A general surface reconstruction method for post-processing of topology optimisation results

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Outline

Context and scientific objectives

Surface Reconstruction strategy for genus 0 open surfaces

Poly-patches strategy for genus N surfaces (open and closed)

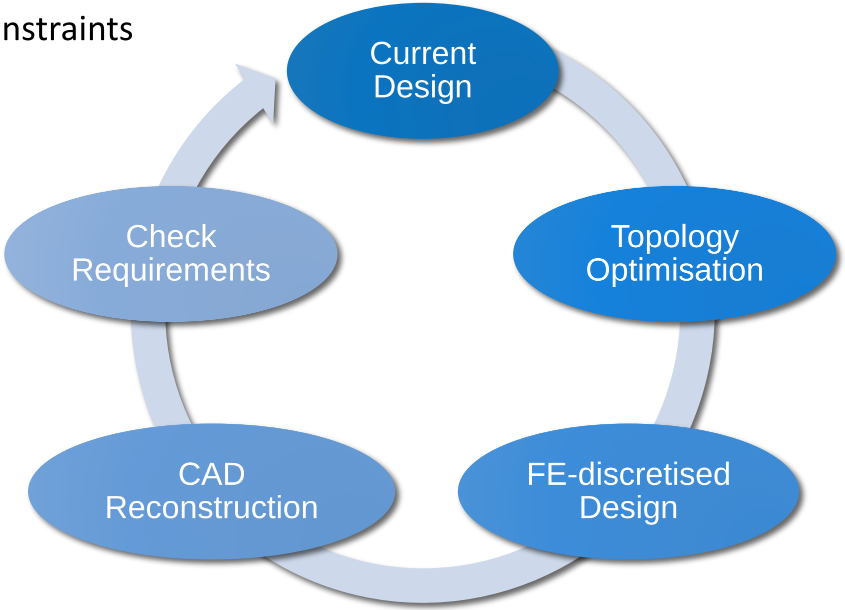
Conclusions and perspectives

Context and scientific objectives

Context

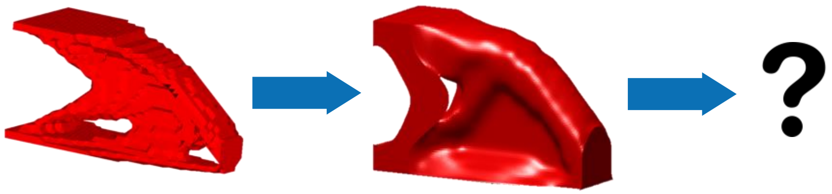
Topology optimisation:

- Optimal distribution of material in a prescribed domain
- Minimise an objective/cost function + meet optimisation constraints

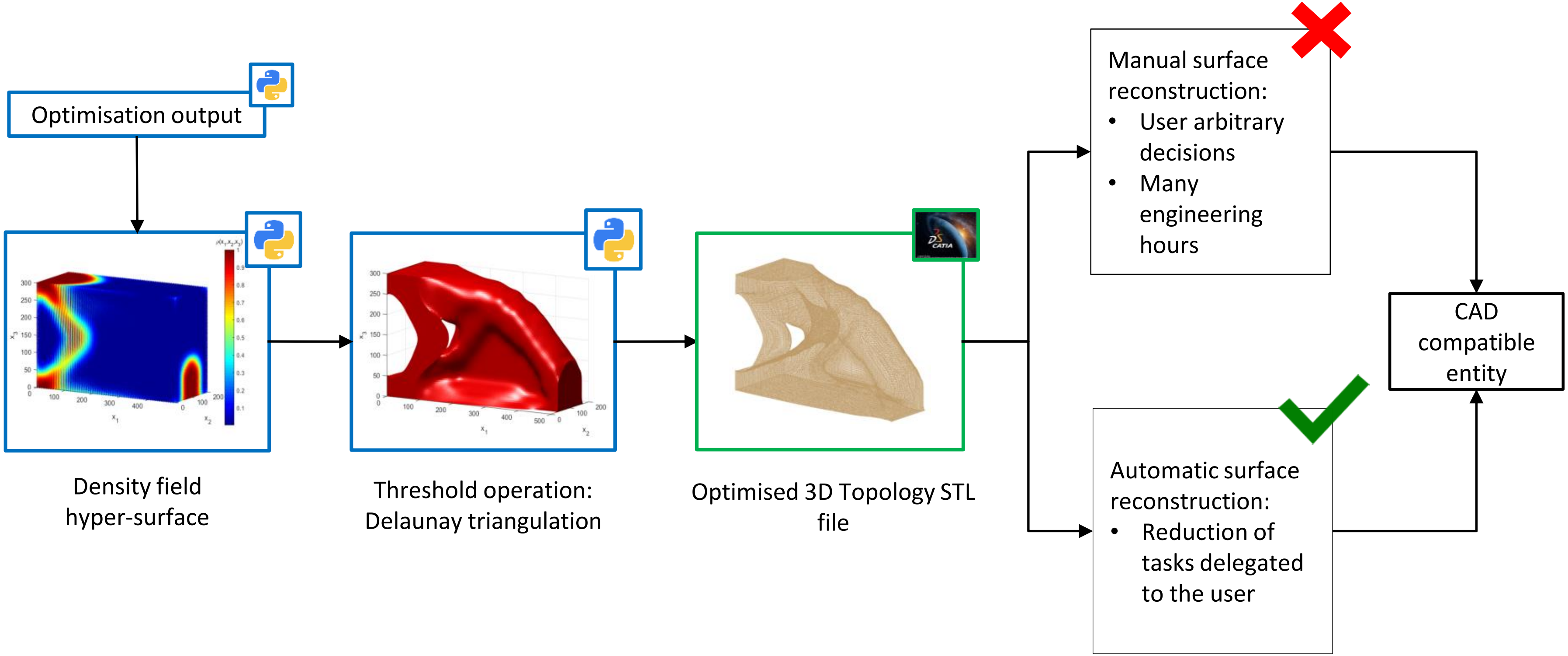


Results of topology optimisation strategy:

- Density field described by element-wise format
- Need to obtain smooth surfaces
- How is it possible to obtain CAD compatible entity?



Objectives

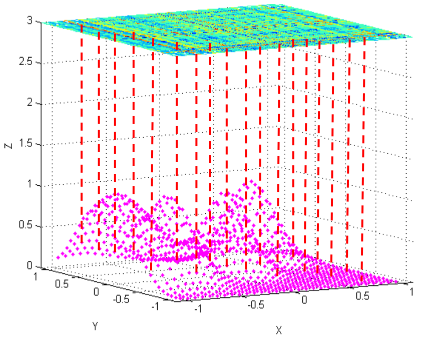


Optimisation output
Density field
hyper-surface
Threshold operation:
Delaunay triangulation
Optimised 3D Topology STL
file
Manual surface
reconstruction:
• User arbitrary
decisions
• Many
engineering
hours
Automatic surface
reconstruction:
• Reduction of
tasks delegated
to the user
CAD
compatible
entity

Surface Reconstruction strategy for genus 0 open surfaces

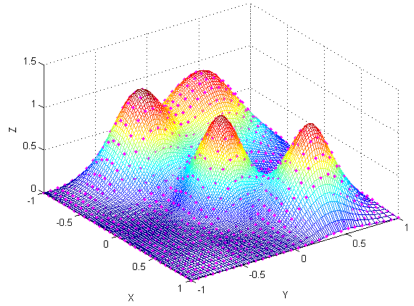
Proposed strategy: main ingredients

Mapping the STL points
Find the planar triangulation P isomorphic to the given triangulated graph G



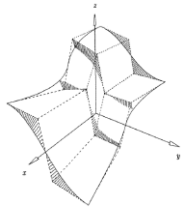
1 - Parameterisation

2 - Fitting

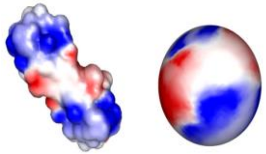


Least squares minimisation
Obtain the optimal set of NURBS parameters (Degrees, Knot vector, Weights)

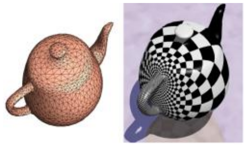
Parameterisation



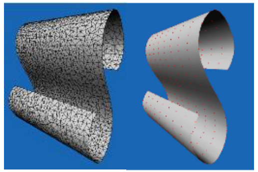
Projection method [Piegl,1995] - open, genus 0, not folded



Mercator's projection method [Rahi,2007] - closed, genus 0



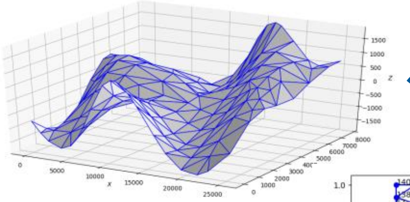
Global conformal method [Gu,2003] - closed, genus N

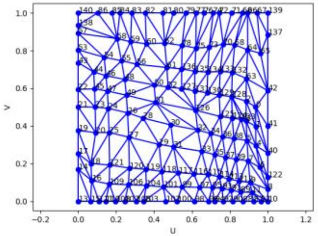


Shape preserving method [Floater,1997] - open, genus 0, folded

Shape preserving method: capabilities and main features

Find (u, v) parameters associated to the Cartesian coordinates of the 3D Euclidean space





Relabel nodes of STL file: **internal nodes** and **boundary nodes** (ordered in anti-clockwise sequence)

Parameterisation of the boundary nodes by chord length method into the boundary of a convex polygon $D \in \mathbb{R}^2$
$[0,1] \times [0,1]$

Expression of each internal node as linear convex combination of neighbours.

- Evaluation of the weights λ_{i,j_k} for each neighbour
- Preserving distances and angles between 3D and 2D

$$\begin{cases} [\Lambda]\{u\} = b_1 \\ [\Lambda]\{v\} = b_2 \end{cases}$$

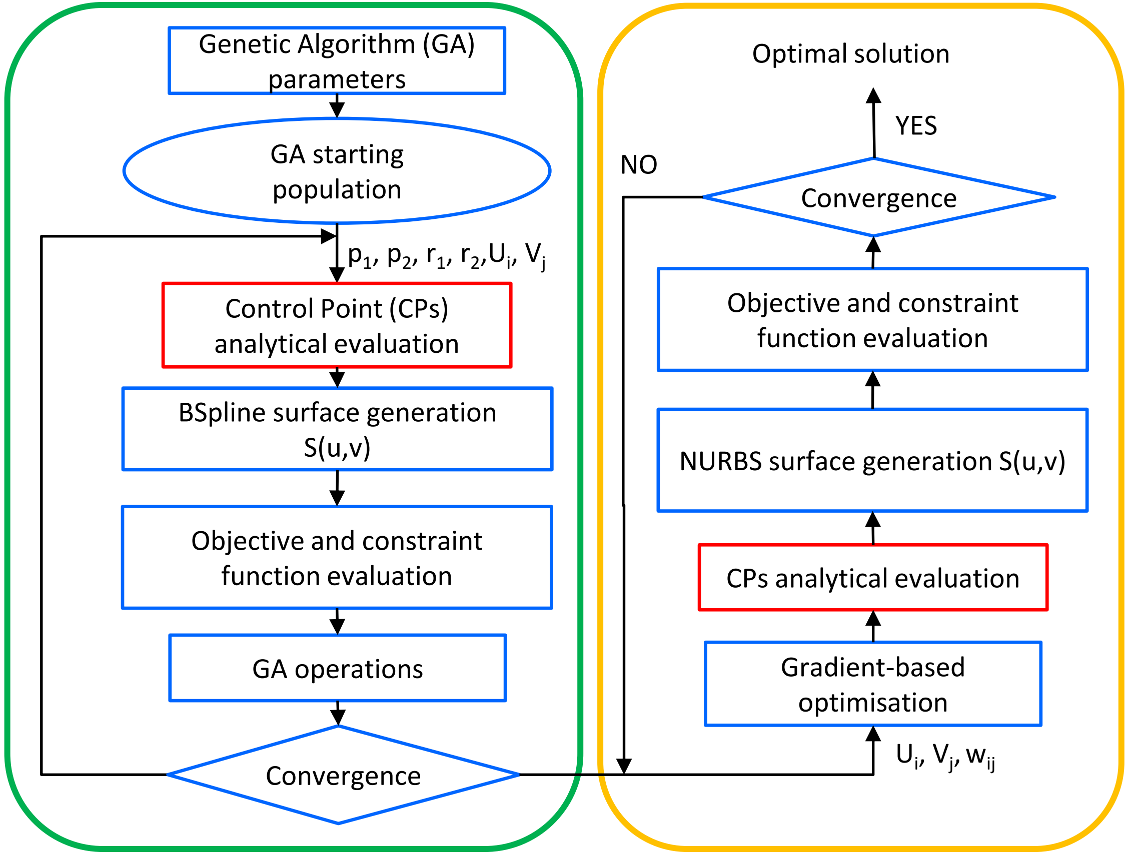
u, v internal nodes

$b1 = \lambda_{i,j}\ u_j \rightarrow u_j$ boundary nodes

$b2 = \lambda_{i,j}\ v_j \rightarrow v_j$ boundary nodes

$[\Lambda] \ \rightarrow \lambda_{i,j_k} = \frac{1}{d_i} \sum_{k=1}^{d_i} \mu_{k,l}$ weights

Surface fitting: Optimisation strategy



Part A

Originality: NURBS surface parameters (degree, CPs number, Knot Vector (KV) components) will be find automatically by the GA (in the literature there are no rules to set these parameters)

Part B

Local refinement of the minimum found by the GA ⇒ improvement of the solution in terms of KV COMPONENTS and WEIGHTS

Problem formulation and numerical aspects: genetic optimisation

Part A

Objective function

$$\min\ f(\mathbf{x}) = \left(\boxed{\sum_{k=0}^{n_{tp}} \|S(u_k, v_k) - Q_k\|^2} + \boxed{\lambda J}\right)^{\frac{1}{r_1+r_2}} \quad \text{such that:}$$

- Distance between BSpline entity and target points
- Thin-plate spline energy functional[Floater,2000]: smoothing term

$$J = \int_{a_1}^{b_1}\int_{a_2}^{b_2} S_{uu}^2 + 2S_{uv}^2 + S_{vv}^2\, du\, dv$$

Constraint function:

Non singularity of Basis Functions (BF) matrix

$$g_1(\mathbf{x}) = \boxed{\dim(BF)} - \boxed{\rho(BF)} \rightarrow BF = [N_u N_v]^T [N_u N_v] + \boxed{\lambda E}$$

- Dimension of the basic functions matrix
- Rank of the matrix of the basic functions matrix
- Smoothing matrix

Design variables

Discrete variables:

- p_1, p_2 → Degrees of the Bspline entity
- r_1, r_2 → Number of non-trivial components of KV

Continuous variables:

- $U_{p1+2,\ldots,}$ $U_{p1+r1+2}$, $V_{p2+2,\ldots,}$ $V_{p2+r2+2}$ → Knot vector components

Design space dimension = 4 + r_1+ r_2

↓

Discrete variables values affect the dimension of the Continuous variables module.

Problem formulation and numerical aspects: genetic optimisation

Part A

GA ERASMUS[Montemurro,2018] **capabilities:**

- Reproduction among individuals: crossover and mutation operations
- Reproduction among species: on individuals with different number of chromosomes
- Penalisation: Automatic Dynamic Penalisation (ADP)
    - Automatically and adaptively updating the coefficients of penalisation
    - Preventing infeasible solutions
    - Efficient exploration of the boundary of the feasible domain

Genotype

	Gene 1	Gene 2
Ch 1	p_1	p_2
Ch 2	r_1	r_2

Individual Standard section

	Gene 1
Ch 1	U_{p1+2}
…	…
Ch r_1	$U_{p1+r1+2}$

Individual Modular Section n. 1

	Gene 1
Ch 1	V_{p2+2}
…	…
Ch r_1	$V_{p2+r2+2}$

Individual Modular Section n. 2

Ch = chromosome

Problem formulation and numerical aspects: deterministic optimisation

Part B

Design variables (only continuous)

$$
\mathbf{x} \begin{cases} \mathbf{U} = \{0, \dots, 0, u_{p+1}, \dots u_n, 1, \dots, 1\} \\ \mathbf{V} = \{0, \dots, 0, v_{q+1}, \dots v_m, 1, \dots, 1\} \\ W = \begin{pmatrix} w_{11} & \cdots & w_{1n_2} \\ \vdots & \ddots & \vdots \\ w_{n_1 1} & \cdots & w_{n_1 n_2} \end{pmatrix} \end{cases}
$$

Objective function

$$
\min f(\mathbf{x}) = \sum_{k=0}^{n_{tp}} \| S(u_k, v_k) - Q_k \|^2 + \lambda J \text{ such that:}
$$

Constraint function

$$
g_1(\mathbf{x}) = \dim(BF) - \rho(BF) \rightarrow BF = [N_u N_v]^T [N_u N_v] + \lambda E
$$

Non trivial KV components

- Numerical evaluation of $\nabla f(\mathbf{x})$ respect to KV

Weights

- Analytical evaluation of $\nabla f(\mathbf{x})$ respect to weights

Focus on the analytical CPs evaluation

Part A

GA optimisation

CPs analytical evaluation

BSpline surface generation S(u,v)

Part B

Gradient-based optimisation

CPs analytical evaluation

NURBS surface generation S(u,v)

CPs analytical evaluation

Evaluation of $\frac{\partial f(\mathbf{x})}{\partial P_{ij}}$

Find the critical point $\nabla f(\mathbf{x}) = 0$

$$\left(\boxed{[\boldsymbol{N_u N_v}]}^{\boldsymbol{T}}[\boldsymbol{N_u N_v}] + \boxed{\boldsymbol{\lambda}}\boxed{\boldsymbol{E}}\right)\boxed{[\boldsymbol{P}]} = [\boldsymbol{N_u N_v}]^{\boldsymbol{T}}\ \boxed{[\boldsymbol{Q}]}$$

Check on the singularity due to the absence of parameters in the knot span

$[N_u N_v]^T [N_u N_v] + \lambda E$ matrix inversion to find $[P]$

- Basis function matrix evaluated at (u_k, v_k)
- Smoothing constant
- Smoothing matrix
- Control points coordinates
- Target points coordinates

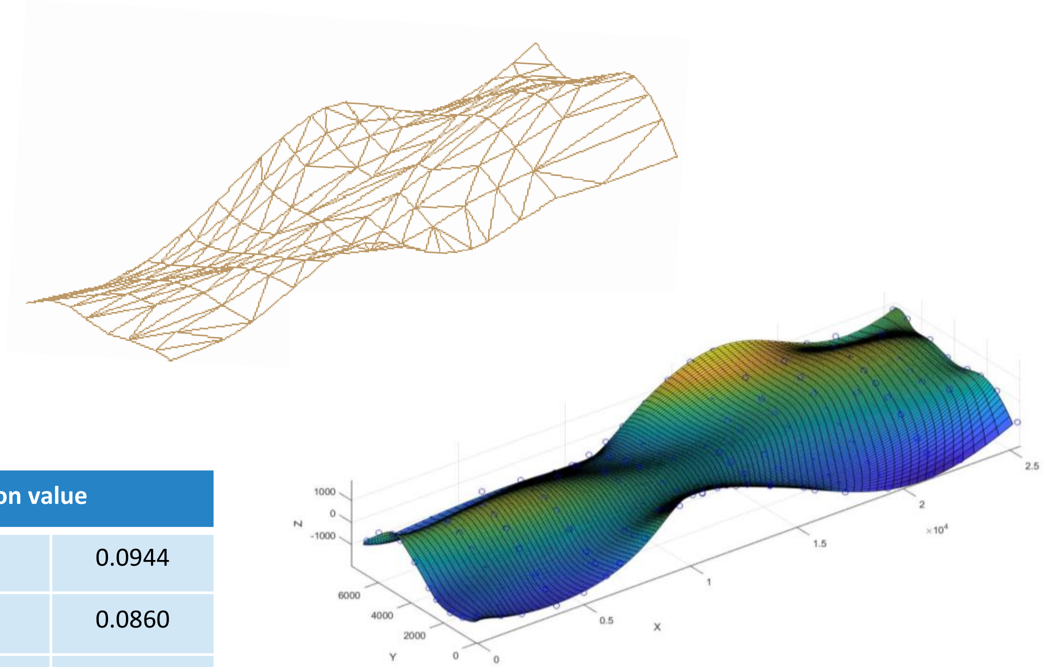
Numerical results: 1st benchmark

	Design variables	Lower - Upper bounds
GA optim	Degree (p)	2 – 6
	N° of KV's components (r)	1 – 17
	KV's components values (U, V)	0.001 – 0.999
Grad optim	Alpha, Beta (KV's components values)	0.001 – 0.999
	Weights (w)	0 – 10

Design space

Optimised values		Objective function value	
Degree (p_1 - p_2)	5 – 5	GA phase	0.0944
N° of KV's components (r_1 - r_2)	1 – 1	Grad KV	0.0860
KV's components values (U, V)	0.374 – 0.599	Grad KV + Weights	0.0844

Optimised design variables at the end of the Surface Reconstruction algorithm and objective function value evolution along the different phases



Results of the Surface Reconstruction strategy

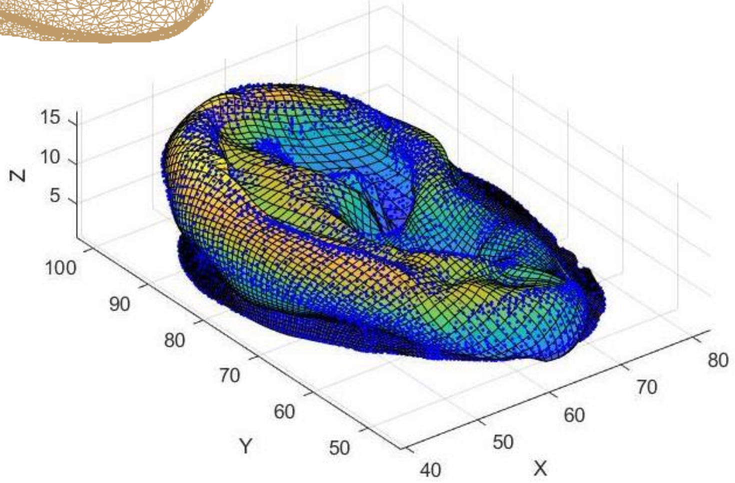
Numerical results: 2nd benchmark

	Design variables	Lower - Upper bounds
GA optim	Degree (p)	2 – 6
	N° of KV's components (r)	16 – 35
	KV's components values (U, V)	0.001 – 0.999
Grad optim	Alpha, Beta (KV's components values)	0.001 – 0.999
	Weights (w)	0 – 10

Design space

Optimised values		Objective function value	
Degree (p_1 - p_2)	2 – 2	GA phase	0.963576
N° of KV's components (r_1 - r_2)	19 – 19	Grad KV	0.954653
		Grad KV + Weights	0.892313

Optimised design variables at the end of the fitting algorithm and objective function value evolution along the different phases



Results of the Surface Reconstruction strategy

Poly-patches strategy for genus N surfaces (open and closed)

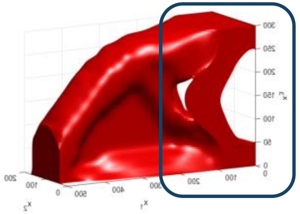
Poly-patches strategy for genus N surfaces (open and closed)

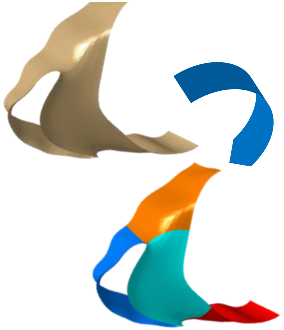
Aim: Application of the Surface Reconstruction strategy to **surfaces** (open and closed) with **holes** (genus > 0)

Strategy:

Manual segmentation

Domain divided into opened patches of genus 0





Parameterisation

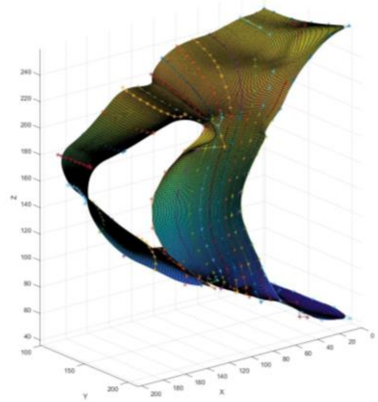
Adjacent patches have same parameters along boundary

Proper roto-translation of patches according to the global reference system

Patch fitting

Automatic calculation of NURBS parameters

Automatic imposition of C0 and C1 continuity condition between patches



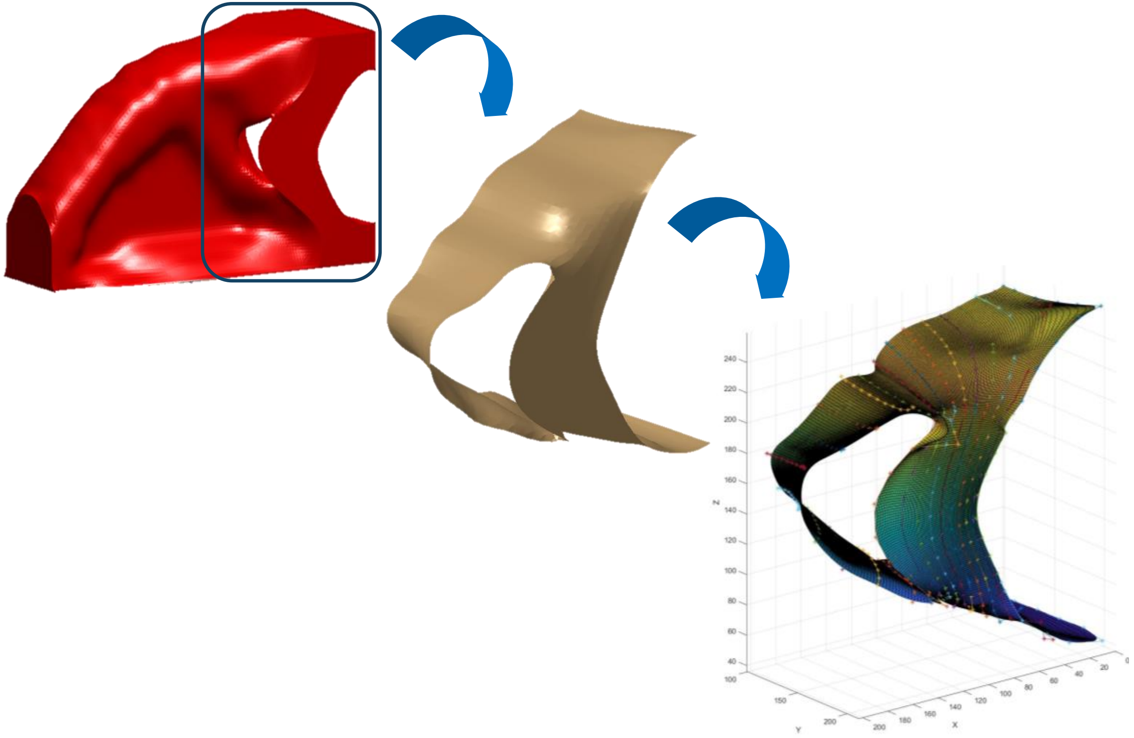
Numerical result

	Design variables	Lower - Upper bounds
GA optim	Degree (p)	2 – 6
	N° of KV's components (r)	4 – 20
	KV's components values (U, V)	0.001 – 0.999
Grad optim	Alpha, Beta (KV's components values)	0.001 – 0.999
	Weights (w)	0 – 10

Design space

Optimised values	
Degree (p_1 - p_2)	2 – 2
N° of KV's components (r_1 - r_2)	8 – 8

Optimised design variables at the end of the fitting algorithm.



Results of the Surface Reconstruction strategy

Conclusions and perspectives

Conclusions and perspectives

Conclusions

- Automatic optimisation of approximation surface parameters
- Reduction of tasks delegated to the user

Perspectives

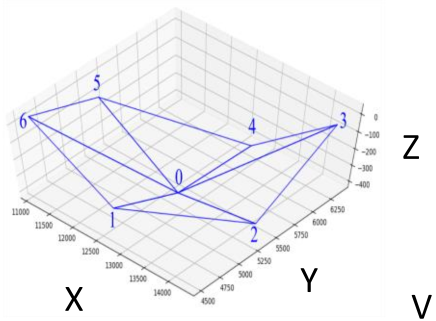
- Mapping methods for genus > 0 surfaces
- Automatic segmentation of the triangulation (STL file)
- Integration of Tspline entities in the surface fitting

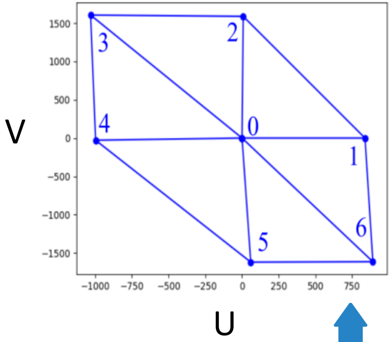
Thank you for your attention

Appendix

Shape preserving method[Floater,1997]

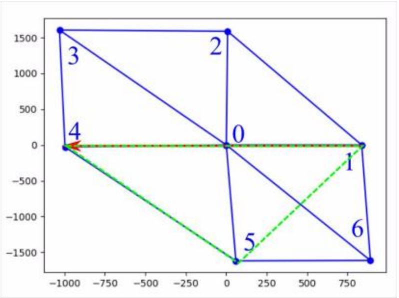
FIRST STEP





$$\begin{cases} \|p_k - p\| = \|x_{j_k} - x_i\| \\ ang(p_k, p, p_{k+1}) = \dfrac{2\pi ang(x_{j_k}, x_j, x_{j_{k+1}})}{\theta_i} \end{cases}$$

SECOND STEP



Barycentric coordinates

$$\begin{cases} \mu_k = \dfrac{\text{area}(p p_r p_{r+1})}{\text{area}(p_k p_r p_{r+1})} \\ \mu_r = \dfrac{\text{area}(p_k p p_{r+1})}{\text{area}(p_k p_r p_{r+1})} \\ \mu_{r+1} = \dfrac{\text{area}(p_k p_r p)}{\text{area}(p_k p_r p_{r+1})} \end{cases}$$

$\mu_{1,1}$	0	$\mu_{3,1}$	$\mu_{4,1}$	0	0
0	$\mu_{2,2}$	0	$\mu_{4,2}$	$\mu_{5,2}$	$\mu_{6,2}$
0	0	$\mu_{3,3}$	0	$\mu_{5,3}$	$\mu_{6,3}$
$\mu_{1,4}$	$\mu_{2,4}$	0	$\mu_{4,4}$	0	0
$\mu_{1,5}$	$\mu_{2,5}$	0	0	$\mu_{5,5}$	0
0	0	$\mu_{3,6}$	0	0	$\mu_{6,6}$

$$\lambda_{i,1} = \frac{1}{6}\sum_{l=1}^{6} \mu_{l,1}$$

...

$$\lambda_{i,6} = \frac{1}{6}\sum_{l=1}^{6} \mu_{l,6}$$

Thin-plate spline energy[Floater,2000]

Adding a smoothing term in the surface approximation of unstructured data aims to find a unique solution.

$$f(\mathbf{x}) = \sum_{k=0}^{n_{tp}} \|S(u_k, v_k) - Q_k\|^2 + \lambda J$$

$$\begin{cases} J = \int_{a_1}^{b_1} \int_{a_2}^{b_2} S_{uu}^2 + 2S_{uv}^2 + S_{vv}^2 \, du \, dv \rightarrow \text{simple thin plate energy functional} \\ \lambda \rightarrow \text{constant measuring the trade off between approximation and smoothing} \end{cases}$$

Find the minimum → normal equations

$$\frac{\partial f(\mathbf{x})}{\partial P_{ij}} = ([N_u N_v]^T [N_u N_v] + \lambda E)[P] - [N_u N_v]^T \, [Q] = 0$$

Where E is a $(n_1 n_2)$ x $(n_1 n_2)$ matrix whose elements are:

$$E_{ijrs} = A_{ijrs} + 2B_{ijrs} + C_{ijrs} \quad \begin{cases} A_{ijrs} = \int_{a_1}^{b_1} N_i''(u) N_j''(u) du \int_{a_2}^{b_2} N_j(v) N_s(v) \, dv \\ B_{ijrs} = \int_{a_1}^{b_1} N_i'(u) N_j'(u) du \int_{a_2}^{b_2} N_j'(v) N_s'(v) dv \\ C_{ijrs} = \int_{a_1}^{b_1} N_i(u) N_j(u) du \int_{a_2}^{b_2} N_j''(v) N_s''(v) dv \end{cases}$$

And λ is:

$$\lambda = \frac{\|([N_u N_v]^T [N_u N_v])^2\|}{\|E^2\|}$$