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Temperature Dependence in the Jiles–Atherton Model for Non-Oriented Electrical Steels: An Engineering Approach

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High operating temperatures modify the magnetic behavior of ferromagnetic cores which may affect the performance of electrical machines. Therefore, a temperature-dependent material model is necessary to model the electrical machine behavior more accurately during the design process. Physics-inspired hysteresis models, such as the Jiles–Atherton (JA) model, seem to be promising candidates to incorporate temperature effects and can be embedded in finite element simulations. In this paper, we have identified the JA model parameters from measurements for a temperature range experienced by non-oriented electrical steels in electrical machines during their operation. Based on the analysis, a parameter reduction has been performed. The proposed approach simplifies the identification procedures by reducing the number of model parameters and does not require any additional material information, such as the Curie temperature. The resulting temperature-dependent JA model is validated against measurements, and the results are in good agreement.

***Index Terms*—Electrical machines, electrical steels, ferromagnetic materials, Jiles–Atherton (JA) model, temperature.**

I. Introduction

MODERN electrical machines may run for a prolonged time at extremely high temperatures which affect the magnetic behavior of the electrical steels used in their cores. Iron losses are usually computed using the Steinmetz equation [1]-based formulae in computer-aided design (CAD) simulations, and the effects of the temperature on the magnetic properties of the ferromagnetic material and iron loss are often neglected. These effects should be taken into account during the CAD of the modern electrical machines to optimize their performance.

The magnetic properties (B–H relationship and iron losses) of the ferromagnetic materials are highly affected by the variation of temperature [2]. Physics-inspired hysteresis models, such as the Jiles–Atherton (JA) model [3], are promising candidates to incorporate temperature effects and can be embedded in finite element (FE) simulations. Although the physical aspects of the JA model and its parameters have been widely discussed [4], this is the most computationally efficient and easy-to-implement hysteresis model available in the literature and can be coupled with the FE simulations to incorporate the effects of hysteresis in field solutions. The JA model has five input parameters (M_S, a, α, c, and k) and the analytical expressions for the identification of these parameters are given in [5]. The details of the JA model and its parameters are given in Section II.

The temperature-dependent JA model was recently presented in the literature [6], [7]. Two additional parameters were introduced, and analytical expressions were proposed for the evolution of the JA model parameters with temperature. In addition to this, some numerical adjustments were applied, such as the value of parameter "c" was constrained in order to keep it within the physical limit, i.e., 0–1. The model was applied to cobalt ferrite and requires parameters to be determined at 0 °K, which is not possible; therefore, mathematical extrapolations were performed. The accuracy of the proposed model is limited, especially around the Curie temperature. A similar model was proposed in [8] which required parameters to be measured at room temperature instead of 0 °K. The model was applied to the 3F3 material. A new method was presented in [9] based on the direct identification of parameters from the measured data at different temperatures and was compared with the model presented in [6] and [7]. The proposed method was applied to NiFe 80/20 material. Although the model predicted good agreement with the measurements, the temperature variations of the parameters could not be explained. Also, the direct relation of the model parameters with the physics of the material could not be established. None of the materials reported in [6]–[8] is electrical steel, and all have Curie temperatures less than 369 °C.

The goal of this paper is to create a temperature-dependent JA approach which can represent the magnetic behavior of non-oriented (NO) electrical steels with suitable accuracy for the temperature range experienced in electrical machines during their operation. The JA model has five parameters (M_S, a, α, c, and k) which *a priori* depend on the temperature [6]–[8]. The idea is to identify a way to reduce the number of parameters needed to model the dependence on temperature. The approach is applied to 35WW300 NO electrical steel ($B_{H=2500\ \mathrm{A/m}} = 1.68$ T and $P_{\mathrm{loss},B=1.5\,\mathrm{T}} = 1.96$ W/kg) and the results are compared with measurements in Section IV.

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II. Jiles–Atherton Model

The JA model is one of the most popular hysteresis models inspired by the physics of ferromagnetism. It explains the hysteresis mechanism with the help of domain wall motion. The two modes of domain wall transitions (bending and translational motion) contribute to reversible and irreversible components of magnetization, respectively. The total magnetization is computed using a differential equation

$$\frac{dM}{dH} = \frac{c}{(1+c)}\frac{dM_{\text{an}}(M_s, a, \alpha)}{dH} + \frac{1}{(1+c)}\frac{(M_{\text{an}}(M_s, a, \alpha) - M)}{\frac{\delta k}{\mu_o} - \alpha(M_{\text{an}}(M_s, a, \alpha) - M)} \quad (1)$$

$$M_{\text{an}}(M_S, a, \alpha) = M_s\left(\coth\left(\frac{H+\alpha M}{a}\right) - \left(\frac{a}{H+\alpha M}\right)\right) \quad (2)$$

where $M_{\text{an}}(M_S, a, \alpha)$ is the anhysteretic magnetization which is computed using Langevin's function (2), M_S is the saturation magnetization, α is the interdomain coupling coefficient, a is a parameter that determines the shape of the anhysteretic curve and has the units of magnetic field, k is the pinning coefficient, and c is the domain wall flexibility coefficient. These are known as the JA model parameters and are identified from the measured major B–H loop. The details of these parameters and the differential equation are given in [3]. One limitation of the JA model is that the five parameters are identified from the major loop, and thus cannot model the inner loops accurately. However, this issue can be resolved using a combination of physical and empirical approaches to represent the experimental data, as presented in [10].

III. Proposed Approach

An engineering approach is presented here to predict the magnetic behavior of the NO electrical steels at different values of operating temperature. The B–H loops ($B_{\max} = 1.5$ T) were measured for 35WW300 NO electrical steel (sample size: 150 mm × 150 mm) at different values of temperature using a high-temperature Brockhaus Single Sheet Tester placed in an electric oven. The electric oven has a maximum operating temperature of 330 °C. Some of these measured B–H loops are shown in Fig. 1 demonstrating the effect of the temperature on the B–H loop's characteristics. The final values of the JA model input parameters are obtained for each of the measured B–H loops using a curve fitting technique, i.e., the nonlinear least squares (NLS) method [11]. The optimization function, used in this case, is the square of the difference between the measured and computed data points. The NLS method is a local search optimization method, which can lead to the solution faster if the initial guess of the JA parameters is carefully estimated. Jiles *et al.* [5] have proposed a method for determining these parameters using the characteristics of the B–H loop, and we have used the same technique in this paper to obtain the initial guess for the NLS method.

This paper targets the application of electrical steels in electrical machines at operating temperatures. Therefore, we have measured the B–H loop at seven temperatures within a limited temperature range (i.e., 50 °C–330 °C) which is well below the Curie temperature of electrical steels ($T_C > 700$ °C).

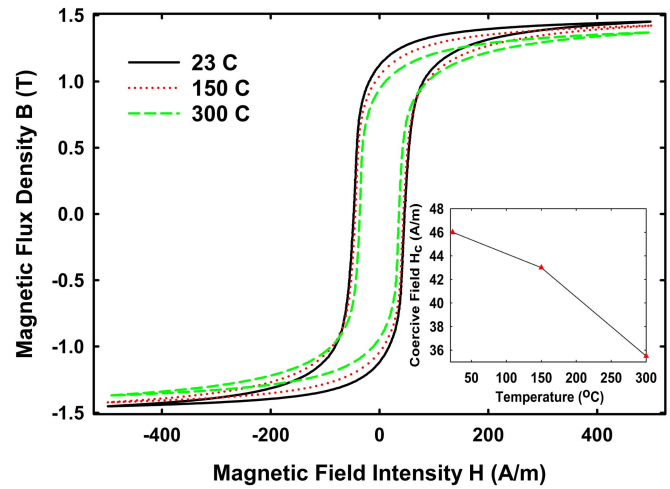

Fig. 1. Effect of temperature on the B–H loops measured at $B_{\max} = 1.5$ T. Inset: variation of coercivity with temperature. The operating frequency is 50 Hz.

As a result, we obtain a total of 35 values for the five JA model parameters, i.e., five at each temperature. Instead of developing the JA model parameters with analytical formulae, we consider the variation of the JA model parameters from the measured B–H loops and perform a primary analysis to reduce the number of parameters based on their thermal evolution.

A. Primary Analysis—Saturation Magnetization (M_S) and Reversibility Factor (c)

The first parameter in our analysis is M_S, which represents the saturation magnetization. It can be seen in Fig. 1 that the saturation magnetization (for constant applied magnetic field intensity, H) decreases with the increase in temperature. The evolution of the parameter M_S identified from measurements at different temperatures for 35WW300 NO electrical steel is shown in Fig. 2(a) and the variation is less than 4%. The temperature dependence of M_S in the studied steel sample correlates with the Weiss molecular theory [2] which states that if the operating temperature is well below the Curie temperature, T_C (i.e., $T/T_C < 0.5$), as in our case, the variation of M_S with temperature could be neglected [2]. Therefore, in this paper, we have assumed that the value of M_S to be constant, i.e., 1.2678×10^6 A/m, which is the value, obtained from the B–H loop measured at 50 °C.

The reversibility factor, c, in the JA model, represents the flexibility of the domain walls [3] and depends on the surface energy of the domain walls, pinning field, and the applied magnetic field intensity, H. The values of c, obtained using the NLS method, are shown in Fig. 2(b) and seem to slightly increase at relatively high temperatures. However, it has been observed that a small variation in c does not have a significant effect on the area of the B–H loop. Hence, the parameter c was also assumed to be temperature independent, i.e., 0.02.

It is important to mention here that M_S and c are identified using the B–H loop measured at the lowest temperature of interest, i.e., 50 °C. The rest of the JA model parameters (i.e., a, α, and k) are then identified from the measured

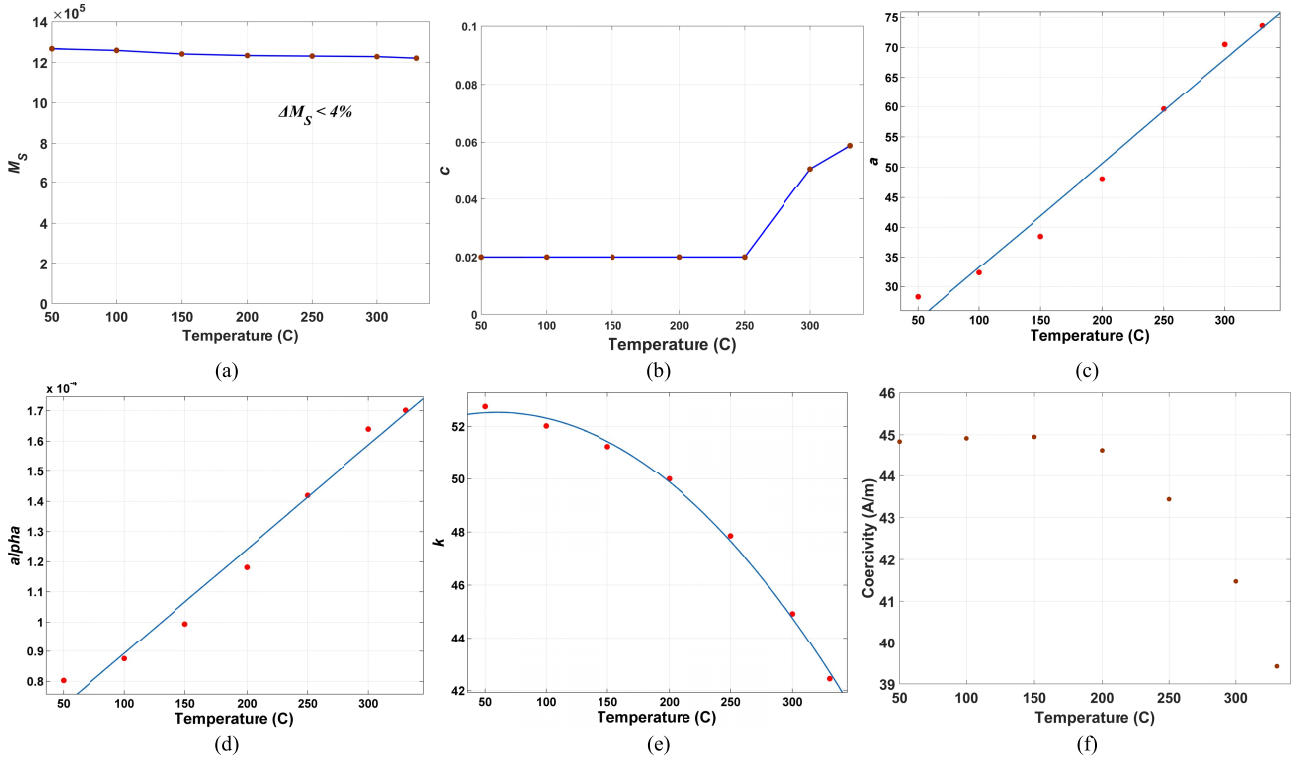

Fig. 2. Variation of the (a) saturation magnetization, M_S, (b) reversibility factor, c, (c) domain density, a, (d) domain coupling coefficient, α (e) pinning factor, k, and (f) measured coercivity of 35WW300 NO electrical steel at seven discrete temperatures. Red dots in (a)–(e): identified values of the JA parameters from the measured data.

B–H loops at other temperatures. The reduction in the number of unknown parameters makes the identification procedure simpler and faster.

B. Domain Density (a) and Domain Coupling Coefficient (α)

The domain density a is related to Boltzmann's energy and has a direct dependence on temperature, i.e., $a = k_B T/\mu_o m$, where k_B is Boltzmann's coefficient, T is the temperature, μ_o is the permeability of free space, and m is the magnetic moment of domains per unit volume. The domain coupling factor α in the JA model is responsible for the interdomain coupling which is due to the exchange interaction at the domain level [3].

Two of the three remaining JA model parameters (i.e., a and α), identified from the measurements, are plotted against temperature in Fig. 2(c) and (d), respectively. It can be seen that both parameters show a linear dependence on the temperature for the given temperature range and can be modeled using the first-order polynomials (i.e., with two unknowns only).

C. Pinning Factor (k)

The coercivity of a ferromagnetic material decreases with temperature [12], and it loses this property completely above the Curie temperature. The parameter k in the JA model represents the hysteresis effect, i.e., the energy required by domain walls to overcome the pinning sites. In the case of soft magnetic materials, the pinning factor k is similar to the coercivity [13] which hints about the decrease in the value of the parameter k with the increase in temperature.

TABLE I

CURVE FITTING OF THE JA PARAMETERS

Parameter	M_s	α	a	k	c
Polynomial Order	*N/A*	1	1	2	*N/A*
Polynomial Equation	a_0	a_0+a_1T	a_0+a_1T	$a_0+a_1T+a_2T^2$	a_0
Number Of Unknowns	1	2	2	3	1
a_0	1.2678×10^6	5.469×10^{-5}	15.8	52.05	0.02
a_1	-	3.469×10^{-7}	0.1742	0.01613	-
a_2	-	-	-	-1.349×10^{-4}	-
R_{Adj}^2	-	0.9712	0.9758	0.9947	-

The parameter k was identified from the measured B–H loops at different temperatures. For the given temperature range, a quadratic dependence (i.e., three unknowns only) of the parameter k on temperature was observed for this material and is shown in Fig. 2(e). The measured values of coercivity at different temperatures are plotted in Fig. 2(f). Both the coercivity and the parameter k show similar decreasing trends against the temperature in the studied steel.

IV. RESULTS AND DISCUSSION

The temperature evolution of the JA model parameters, described in Section III, is fitted to their respective polynomial equations, and the details of the number of unknowns for each parameter are given in Table I. The unknowns' values and the quality of the fit for all the parameters are also given. It can be seen that the proposed approach not only eases the process

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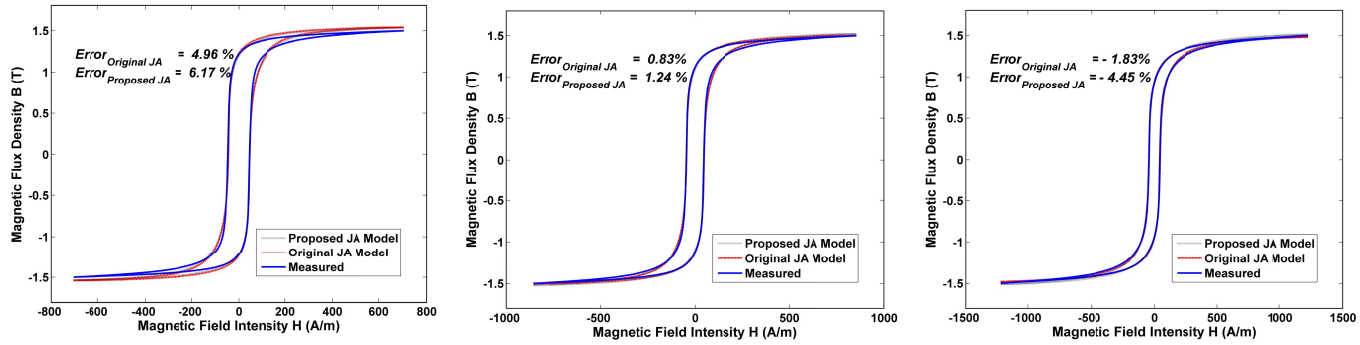

(a) (b) (c)

Fig. 3. Measured and computed B–H loops at (a) 50 °C, (b) 150 °C, and (c) 300 °C, using the identified JA model parameters (the original JA model) and the computed JA model parameters (the proposed JA model).

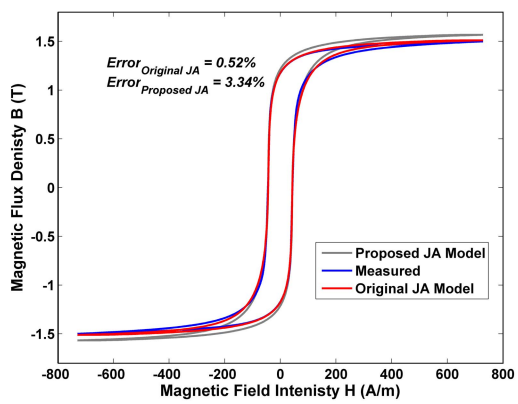

Fig. 4. Measured and computed B–H loops for B35AV1900 NO electrical steel at 150 °C, using the identified JA model parameters (the original JA model) and the computed JA model parameters (the proposed JA model). The parameters of the original JA model are $M_S = 1.2594 \times 106$ A/m, $c = 0.02, a = 27.1854$ A/m, $\alpha = 7.7459 \times 10^{-5}$, and $k = 50.443$ A/m, and for the proposed approach, $M_S = 1.2594 \times 106$ A/m, $c = 0.02, a(T) = 13.92 + 0.1189$ T A/m, $\alpha(T) = 4.8452 \times 10^{-5} + 2.549 \times 10^{-7}$ T, and $k(T) = 50.2 + 0.00842T - -6.507 \times 10^{-7}T^2$ A/m.

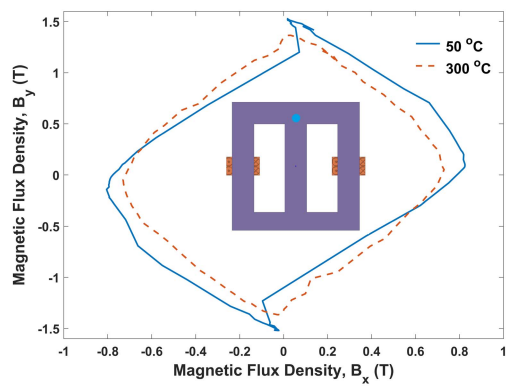

Fig. 5. Computed rotational fields at the point (marked blue) in the T section of the ferromagnetic core at 50 °C and 300 °C.

of identification of the JA model parameters but also helps in reducing the number of unknowns. No additional material information is needed.

The next step is to use these unknowns in their respective polynomial equations to compute the values of the JA model parameters which can then be used to predict the B–H loops of ferromagnetic materials at different temperatures. The measured and computed hysteresis loops of 35WW300 NO electrical steel are compared in Fig. 3 for different temperatures (50 °C, 150 °C, and 300 °C). We can see that as the temperature increases, the area and the width of the $B - H$ loop decrease.

An error metric is defined to examine the accuracy of the predictions and is given in the following equation:

$$\text{Error}\,(\%) = \frac{\text{ELD}_{\text{computed}} - \text{ELD}_{\text{measured}}}{\text{ELD}_{\text{measured}}} \times 100\% \qquad (3)$$

where ELD is the energy loss density that is computed by calculating the area of the loop. The proposed approach gives reasonable estimates for the given temperature range, and the errors in ELD are well within 6%. The errors obtained using the proposed model are slightly higher than that of the original model, because the proposed approach approximates the JA parameters requiring fewer measurements which affects the accuracy. The same approach was applied to a similar material, i.e., B35AV1900 NO electrical steel ($B_{H=2500\,\text{A/m}} = 1.68$ T and $P_{\text{loss},B=1.5\,\text{T}} = 1.91$ W/kg) and similar results were obtained (Fig. 4 shows a sample result at 150 ^{O}C).

A. Application Example—TEAM 32 Problem

Case 3 of TEAM 32 problem (details can be found in [14]) has been solved in 2-D using the vector JA model [15] in Infolytica's MagNet [16]. We have computed and used two sets of the JA parameters for 35WW300 NO electrical steels at two different temperatures (i.e., 50 °C and 300 °C) using the data given in Table I. The B-field solutions at the point $x = 0$ mm, $y = 61.5$ mm (the model is centered at the origin) in the "T" section of the ferromagnetic core are plotted for two temperatures in Fig. 5. We have applied the proposed approach in the TEAM problem to demonstrate that the proposed empirical approach can be used in thermally coupled electromagnetic simulations for the accurate design of modern electrical machines.

V. Conclusion

The temperature dependence of the JA model parameters was studied for NO electrical steels. A simple engineering approach has been presented to extend the JA model to incorporate the effects of temperature. The proposed approach makes use of a minimal amount of the experimental data, i.e., B–H loops measured at only three different temperatures are sufficient for model predictions within a given temperature range with reasonable accuracy. The JA parameters are identified from these measured B–H loops. The values of the parameters M_S and c are obtained at the lowest temperature of interest from the measured data and are kept constant. The remaining JA model parameters (a, α, and k) are then identified at different temperatures to fit the respective polynomials.

The proposed approach offers two advantages. The identification of the JA model parameters has been simplified by reducing the number of parameters. The JA model parameter information at extremely low temperatures (i.e., 0 °K) is not required. Since this is an experimental approach, the identification of the JA model parameters using analytical formulae, as previously described in the literature, may be ignored. The proposed approach has only been tested for NO electrical steels for a limited temperature range which is a typical range of thermal operation of an electrical machine.

Acknowledgment

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