Effect of plastic anisotropy on the prediction of the ductility for HCP sheet metals

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Abstract - Due to their lightness, low stiffness and high strength, Hexagonal Closed Packed (HCP) materials are widely used in aeronautic and aerospace industries. In this paper, the ductility limit of HCP sheet materials at room temperature (25° C) is predicted by coupling the Cazacu yield function and the Marciniak and Kuczyński (MK) necking criterion. Based on transformed principal stresses, the phenomenological constitutive model of Cazacu is used to take into account the initial plastic anisotropy and strength differential (SD) effects. For plane stress and orthotropic symmetry, two linear transformations are required to use a number of anisotropy coefficients which are more suitable for practical applications. Under these circumstances, a prediction of formability for HCP sheet materials with more than one linear transformation is performed using the numerical tool Mathematica.

Key words: HCP sheet materials / FLD / Cazacu yield criterion / MK necking criterion / Linear transformation.

1. Introduction

The study of the mechanical behavior of hexagonal closed packed (HCP) materials becomes a major challenge for the researchers. Consequently, predicting the formability for these materials is very interesting. Furthermore, HCP materials are characterized by a low formability at room temperature (25°C) and their great asymmetry between yielding in tensile and compression. The most robust macroscopic yield function for these materials is proposed by Cazacu et al. [1] to describe with accuracy both the asymmetry in yielding between tension and compression, and the strong anisotropy based on linear transformation. Many researchers are interested by the phenomenological constitutive model of Cazacu such as Barlat et al. [2] who described the plastic anisotropy using more than one linear transformation and Plunkett et al. [3] who used this theory for HCP sheet materials. The plastic deformations, under which a sheet metal can be subjected in such operation, are limited by the onset of localized necking. The knowledge of the limit deformations and their characterization of sheet metals represent a big problem for both academic and industrial applications. To predict the incipience of localized necking, the concept of forming limit diagram (FLD) is introduced and will be applied in the present contribution. In the literature, several predictive approaches have been developed to predict the FLD. The Marciniak and Kuczyński (MK) criterion, introduced in [4], seems to be one of the most popular and pragmatic necking criteria developed in the literature. It will be used in the current work.

To predict a FLD for HCP sheet materials, a coupling between Cazacu yield function using more than one linear transformation and MK necking criterion is considered in the present work.

2. Cazacu-MK model

In the present section, Cazacu yield function and MK imperfection model are described.

– **Cazacu yield function**

Because strain localization occurs at relatively large strains, elasticity may be neglected and, hence, the behavior is taken rigid–plastic.

Due to the strong anisotropy of HCP materials, a linear transformation $\boldsymbol{\varphi}$ on the deviator stress $\mathbf{s}$ has been used. This transformation is described by a double contraction between an orthotropic 4th order tensor $\mathbf{L}$ and tensor $\mathbf{s}$:

$$\boldsymbol{\varphi} = \mathbf{L} : \mathbf{s} = \mathbf{L} : \mathbf{T} : \boldsymbol{\sigma} \tag{1}$$

where $\mathbf{T}$ is the matrix transforming from the Cauchy stress tensor $\boldsymbol{\sigma}$ to its deviator $\mathbf{s}$. The Cazacu yield function F is presented as follow using the principal values of the linear transformation $\boldsymbol{\varphi}$.

$$F = \left(|\varphi_1| - k\varphi_1\right)^a + \left(|\varphi_2| - k\varphi_2\right)^a + \left(|\varphi_3| - k\varphi_3\right)^a \tag{2}$$

where the material parameter k is the strength differential (SD) with a physical significance which may be admitted from uniaxial tests and the material constant a presents the homogeneity coefficient.

For more than one linear transformation, the used yield function is described as below:

$$F\left(\varphi_1^{(n)}, \varphi_2^{(n)}, \varphi_3^{(n)}\right) = \sum_{n=1}^{i} \left(\sum_{m=1}^{3} \left(\left|\varphi_m^{(n)}\right| - k\varphi_m^{(n)} \right)^a \right) \tag{3}$$

where $\boldsymbol{\varphi}^{(n)} = \mathbf{L}^{(n)} : \mathbf{s}$ and n is the number of linear transformations.

The equivalent stress $\tilde{\sigma}$ is described using the yield function F and a parameter B which is calculated using $\mathbf{L}$ components ([1]) :

$$\tilde{\sigma} = B.F^{\frac{1}{a}} \tag{4}$$

To completely describe the behavior law of HCP materials, the Eqs. (1)-(4) describing the Cazacu yield function are completed by the flow rule (Eq. 5) and the consistency condition (Eq. (6)):

$$\dot{\boldsymbol{\varepsilon}} = \dot{\lambda} \frac{\partial F}{\partial \boldsymbol{\sigma}} \tag{5}$$

$$\tilde{\sigma} - Y = 0 \tag{6}$$

where $\dot{\boldsymbol{\varepsilon}}$ is the strain rate and $\dot{\lambda}$ is the plastic multiplier (equal to the equivalent strain rate $\dot{\tilde{\varepsilon}}$). However, Y is the yield stress defined by the following isotropic hardening law:

$$Y\left(\tilde{\varepsilon}_p\right) = \alpha\left(\sigma_0 + \sigma_{sat}\left(1 - e^{-n_v \tilde{\varepsilon}_p}\right)\right) + (1-\alpha)\left(K\left(\varepsilon_0 + \tilde{\varepsilon}_p\right)^{n_s}\right) \tag{7}$$

If $\alpha = 0$, the hardening law is that of Swift and its parameters are K, $\boldsymbol{\varepsilon_0}$ and n_s, but if $\alpha = 1$ the hardening law is that of Voce such as σ_0, σ_{sat} and n_v are its parameters.

– **MK localized necking criterion**

The theoretical MK model is considered as shown in figure 1

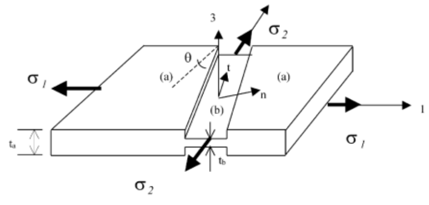

Figure 1. Concept of the MK criterion [5]

The homogeneous zone and the band are referenced by letters a and b, respectively. However, angle θ defines the orientation of the groove compared to the major strain direction. Symbols t_a^0, t_b^0 and t_a, t_b represents the initial and the actual thicknesses of the homogeneous zone and the groove, respectively. The initial imperfection factor f_0 is defined as the ratio t_b^0 / t_a^0. The MK analysis is based on three main equations, the first one is the constitutive Eq. 4. The second one is equilibrium balance across the groove zone presented as below:

$$\boldsymbol{\sigma}_a.\vec{\mathbf{n}}\, t_a = \boldsymbol{\sigma}_b.\vec{\mathbf{n}}\, t_b \tag{8}$$

Where $\vec{\mathbf{n}}$ is the current normal to the imperfection. Maxwell's compatibility condition, for the velocity field, states that there exists a vector $\vec{\beta}$ such that the jump in $\mathbf{A}$ reads:

$$[\mathbf{A}] = \mathrm{sym}\left(\vec{\beta} \otimes \vec{\mathbf{n}}\right) \tag{9}$$

where $\otimes$ stand for the vector product and sym is the symmetric part of this product. Using Maxwell's compatibility, the third equation is the geometrical compatibility:

$$\dot{\boldsymbol{\varepsilon}}_b = \dot{\boldsymbol{\varepsilon}}_a + [\mathbf{A}] \tag{10}$$

where $\dot{\boldsymbol{\varepsilon}}_a$ and $\dot{\boldsymbol{\varepsilon}}_b$ are respectively the strain rate in homogenous and b-zone which are the symmetric part for gradient deformation.

3. Results and discussions

The aim of this section is to predict the formability of rolled magnesium alloy sheets AZ31. First of all, the effect of the used hardening law on the FLDs is analysed in Fig. 2. In this figure, our predictions with the Swift and the Voce hardening laws are compared with other results from the literature ([6]) which ε_1 and ε_2 designates respectively major and minor strains. In the left-hand side of the FLD, the prediction with Voce hardening law leads to limit strains lower than the ones predicted by the Swift hardening law. This observation is probably due to the use of the stress saturation σ_{sat} in the Voce law. The results show that Voce hardening law is in a good agreement with experimental study (better than the Swift law) which give an average error equals to 2.83%. Fig. 3 and Fig. 4 show respectively the influence of linear transformations on forming limit stress diagram (FLSD) and FLD. Regular shapes of forming limit curves are found from these figures (Figs. (3)-(4)). Owing to increasing and decreasing respectively strain and stress values for homogeneous zone as shown in Figs. (3)-(4), one can prove that using $n > 1$, requires to obtain a deformed sheet metal with a minimum of constraints during tensile test better than using $n = 1$. Unfortunately, experimental study using more than one yield surface is extremely important for HCP materials to predict the best FLD for $n > 1$. However, we can estimate based only on numerical study that the closest results of FLD and FLSD using one linear transformation are defined by using $n = 2$. As summaries Barlat et al [2] proved that the best predicted behaviour law is obtained for $n = 2$, and the previous finding for this paper privilege $n = 2$ to predict best FLD and FLSD.

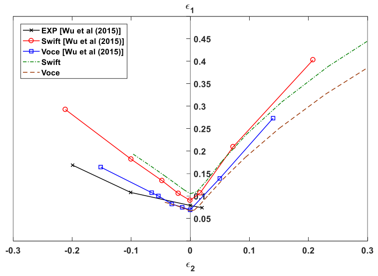

Figure 2. Effect of the hardening model on the FLD predictions (comparison with experimental results).

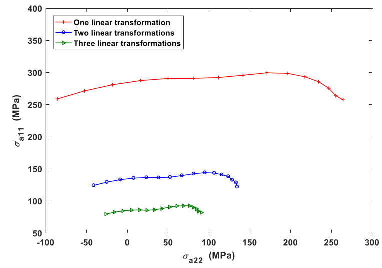

Figure 3. Effect of the number of linear transformations on the shape and the level of FLSDs (σ_{a11} and σ_{a22} designate the xx and yy component for homogeneous stress)

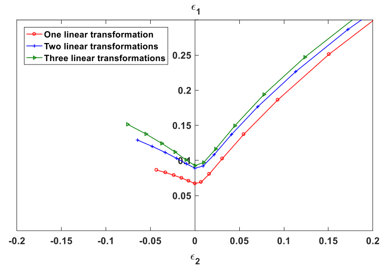

Figure 4. Effect of the number of linear transformations on the shape and the level of FLSDs.

4. Conclusions

In the present work, a numerical model based on the coupling between Cazacu yield function and MK necking criterion is developed to predict the forming limit diagram for HCP sheet materials. It can generally be concluded that the calculation of the FLD and FLSD is strongly influenced by the selected number of linear transformation. It is clearly seen that predicting FLD and FLSD for $n = 2$ is more convincing to describe the whole set of forming limit curves but this conclusion remains temporary until validation by an experimental study.

5. Reference

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