4.2). See Sec. 5.2 for details regarding the simulated system.

Then, due to the linearization of the trigonometric functions given in Eq. (2), it is possible isolate from the selected windows the exact harmonic contribution $G_n(f) = \mathcal{FT}[g_n(t)](f)$ of the nth harmonic in the frequency domain. However, as the information we try to infer from these windows is related to the nth harmonic, the nth harmonic of the input signal has to be used in the estimation process. The function $G_n(f)$ can thus be directly by using the Welch method described in Section 3 over the selected temporal windows and using the nth harmonic $x_n(t) = \sin[n\phi(t)]$ as input instead of the actual input $x(t) = \sin[\phi(t)]$. This is illustrated in Fig. 3 where the parts of $G_1(f)$ estimated by each window are plotted. Formally, using notations of Eq. (6), what has been done to estimate $G_n(f)$ is:

$$\hat{G}_n(f) = \frac{\hat{\Gamma}_{x_n y}(f)}{\hat{\Gamma}_{x_n x_n}(f)} = \frac{\sum_{p=1}^P \hat{\Gamma}_{x_n y}^p(f)}{\sum_{p=1}^P \hat{\Gamma}_{x_n x_n}^p(f)} \qquad (7)$$

From the estimated $\hat{G}_n(f)$, it is then possible to recover the kernels $H_n(f)$ of the PHM by inverting the linear combination defined by the matrix $\mathbf{A}$ as in Sec. 2.

Note that contrary to the classical ESS implementations (Novak et al., 2010, 2015; Rébillat et al., 2011, 2016), this method does not need an ESS satisfying any fundamental phase property as in Eq. (4). The only condition that needs to be satisfied here is related to the function $\phi(t)$ that needs to be known explicitly in order to determine the frequency bandwidth of each segment and thus the windows lengths that suit the Welch method.

4.2 Design of temporal windows for the CPSD estimator

As stated previously, a temporal window series relative to an harmonic of interest is said to be acceptable if : for all the segment of this temporal window series, the spectral content relative to the harmonic of interest is not overlapping with the spectral content of other harmonics (see Fig. 2). Then, it is possible to build the maximum length of each temporal window by building rectangles around each harmonic straight lines. Mathematically, it is demonstrated by recurrence that when focusing on the harmonic n, the pth temporal window of the series of temporal windows starting at frequency f_0^n and with no overlap should stop at a time such that the frequency of the input signal corresponds at this time to:

$$f_p^n = f_0^n \left(\frac{n+1}{n}\right)^p \qquad (8)$$

This grid is defined in the frequency domain. It is then necessary to go back to the temporal domain by making use of the instantaneous frequency derived from the phase function $\phi(t)$. If we consider the phase given in Eq. (3), this gives the following series of times:

$$t_p^n = \frac{T}{\ln(f_2/f_1)} \left[\ln(f_0^n/f_1) + p \ln\left(\frac{n+1}{n}\right)\right] \qquad (9)$$

Note that this interestingly leads to regularly spaces times for this particular case of an ESS. Additionally, the step is the same as the maximum length of temporal window in the ESS method, see section 5.2. This will lead to an equivalent noise rejection for the two method.

If we consider now the phase of a linear sweep, *i.e.* a sweep with the instantaneous frequency such that :

$$f(t) = f_1 + \frac{f_2 - f_1}{T} t \qquad (10)$$

then the associated series of time is given by :

$$t_p^n = \frac{(\frac{n+1}{n})^{p-1}}{f_2 - f_1} f_1 T \qquad (11)$$

and the series of "optimal windows" is defined by

$$T_p^n = t_p^n + t_{p-1}^n = \frac{1}{n} \left(\frac{n+1}{n}\right)^{p-1} \frac{f_1 T}{f_2 - f_1} \qquad (12)$$

To remain simple, we keep a grid regularly spaced in time for each order. Therefore, the step of the temporal grid

associated to the linear sweep is defined by the smallest time window, which is T_1^n in this case.

4.3 Regularization procedure

The computation of the inverse filter $x_{-1}(t)$ (see Sec. 2) is crucial to reject unwanted harmonics. Because the CPSD estimation is based on the temporal windowing of the signals, the regularization procedure proposed by Rébillat et al. (2011, 2016) applies naturally here. Thus, the following regularization procedure is performed in order to reject efficiently unwanted harmonics:

$$\hat{\Gamma}_{x_n x_n}(f) = \begin{cases} \hat{\Gamma}_{x_n x_n}(f) \text{ if } nf_1 < f < nf_2 \\ \hat{\Gamma}_{x_n x_n}(f) + \epsilon_x \times \max_f \left[\Gamma_{x_n x_n}(f) \right] \text{ else.} \end{cases}$$

with ϵ_x an arbitrary factor. However, this will only affect the exterior part of the frequencies played by the input signal. This will not reject the higher harmonics which are within the interval of the frequency played (see Fig. 3). Therefore, an additional regularization operation is performed for each segment to reduce the energy of the partial part $\hat{\Gamma}_{x_n y}^p(f)$ of the quantity $\hat{\Gamma}_{x_n y}(f)$ outside of the frequency range relative to this segment (see Eq. (7)):

$$\hat{\Gamma}_{x_n y}^p(f) = \begin{cases} \hat{\Gamma}_{x_n y}^p(f) \text{ if } f_{p-1}^n < f < f_p^n \\ \hat{\Gamma}_{x_n y}^p(f)/\epsilon_y \text{ else.} \end{cases}$$

with ϵ_y an arbitrary factor. Both ϵ_x and ϵ_y are taken equal to 10^{10}. This effect is illustrated on Fig. 3. In each figures, 3 zones can be distinguished: the estimated part, the rejected harmonics, and the noise floor. The top figure is the straight application of the method whereas the bottom figure includes the regularization procedure proposed here. As can be seen from this figure, the proposed regularization really helps in rejecting unwanted harmonics components.

5. COMPARISON ON A SIMULATED SYSTEM

5.1 Implementation details

Demonstration of the efficiency of the CPSD method is done with two kind of ASS : an exponential sine sweep to compare results with the classical ESS method; and a linear sine sweep (LSS) to illustrate the potential of the method. All the ASS are 5 seconds long and starting at frequency 200 Hz and rising up to 10 kHz with a sampling frequency of 96 kHz. For the CPSD method, the classical value of 50% of overlapping segments pondered with Hanning windows has been retained.

For the ESS, the series of time used in CPSD method is defined according to Eq. (9). The step has been multiplied by a *security factor* equal to $3/4$ which reduces the length of the time windows. This is done to avoid the spectral content from the next upper harmonic to be too close to the spectral content of interest. Because of the overlapping of 50%, the final series of time is given by:

$$t_p = \frac{1}{2} \times \frac{3}{4} \times p \times \frac{T \ln[(N+1)/(N)]}{\ln(f_2/f_1)} \qquad (13)$$

Note that the pth segment is composed by the points $[t_p, t_{p+2}]$ because of the 50 % overlapping.

In the same way, the series of times relative to the LSS for the CPSD method is (see Sec. 4.2) :

$$t_p^n = \frac{1}{2} \times \frac{3}{4} \frac{1}{n} \frac{f_1 T}{f_2 - f_1} \qquad (14)$$

Note that this time, we used a different grid for each order in order n to keep a good precision in the low frequency range.

In the ESS method, after deconvolution of the output signal, each harmonic impulse response is localized in advance from the linear part and is extracted by a simple temporal windowing of length T_{cut} (see Sec. 2). As stated in Sec. 4.1 and 3.2, this parameter has an equivalent role than the step of the CPSD series of time. Then the value $T_{\text{cut}} = 3/4 T \ln((N+1)/N)/\ln(f_2/f_1) \approx 0.21\,\text{s}$ is kept. Note that for the linear case, the equivalent value of T_{cut} extracted from the CPSD method is $T_{\text{cut}} = 3/4 T f_1/(n(f_2 - f_1)) \approx 0.076\,\text{s}$ ($n = 1$) or $0.019\,\text{s}$ ($n = 4$). Because the windows are smaller for the LSS, the noise will have a lower influence on the estimated kernels in comparison with the ESS.

5.2 Simulated system

We simulated a PHM system of order 4. The linear part is modeled by a Chebyshev filter of Type II of order 4 with a cutoff frequency of 2 kHz. The harmonic parts are modeled with ARMA filters of order 2 defined by the coefficients given in Tab. 1.

Table 1. Coefficients of the ARMA filters used
to model high order kernels

Order	a_0	a_1	a_2	b_0	b_2	b_2
2	1.0000	-1.8996	0.9025	0.0500	-0.0951	0.0470
3	1.0000	-1.9075	0.9409	0.0250	-0.0474	0.0226
4	1.0000	-1.8471	0.8649	0.0087	-0.0115	0.0056

We also add a Gaussian white noise to the output signal $y(t)$. Numerous SNR , PHM order N and different length of sweep have been tested showing identical results. We kept the representative value of $N = 4$, $SNR = 40\,\text{dB}$ and a duration of 5 s to present results. However a small discussion about computation time as a function of the sweep length and PHM order is given later.

5.3 Results

The PHM Kernels estimated through the classical ESS method and the proposed Welch Sine Sweep Method for the two kind of sweeps versus the theoretical ones are presented in the frequency domain in Fig. 4. From this figure, it can be seen that as expected both methods provides estimated Kernels that are in good agreement with the theoretical ones. The PHM Kernels estimation through a LSS is only possible with the CPSD method. As expected, the noise is more rejected with this sweep than with the ESS as the windows used are smaller. Note that because the LSS spends more time in high frequency than in low frequency as compared to the ESS, the estimation obtained with the LSS is better in high frequencies than in low frequencies. Finally, note that the ESS method only requires a single deconvolution whereas the CPSD method need as much FFT as the number of temporal windows
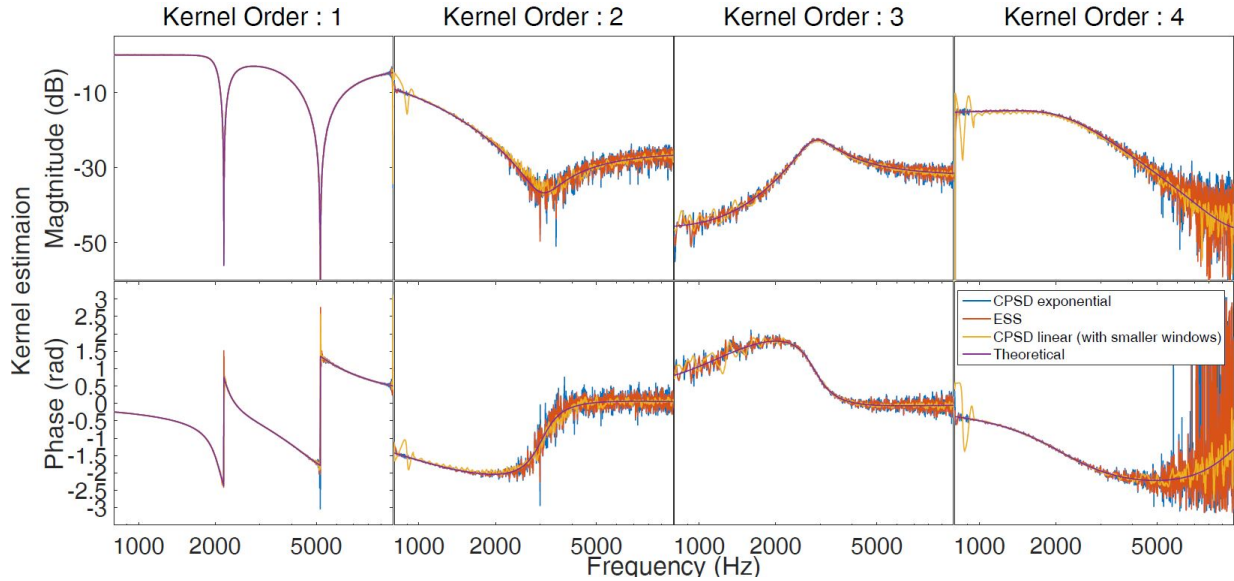
Fig. 4. PHM Kernels estimated through the classical exponential sine sweep method (ESS) and the proposed Welch Sine Sweep Method (CPSD) for two kind of sweeps versus theoretical ones in the frequency domain ($SNR = 40$ dB).

being used. Then the longer is the signal, the higher is the computational cost of the CPSD method. Also, because the largest acceptable length of window depends on the maximum order N of the PHM, the more complex is the system, the higher is the computational cost of the CPSD method. Some computation times are given in Tab. 2.

Table 2. Comparison of computation times between CPSD (ESS) and ESS method.

Method	$N = 4$ $T = 1\,\text{s}$	$N = 4$ $T = 5\,\text{s}$	$N = 7$ $T = 1\,\text{s}$	$N = 7$ $T = 5\,\text{s}$
ESS	0.016	0.050	0.017	0.049
CPSD	0.168	0.427	0.277	0.957

6. CONCLUSION AND PERSPECTIVES

In this article, a new method to estimate a a Parallel Hammerstein Models based on the Welch estimator of the cross power spectral density is proposed and validated. This method is compared to the classical ESS method on a simulated system with noise. It is shown that the proposed method releases all constraints on the input sweep in comparison with the classic ESS method at the cost of a longer computational time. Results provided by both methods are found to be almost identical. Because constraints on sine sweep are released and it is harmonic based, this new method is very attractive as it could be used with contextual sweeps (related to the power spectral density of the environmental noise for example) or be the basis of a nonlinear model estimation with input harmonic signals such as music, which corresponds to a strong demand of professionals in loudspeaker measurement.

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