A game-based meta-heuristic for a fuzzy bi-objective reliable hub location problem

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A B S T R A C T

Nowadays, offering fast and reliable delivery service has become a vital issue associated with all shipment delivery systems. Due to unpredictable variability in travel times, configuration of transportation systems plays a key role in ensuring of meeting the delivery service requirement. This paper tries to investigate the effect of delivery service requirement on the configuration of the transportation system through a hub-and-spoke network. The primary goal of this paper is to study a bi-objective single allocation p-hub center-median problem (BSpHCMP) by taking into account the uncertainty in flows, costs, times and hub operations. The proposed problem is modeled through a bi-objective mixed-integer non-linear programming (BMINLP) formulation that simultaneously locates p hubs, allocates spokes to the located hubs, and assigns different transportation mode to the hub-to-hub links. Then, a fuzzy-queuing approach is used to model the uncertainties in the network. Additionally, an efficient and powerful evolutionary algorithm based on game theory and invasive weed optimization algorithm was developed to solve the proposed BSpHCMP model and obtain near optimal Pareto solutions. Several experiments besides a real transportation case show the applicability of the proposed model as well as the superiority of the proposed solution approaches compared to NSGA-II and PAES algorithms.

1. Introduction

Hub location problems (HLPs) arise in systems with the need of transporting flows (e.g., goods or passengers) between origin–destination (OD) nodes (spokes). In such systems, a direct transportation of the flows between spokes is neither practical nor costly efficient. Therefore, one may use a specific network structure called hub-and-spoke network, wherein the hubs are intermediate facilities in these networks whose duties are consolidating the flows from the origins, transferring the flows between hubs, and distributing the flows to the destinations. Transferring the flows via the hub-to-hub links allow us to exploit transportation flow economies (Ernst et al., 2009). Spokes can be allocated to one or more hubs based on single or multiple allocation strategies. A common assumption is that the hubs are fully interconnected, while there is no connection between spokes. Therefore, all flows must pass at least one hub on their route.

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In HLPs, the considered objectives are mainly median and center. The hub median problem is to locate a set of hubs and to allocate spokes to the located hubs with the objective of minimizing the total transportation cost. The hub median problems are applied to airline and telecommunication systems. A drawback of median design of a hub network is when maximum OD distances are excessively large. To overcome this weakness, hub center problems arise when the main objective is to minimize the maximum distance or cost between OD pairs. This objective is significantly important for the shipment delivery systems as well as delivery of perishable or time sensitive items (Campbell et al., 2007).

In shipment delivery systems, most customers are looking for companies that offer fast, cheap and reliable delivery service as well as guarantee that when deliveries will be made. However, these companies have to operate a huge volume of shipments each day (Sim et al., 2009). To operate a huge volume of deliveries between many pairs of OD nodes, companies need to design an efficient and reliable hub-and-spoke network not only to meet delivery requirements, but also to offer cheap deliveries (Grove and O'Kelly, 1986; Hall, 1989; Sim et al., 2009). Shipment delivery systems through hub-and-spoke topology benefit from economies of scales by transferring large volume of flows through hub-to-hub links that result in faster delivery as well as cheaper transportation. To this aim, the hub network should be designed respecting median and center objectives.

The considered hub location problem in this paper is called the bi-objective single allocation *p*-hub center-median problem (BS*p*HCMP), where the hubs and the connection links have infinite capacity. The aim is to locate *p* hubs and allocate the spokes to exactly one of the located hubs in such a way that the total transportation cost (i.e., median objective) besides maximum transportation time between any OD pair (i.e., center objective) is simultaneously minimized. After designing the hub network, a company may evaluate alternative transportation modes for those OD pairs that are not tightly limited by the time service requirement. For example, a single or a combination of slower but lower cost transportation modes may be used between some particular OD pairs while their delivery requirement is still met.

Different attributes of such problem, like size, responsibilities, services, and the nature of spoke allocation have made designing it quite complicated. Owning to the complexity and several components of the hub network, a little uncertainty may cause a huge disruption in the network, which imposes huge costs and hard-to-recover effects on the network (Cui et al., 2010). These uncertainties (natural disasters, terrorist attacks, unexpected work overloads, shortages, labor strikes, etc.) can affect not only costs, demands and transportation times, but also hub operations. That is, disruptions can occur and the hubs may become temporarily unavailable to provide service. In these situations, retrieving of the hubs yields extra costs and time in the network.

Hub's disruptions particularly affect the center objective, where the flows entering the hub must wait until the hub is retrieved then receive services. These waiting times at the hubs lead to higher delivery time and increase customer dissatisfaction. Since the flow entering a hub is uncertain, the queuing theory is needed to analyze the waiting time of the flows at the hubs. Therefore, the uncertainties of flows, costs, times and hub operations should be taken into account in network design phase (for uncertainties in network designs see Mohammadi et al., 2011a, 2011b, 2013, 2014a; Contreras et al., 2011; Ishfaq and Sox, 2012; Yang et al., 2013a, 2013b, 2014; Mousazadeh et al., 2015; Zahiri et al. 2014a; 2004b, 2015).

Finally, the primary goal of this paper is to address the BS*p*HCMP considering uncertainty in flows, costs, times and hub operations. We model the proposed problem through a bi-objective mixed-integer non-linear programming (BMINLP) formulation that simultaneously locates *p* hubs, allocates spokes to the located hubs, and assigns different transportation mode to the hub-to-hub links. The median objective minimizes the sum of total transportation cost and cost of locating hubs and assigning different transportation modes across the network. The center objective minimizes the maximum transportation time between each OD pair in the network.

One of the most challenging issues in HLPs is how to solve them, since they are known to be NP-hard (Alumur and Kara, 2008). Solving the proposed BS*p*HCMP, compared with its single-objective version, has not been so extensively studied in the literature, and so far a few papers have developed multi-objective evolutionary algorithms to deal with the multi-objective HLPs (e.g., Mohammadi et al., 2013, 2014a and references herein). These articles mainly lack for efficient and fail to propose powerful algorithms that are able to find near optimal solutions for a bi-objective HLP.

Accordingly, the second purpose of this paper is to propose an efficient and powerful evolutionary algorithm, based on game theory and invasive weed optimization algorithm (Mehrabian and Lucas, 2006) to solve the proposed BS*p*HCMP model and obtain near optimal Pareto solutions.

The rest of this paper is organized as follows. Section 2 reviews prior researches, first, in the area of center-median HLPs under uncertainty and, second, in domain of evolutionary algorithms to solve HLP models. Section 3 describes the modeling framework and presents the BMINLP formulation. Section 4 describes the solution approach and develops a meta-heuristic algorithm. Computational experiments with comprehensive sensitivity analyses are provided in Section 5. Finally, the conclusion is presented in Section 6.

2. Literature review

This section is organized in two sections. First, Section 2.1 provides a brief review of HLPs considering uncertainty and disruption in the hub network design. Next, some relevant papers developing meta-heuristic algorithms are reviewed in Section 2.2.

2.1. HLPs and uncertainty

Campbell (1994) proposed the first formulation for *p*-hub center problem (*p*HCP) as a quadratic programming model. Afterwards, several alternative linear formulations for the single allocation *p*HCP were proposed by Kara and Tansel (2000). Ernst et al. (2000) modeled both single and multiple allocation *p*HCP through a new mixed-integer linear programming formulation based on the concept of the radius of hubs. Kara and Tansel (2001) incorporated an operational-level constraint to the *p*HCP which flows departing from a hub cannot leave until all flows entering the hub have arrived. They called this problem as the latest arrival hub location problem.

Wagner (2004) explained that the solution of the min–max version of the latest arrival hub location problem is similar to the *p*HCP because the route that determines the longest path in the network is the one where the transient or waiting times at the hubs are zero. Yaman et al. (2007) presented a new version of the latest arrival hub location problem by considering stop overs and between the spoke and hubs. Campbell et al. (2007) studied the single and multiple allocation versions of the *p*HCP, and concluded that several special cases of the *p*HCP and latest arrival HLP can be solved in polynomial time.

For the first time, O'Kelly (1987) formulated a single allocation *p*-hub median problem (*p*HMP) as a quadratic integer program. However, this formulation resulted in a very difficult problem to be solved. Next, Campbell (1994) provided new formulation for the *p*-hub median problem as an integer program, but this formulation contained many variables and constraints. Yaman (2011) proposed three different formulations for the uncapacitated *r*-allocation *p*HMP. For more details on *p*HCP and *p*HMP formulations, interested readers are referred to Alumur and Kara (2008) and Campbell et al. (2002) and more recently Zanjirani Farahani et al. (2013) for surveys on HLPs.

To the best of our knowledge, a few papers have studied the *p*HCP and *p*HMP with uncertainty in flows, costs, and transportation time. Sim et al. (2009) introduced a stochastic *p*HCP (S*p*HCP) utilizing a chance-constraint method to model the minimum delivery service requirement by taking the variability in transportation times into account. Yang et al. (2013a) presented a new risk aversion *p*HCP with fuzzy travel times by adopting value-at-risk (VaR) criterion in the formulation of objective function. In order to solve and validate the model, they first turned the original VaR *p*HCP into its equivalent parametric mixed-integer programming problem, and then developed a hybrid algorithm by incorporating genetic algorithm and local search (GALS) to solve the parametric mixed-integer programming problem. Yang et al. (2013b) proposed a new *p*HCP with normal fuzzy travel time, in which the main goal is to maximize the credibility of fuzzy travel times not exceeding a predetermined acceptable efficient time point along all paths on a network. Due to complexity of the proposed model, they applied an approximation approach (AA) to discretize fuzzy travel times and reformulate the original problem as a mixed-integer programming problem subject to logical constraints. Next, they took advantage of the structural characteristics to develop a parametric decomposition method to divide the approximate *p*HCP into two mixed-integer programming subproblems. Finally, the authors developed an improved hybrid particle swarm optimization (PSO) algorithm by combining PSO with genetic operators and local search (LS) to update and improve particles for the subproblems. In another work, Yang et al. (2014) reduced the uncertainty embedded in secondary possibility distribution of a type-2 fuzzy variable by fuzzy integral and applied the proposed reduction method to *p*HCP. They also developed a robust optimization method to take uncertainty in travel times into account by employing parametric possibility distributions.

Contreras et al. (2011) studied stochastic uncapacitated HLPs, where flows and transportation costs considered to be uncertain. They showed that the stochastic problems with uncertain flows or dependent transportation costs are equivalent to their associated deterministic expected value problem (EVP), in which random variables are replaced with their expectations. Mohammadi et al. (2013) developed a stochastic bi-objective multi-mode transportation model for hub covering problem. They considered the transportation time between each pair of nodes as an uncertain parameter that is also influenced by a risk factor in the network. Similar to Contreras et al. (2011), Adibi and Razmi (2015) developed a 2-stage stochastic programming for formulating stochastic uncapacitated multiple-allocation HLP. They considered three cases, wherein, (1) flow is stochastic, (2) cost is stochastic, and (3) both flow and cost are stochastic. Unlike Contreras et al. (2011), the authors concluded that considering uncertainty into formulation could result in different solutions.

To the best of our knowledge, there is no paper considering uncertainty in hub operations that affect the center objective. Most of papers considering uncertainty in the location–allocation problem have studied disruption particularly in facility location problems (FLPs). Two reliability models called reliable *p*-median and reliable uncapacitated fixed-charge location were studied by Snyder and Daskin (2005). In their model, the uncertainty resulted in unavailability of the facilities and each customer is assigned not only to a primary facility but also to a number of backup facilities, in which at least one facility must be available. If the current facility fails, the customer is served by the next available backup facility. They also consider that the failure probabilities are equal and mutually independent. Li et al. (2010) studied a FLP in a three-level supply chain problem by taking random disruption for both suppliers and retailers into account. Their models determine the optimal locations of retailers, customer assignations and inventory policy. Peng et al. (2011) introduced the *p*-robustness criterion so that the designed network performs well in both disrupted and normal conditions. Zheng and Ling (2013) proposed a multi-objective fuzzy optimization problem of emergency transportation planning, in which disruption is occurred due to natural phenomena and took into consideration three transportation modes: air, rail, and road.

There are only two studies that have studied disruption in the hub network. Parvaresh et al. (2012) formulated a bi-level multiple allocation *p*HMP under intentional disruptions with bi-objective functions at an upper level and a single objective at a lower level. In their model, the leader aims at identifying the location of hubs so that minimize normal and worst-case transportation costs. Finally, the worst-case scenario is modeled in a lower level where the follower's objective is to identify the hubs that if it is lost, it will mostly increase the transportation cost. Additionally, they developed two multi-objective meta-heuristics based on simulated annealing and tabu search to solve their proposed model. In similar work, Parvaresh et al. (2013) developed a multiple allocation *p*-hub median problem under intentional disruptions using different definitions of a failure probability of the hub in comparison to their previous work.

2.2. Solution algorithms

Regarding evolutionary algorithms to solve large-sized instances of the hub location problems, a tabu search method by Klincewicz (1992) and Skorin-Kapov and Skorin-Kapov (1994) and a simulated annealing heuristic by Ernst and Krishnamoorthy (1999) were proposed for *p*HMPs. Also for *p*HCPs, a tabu search based heuristic was presented by Pamuk and Sepil (2001). Chen (2007) proposed two approaches to determine the upper bound for the number of hubs along with a hybrid heuristic based on the simulated annealing method, tabu list, and improvement procedures to solve the proposed uncapacitated single allocation HLP (USAHLP). The computational results demonstrated that the proposed hybrid heuristic outperforms a genetic algorithm and a simulated annealing method in solving USAHLP. Cunha and Silva (2007) proposed a heuristic based on genetic algorithm to solve the problem of configuring hub-and-spoke networks for trucking companies that operate less-than-truckload (LTL) services in Brazil. Their genetic algorithm incorporated an efficient local improvement procedure that was applied to each generated individual of the population.

Randall (2008) applied ant colony optimization in order to solve a capacitated single allocation HLP. Calik et al. (2009) presented a tabu search heuristic for the hub covering problem. In addition, Qu and Weng (2009) developed a path relinking approach for hub maximal covering location problem. Silva and Cunha (2009) proposed three variants of a simple and efficient multi-start tabu search heuristic, as well as a two stage integrated tabu search heuristic. Yang et al. (2013a) developed a hybrid algorithm by incorporating genetic algorithm and local search (GALS) to solve the parametric mixed-integer programming model of a *p*HCP. In our designed GALS, the GA was used to perform global search, while LS strategy is applied to each generated individual (or chromosome) of the population. Yang et al. (2013b) designed an improved hybrid particle swarm optimization (PSO) algorithm by combining PSO with genetic operators and local search (LS) to update and improve particles. They also evaluated the improved hybrid PSO algorithm against the other two solution methods, genetic algorithm (GA) and PSO without LS components.

Saboury et al. (2013) proposed two hybrid heuristics algorithms to solve the problem, namely SA_{VNS} and TS_{VNS} which incorporated a variable neighborhood search (VNS) algorithm into the framework of simulated annealing (SA) and tabu search (TS). Their proposed algorithms were able to easily obtain the optimal solutions for 24 small instances existing in the literature in addition to efficiently solving new generated medium and large instances. Bashiri et al. (2013) proposed a hybrid approach based on genetic algorithm and fuzzy VIKOR to solve a new fuzzy pHCP. Mohammadi et al. (2013) proposed a hybrid multi-objective imperialist competitive algorithm (MOICA) incorporating with crossover operator of genetic algorithm to solve a hub covering location problem. They also provided a new continuous solution representation for the HLP. In a similar work, Mohammadi et al. (2014a) developed two different meta-heuristic algorithms, namely ICA and SA, to solve a multi-objective sustainable HLP.

Peiró et al. (2014) proposed a heuristic for the uncapacitated *r*-allocation *p*HMP. Similarly, Martí et al. (2015) presented a scatter search implementation for an NP-hard variant of the classic *p*HMP. Specifically, they tackled the uncapacitated *r*-allocation *p*HMP that consisted of minimizing the cost of transporting the traffics between nodes of a network through special facilities that acted as transshipment points.

Table 1
Review of related works.

Author	Year	Model formulation								Solution algorithm				
		Objective			Uncertainty					Objective		Mechanism		
		Center	Median	Covering	Flow	Cost	Time	Hub operation	Mixed	Single	Multi	Heuristic	Meta-heuristic	Hybrid
Kara and Tansel	2000	√								√				
Ernst et al.	2000	√								√				
Kara and Tansel	2001	√								√				
Wagner	2004	√								√				
Yaman et al.	2007	√								√				
Campbell et al.	2007	√								√				
Chen	2007		√							√		√		√
Cunha and Silva	2007		√							√			√	
Sim et al.	2009	√					√			√		√		
Ghodsi et al.	2010			√						√			√	
Mohammadi et al.	2010			√						√			√	√
Yaman	2011	√								√				
Contreras et al.	2011		√		√	√				√		√		
Mohammadi et al.	2011a			√	√					√			√	
Mohammadi et al.	2011b	√		√							√		√	
Parvaresh et al.	2012		√					√		√			√	
Yang et al.	2013a	√					√			√			√	√
Yang et al.	2013b	√					√			√			√	√
Mohammadi et al.	2013	√	√	√		√					√		√	
Parvaresh et al.	2013		√					√		√			√	
Saboury et al.	2013		√							√		√		√
Bashiri et al.	2013	√				√	√			√			√	
Yang et al.	2014	√					√			√			√	√
Mohammadi et al.	2014a		√	√	√	√	√		√		√		√	
Peiró et al.	2014		√							√		√		
Adibi and Razmi	2015		√		√	√				√				
Martí et al.	2015		√							√			√	
This paper		√	√		√	√	√	√	√		√	√	√	√

For many optimization problems, it has been demonstrated that it is essential to involve some improvement strategies into a heuristic method to yield effective optimization tools (see Maric et al., 2013; Goh et al., 2009; Ribeiro and Hansen, 2002; Ishibuchi et al., 2009; Yang and Liu, 2014; Asl-Najafi et al., 2015). In the literature, there are several heuristic and meta-heuristic algorithms in different fields such as genetic algorithm for inspection planning (Mohammadi et al., 2015), teaching–learning-based optimization algorithm for realistic flowshop rescheduling problems (Li et al., 2015), GRASP algorithm for humanitarian relief transportation problem (Talebian-Sharif and Salari, 2015), ant colony optimization in solving JIT scheduling problem (Khalouli et al., 2010), multi-start path relinking algorithm in vehicle routing problem (Tan et al., 2001, Lacomme et al., 2015), tabu search algorithm for the maximum independent set problem (Jin and Hao, 2015), variable neighborhood search in flowshop scheduling problem (Giannopoulos et al., 2012), simulated annealing and particle swarm optimization for track train timetabling and HLP (Jamili et al., 2012; Sedehzadeh et al., 2014), swarm intelligence in green logistics (Zhang et al., 2015), imperialist competitive algorithm in healthcare network design, HLP, redundancy allocation problems and reverse logistics (Ghodsi et al., 2010; Mohammadi et al., 2010, 2011a, 2011b, 2013, 2014b; Azizmohammadi et al., 2013; Zahiri et al., 2014a, 2014b; Vahdani and Mohammadi, 2015; Sedehzadeh et al., 2015), and invasive weed optimization algorithm in hub location problem (Niakan et al., 2014).

In this paper, we proposed a new multi-objective meta-heuristic algorithm, i.e. bi-objective invasive weed optimization algorithm (BOIWOA) which is originally developed by Mehrabian and Lucas (2006) and have been successfully applied to a number of problems in engineering interests. In addition, this algorithm is hybridized with game theory to obtain near optimal Pareto solutions.

Some most related works in terms of model formulation and solution algorithms have been listed in Table 1. According to Table 1, the contributions of this paper are elaborated as follows:

- Developing a new bi-objective center-median HLP.
- Considering uncertainties in costs, transportation time and hub operations.
- Modeling the hubs as queuing systems.
- Considering disruption (breakdown) at hub's queuing system.
- Considering mixed uncertainty in flows (i.e., stochastic flow with fuzzy levels).
- Developing a hybrid meta-heuristic algorithm based on invasive weed optimization (IWO) algorithm, variable neighborhood search (VNS) algorithm, and game theory technique.
- Considering game-based rewarding procedure to evaluate the solutions of the algorithm.

3. Modeling framework and mathematical formulation

This section presents a modeling framework for the design of BSpHCMP. The proposed BSpHCMP is represented by a graph in which nodes represent demand points and arcs represent different modes of transportation between the nodes. A p number of nodes are located as hubs and the remained ones (spokes) are allocated to only one of the located hubs. The set of arcs can include of different modes of transportation, such as road, rail and air. In this modeling framework, a shipment between an origin node i and a destination node j can be traveled either through a pair of hubs (k, l) or through a single hub. The travel of shipments between each pair of O–D nodes consists of three parts. The first part is consolidation when shipments from the origin node i are consolidated at the origin hub k. The second part is transferring when shipments are transferred between the origin hub k and destination hub l. The third part is distribution when shipments are distributed to the destination node j. In the case of a single hub shipment ($k=l$), the travel consists of only two parts: consolidation and distribution.

The transferring part is the line haul where shipments are transferred in larger quantities and with higher frequencies. Using different transportation modes for the hub-to-hub links is an aid to define competitive and efficient delivery requirement. The choice of a specific transportation mode m for hub-to-hub travel is represented by the binary variable X_{iklj}^{m}. Although a specific mode may be selected for the hub-to-hub transfer of shipments for a specific O–D pair, other O–D pairs may ship through the same hubs using a different mode of transportation. It should be noted that for each O–D pair, only one mode is selected. Also note that shipments are consolidated and distributed using transportation mode $m=1$.

Although most of papers have taken into account only travel time on the connection links for the path $i\rightarrow k\rightarrow l\rightarrow j$, it is noteworthy that arrival shipments cannot be quickly transferred and need to be processed. For example, arrival shipments must be unloaded, sorted, packaged and loaded before transferring to their destinations. Hence, shipments must spend time at the hub(s). In addition, due to some recourse limitations at the hub(s), all shipments cannot be processed at the same time and must wait to be processed. Therefore, the total travel time is the sum of transportation time on the links and the time spent at the hub(s). The recourse limitation at the hub(s) causes shipment delays if the average shipment arrival rate gets closer to the processing rate at these operations. These delays increase as more and more shipments are consolidated in the hub to take advantage of the economies of scale. As these delays significantly affect the delivery time requirement, spent time at the hubs should be calculated and taken into account.

Since the hub operations are affected by uncertainties, the waiting time at hubs are also uncertain. In other words, the waiting time of the flows directly depends on the disruption at hubs. Section 3.1 will discuss how to calculate and analyze hub waiting time under disruption.

3.1. Hub waiting time under disruption

In this paper, arrival shipments to a hub should wait to be processed and then be transferred to their destinations. This waiting time contribute a high value to the transportation time between each O–D pair (Mohammadi et al., 2011a, 2014a) and significantly affect the center objective. Therefore, in order to establish a delivery requirement and reach center objective, the waiting time at hubs needs to be considered. Since the flow between each pair of O–D nodes are not deterministic, a queuing approach is considered to analyze the waiting time. In this way, accounting for uncertain amount of flows and calculation of waiting times through queue theory makes the proposed model more attractive in practice.

A number of papers have already justified the use of queuing approach to analyze the operation of hubs by using both empirical and simulation procedures (see e.g. van Woensel and Vandaele, 2006; van Woensel et al., 2006; van Woensel and Cruz, 2009; Mohammadi et al., 2011a, 2014a); where validation results proved that in order to calculate the waiting time of flows in the hub, queuing models can adequately be applied. Based on these validations, a Poisson process was suggested as a good representation of arrival rates where there is always a variation around scheduled arrival times (Peterson et al., 1995). Generally, Poisson process can be considered in other service systems where arrival rates and capacity levels vary significantly over time.

Despite of literature and in order to model the uncertainty in the hub operations, we consider that the queue system at each hub is stochastically disrupted and again retrieved with specific rates. In case of M/M/c queue system, service times are assumed to be independent and identically distributed exponentials with rate μ. During disruptions, the number of operational servers decreases from c to c' and the service rates of all servers drop from μ to $\mu'\geq 0$. As soon as the hub is retrieved, the number of working servers and their service rates are restored to c and μ, respectively. We assume that disruptions arrive according to a Poisson process with rate v, and the retrieve times are *i.i.d.* exponentials with rate r. The flow arrivals are in accordance with a homogeneous Poisson process with intensity λ.

Within the context of traditional queuing theory, the inter-arrival times and service times are required to follow certain probability distributions. However, in many practical applications, the statistical information may be obtained subjectively, i.e., the arrival pattern and service pattern are more suitably described by linguistic terms such as fast, slow (or) moderate, rather than by probability distributions. Therefore, fuzzy queues are much more realistic than the commonly used crisp queues (Li and Lee, 1989; Negi and Lee, 1992; Prade, 1980). Accordingly, in this paper, we develop a method that is able to provide fuzzy performance measures for a queue with fuzzified exponential arrival rate and service rate. Therefore, a FM/FM/1 queue system is considered for all located hubs, in which the fuzzy mean waiting time, at a hub, when $\mu'=0$, can be calculated as Eq. (1) (Baykal-Gursoy et al., 2009). In addition, a trapezoidal fuzzy numbering is considered for the shipment flow, arrival and service rates. The following notation is used to present the queuing system:

Sets:

$i,j\in\{1,2,\ldots,I\}$	Set of spokes.
$k,l\in H;H\in I$	Set of hubs.

Parameters:

$\tilde{s}_{ij}$	Fuzzy shipment between spoke i and spoke j.
$\tilde{\mu}_k$	Fuzzy service time at hub k.
v_k	Disruption rate at hub k.
r_k	Retrieve time rate at hub k.

Variables:

X_{ik}	1 if spoke i is allocated to hub k; 0 otherwise. If $X_{kk}=1$, this indicates that node k has been located as a hub.
$\tilde{\lambda}_k$	Arrival rate of shipments to the hub k.
$\tilde{W}_k$	Mean waiting time at hub k.

$$\tilde{W}_k=\frac{(r_k+v_k)^2+\tilde{\mu}_k v_k}{(r_k+v_k)\left(r_k\left(\tilde{\mu}_k-\tilde{\lambda}_k\right)-\tilde{\lambda}_k v_k\right)}\forall k \quad (1)$$

3.2. Mathematical formulation: BMINLP

According to the modeling framework and the explanations in Section 3.1, this section provides a bi-objective mixed-integer nonlinear programming (BMINLP) formulation for the proposed BS*p*HCMP in order to investigate the effect of delivery service requirement and different sources of uncertainties on the configuration of the hub-and-spoke network and to analyze the tradeoff between center and median objectives. The center objective is reached by minimizing the maximum travel time between each O–D pair that is the sum of transportation time on the links and waiting time at the hubs. The transportation time and waiting time are represented by fuzzy and fuzzy-stochastic numbering, respectively. The median objective is also obtained by minimizing the sum of the total transportation cost and fixed cost of locating the hubs. We present a bi-objective mathematical model that simultaneously determines the location of p hubs, allocates spokes to the located hubs, and assigns different transportation modes to the hub-to-hub links. Necessary notations are first listed, and the proposed BMINLP model is then presented as follows:

Sets:

$m \in M$	Set of transportation modes.

Parameters:

$\tilde{c}_{kl}^{m}$	Unit transportation cost over the link from node k to node l.
$\tilde{f}_k$	Fixed cost of locating a hub at node k.
$\tilde{t}_{kl}^{m}$	Transportation time on the link from node k to node l.
σ_{kl}	Hub-to-hub cost discount factor.
δ_{kl}	Hub-to-hub time discount factor.

Variables:

X_{iklj}^{m}	1 if there exists a path in the network from spoke i to spoke j through hub k first then hub l by hub-to-hub transportation mode m.
β	Travel time of the longest path in the network.

The BMINLP model is proposed as follows:

$$\min \sum_i \sum_j \sum_k \sum_l \tilde{s}_{ij}\left(\tilde{c}_{ik}^{1} + \sigma_{kl}\tilde{c}_{kl}^{m} + \tilde{c}_{lj}^{1}\right) X_{iklj}^{m} + \sum_k f_k X_{kk} \tag{2}$$

$$\min \beta \tag{3}$$

s.t.

$$\left(\tilde{t}_{ik}^{1} + \tilde{W}_k + \delta_{kl}\tilde{t}_{kl}^{m} + \tilde{W}_l + \tilde{t}_{lj}^{1}\right) X_{iklj}^{m} \leq \beta \qquad \forall i,k,l,j \tag{4}$$

$$\sum_k X_{kk} = p \tag{5}$$

$$X_{ik} \leq X_{kk} \qquad \forall i,k \tag{6}$$

$$\sum_k X_{ik} = 1 \qquad \forall i \tag{7}$$

$$\sum_m X_{iklj}^{m} \geq X_{ik} + X_{lj} - 1 \qquad \forall i,k,l,j \tag{8}$$

$$X_{ik}, X_{iklj}^{m} \in \{0,1\} \qquad \forall i,k,l,j \tag{9}$$

$$\beta \geq 0 \tag{10}$$

Objective function (2) is the median objective that minimizes the sum of the total transportation cost and fixed cost of locating the hubs. Objective function (3) and constraint (4) address center objective by minimizing the maximum travel time between each O–D pair in the network. The value of β is set and offered as the delivery service requirement. Constraint (5) ensures that exactly p hubs must be located in the network, while constraints (6) state that a non-hub node i can only be assigned to a located hub at node k. Constraint (7) imposes the single allocation assumption. Constraint (8) ensures valid transportation mode assignment between each pair of located hubs. Finally, constraints (9) and (10) are domain constraints.

4. Proposed solution approach

The proposed model in Section 3.2 is a bi-objective fuzzy possibilistic BMINLP (FBMINLP) model. To solve this model, a two-phase approach is developed in this section. The first phase converts the proposed FBMINLP model into an equivalent auxiliary crisp model by applying an efficient solution methodology resulted from fuzzy possibilistic programming. Second, a new multi-objective meta-heuristic algorithm based on fuzzy invasive weed optimization (FIWO) algorithm, variable neighborhood (VNS) algorithm and game theory is developed to find near optimal Pareto solutions of the proposed BSpHCMP.

4.1. Fuzzy possibilistic programming

In order to convert the proposed FBMINLP model into an equivalent auxiliary crisp model, an efficient possibilistic method proposed by Jimenez et al. (2007) is utilized as follows.

By assuming that $\tilde{A} = [a^1, a^2, a^3, a^4]$ is a trapezoidal fuzzy number, the membership function of A is provided as Eq. (11).

$$6_a(x) = \begin{cases} f_a(x) = \frac{x-a^1}{a^2-a^1}, & a^1 \leq x \leq a^2 \\ 1, & a^2 \leq a \leq a^3 \\ g_a(x) = \frac{a^4-a}{a^4-a^3}, & a^3 \leq x \leq a^4 \\ 0 & a^1 > x, a^4 < x \end{cases} \tag{11}$$

The following fuzzy mathematical programming model (12) is considered, in which, all parameters are defined as trapezoidal fuzzy numbers:

$$\begin{aligned} &\min Z = \tilde{C}^T X \\ &\text{s.t.} \\ &A_i X \geq B_i \quad i = 1,\ldots,l \\ &A_i Y = B_i \quad i = l+1,\ldots,I \\ &X, Y \geq 0 \text{ (or integer)} \end{aligned} \tag{12}$$

An equivalent crisp parametric model (i.e., model (13)) can be found from the model (12) (Jimenez et al., 2007).

$$\begin{aligned} &\min z = EV\left(\tilde{C}\right)^T X \\ &\text{s.t.} \\ &\left[(1-\alpha)E_2^{A_i} + \alpha E_1^{A_i}\right] X \geq \alpha E_2^{B_i} + (1-\alpha)E_1^{B_i} \quad i = 1,\ldots,l \\ &\left[\left(1-\frac{\alpha}{2}\right)E_2^{A_i} + \frac{\alpha}{2}E_1^{A_i}\right] Y \geq \frac{\alpha}{2}E_2^{B_i} + \left(1-\frac{\alpha}{2}\right)E_1^{B_i} \quad i = l+1,\ldots,I \\ &\left[\frac{\alpha}{2}E_2^{A_i} + \left(1-\frac{\alpha}{2}\right)E_1^{A_i}\right] Y \leq \left(1-\frac{\alpha}{2}\right)E_2^{B_i} + \frac{\alpha}{2}E_1^{B_i} \quad i = l+1,\ldots,I \\ &X, Y \geq 0 \text{ (or integer)} \end{aligned} \tag{13}$$

where

$$EV(\tilde{C}) = \frac{C^1 + C^2 + C^3 + C^4}{4}, E_1^A = \frac{A^1 + A^2}{2}, E_2^A = \frac{A^3 + A^4}{2},$$
$$E_1^B = \frac{B^1 + B^2}{2} \quad \text{and} \quad E_2^B = \frac{B^3 + B^4}{2}$$

For more details on the method, the interested readers are referred to Jimenez et al. (2007). According to the above descriptions, the equivalent auxiliary crisp model can be formulated as follows:

$$\begin{aligned} \min \sum_i \sum_j \sum_k \sum_l & \left(\frac{s_{ij}^{\ 1} + s_{ij}^{\ 2} + s_{ij}^{\ 3} + s_{ij}^{\ 4}}{4}\right)\left\{\left(\frac{c_{ik}^{1}1 + c_{ik}^{1}2 + c_{ik}^{1}3 + c_{ik}^{1}4}{4}\right)\right. \\ & \left. + \sigma\left(\frac{c_{kl}^{m}1 + c_{kl}^{m}2 + c_{kl}^{m}3 + c_{kl}^{m}4}{4}\right) + \left(\frac{c_{lj}^{1}1 + c_{lj}^{1}2 + c_{lj}^{1}3 + c_{lj}^{1}4}{4}\right)\right\} X_{iklj}^{m} \\ & + \sum_k \left(\frac{f_k^{\ 1} + f_k^{\ 2} + f_k^{\ 3} + f_k^{\ 4}}{4}\right) X_{kk} \end{aligned} \tag{14}$$

$$\min \beta \tag{15}$$

s.t.

$$\Bigg(\Big[\alpha \quad \frac{t_{ik}^{1}3+t_{ik}^{1}4}{2}\Big) + (1-\alpha) \quad \frac{t_{ik}^{1}1+t_{ik}^{1}2}{2}\Big)\Big] + \Big[\alpha E_2^{W_k} + (1-\alpha)E_1^{W_k}\Big]$$

$$+\delta\Big[\alpha\Big(\frac{t_{kl}^{m}3+t_{kl}^{m}4}{2}\Big) + (1-\alpha)\Big(\frac{t_{kl}^{m}1+t_{kl}^{m}2}{2}\Big)\Big] \qquad \forall i,k,l,j$$

$$+\Big[\alpha E_2^{W_l} + (1-\alpha)E_1^{W_l}\Big]$$

$$+\Big[\alpha\Big(\frac{t_{lj}^{1}3+t_{lj}^{1}4}{2}\Big) + (1-\alpha) \quad \frac{t_{lj}^{1}1+t_{lj}^{1}2}{2}\Big)\Big]\Bigg) X_{iklj}^{m} \le \beta \tag{16}$$

$$E_1^{W_k} = \frac{(r_k+v_k)^2 + [\mu_k^4 - (\mu_k^4-\mu_k^3)\alpha]v_k}{(r_k+v_k)\Big(r_k\Big([\mu_k^4-(\mu_k^4-\mu_k^3)\alpha] - \Big[\Big(\lambda_k^2-\lambda_k^1\Big)\alpha+\lambda_k^1\Big]\Big) - \Big[\Big(\lambda_k^2-\lambda_k^1\Big)\alpha+\lambda_k^1\Big]v_k\Big)} \forall k \tag{17}$$

$$E_2^{W_k} = \frac{(r_k+v_k)^2 + [(\mu_k^2-\mu_k^1)\alpha+\mu_k^1]v_k}{(r_k+v_k)\Big(r_k\Big([(\mu_k^2-\mu_k^1)\alpha+\mu_k^1] - \Big[\lambda_k^4-\alpha\Big(\lambda_k^4-\lambda_k^3\Big)\Big]\Big) - \Big[\lambda_k^4-\alpha\Big(\lambda_k^4-\lambda_k^3\Big)\Big]v_k\Big)} \forall k \tag{18}$$

$$\lambda_k^1 = \sum_{i=1}^{N}\sum_{j=1}^{N}\Big(s_{ij}^1+s_{ji}^1\Big)X_{ik}\,\forall k \tag{19}$$

$$\lambda_k^2 = \sum_{i=1}^{N}\sum_{j=1}^{N}\Big(s_{ij}^2+s_{ji}^2\Big)X_{ik}\,\forall k \tag{20}$$

$$\lambda_k^3 = \sum_{i=1}^{N}\sum_{j=1}^{N}\Big(s_{ij}^3+s_{ji}^3\Big)X_{ik}\,\forall k \tag{21}$$

$$\lambda_k^4 = \sum_{i=1}^{N}\sum_{j=1}^{N}\Big(s_{ij}^4+s_{ji}^4\Big)X_{ik}\,\forall k \tag{22}$$

Constraints (5)–(10) where the value of $E_2^{W_k}$, is obtained by α-cut maximum and minimum value of λ_k and μ_k, respectively. Also, the value of $E_1^{W_k}$, is obtained by α-cut minimum and maximum value of λ_k and μ_k, respectively.

4.2. Hybrid solution algorithm

There are various solution algorithms in the literature to solve wide range of engineering problems (Alfi et al., 2013; Shokri-Ghaleh and Alfi, 2014; Khooban et al., 2013; Darabi et al., 2012; Alfi and Modares, 2011; Alfi and Fateh, 2011; Vahdani and Mohammadi, 2015; and lots of other papers). Among them, in this section, we proposed a new multi-objective meta-heuristic algorithm based on fuzzy invasive weed optimization (FIWO) algorithm, variable neighborhood (VNS) algorithm and game theory, so called GVIWO algorithm. The IWO algorithm was originally developed by Mehrabian and Lucas, (2006) and has been successfully applied to a number of problems of engineering domains. First of all, a brief definition of multi-objective problems (MOPs) is provided and Pareto solutions are described. Next, the game theory approach is combined with MOP to introduce a new variant of collaborative meta-heuristic algorithm.

4.2.1. Multi-objective problems: MOPs

A mathematical programming problem, optimizing multiple conflicting objective functions simultaneously under given constraints, is called an MOP. Unlike single-objective problems (SOP), in MOPs, there is not a solution that is the best or the global optimum with respect to all objectives. Hence, in these problems, a set of solutions depending on non-dominance criterion are found, which are called Pareto solutions. In the following, we describe summary of basic definitions of Pareto optimality and fuzzy domination (Kundu et al., 2011). Let $c_i = (c_{i1}, c_{i2}, \ldots, c_{in})$, $i = 1, \ldots, k$ denote an n dimensional coefficient row vector of the ith objective function. Hence, the MOP is represented as model (23):

$$\min Z(x) = Cx$$
$$\text{s.t.}$$
$$Ax \le B; x \ge 0 \tag{23}$$

where C denotes a coefficient matrix of the objective functions.

Definition 1. (Pareto-optimality): consider $\vec{x} \in X$ as a sample solution vector.

The solution vector $\vec{x}$ is determined to be non-dominated based on set $X' \subseteq X$if and only if there is no solution in X'which can dominate $\vec{x}$.

(1) The solution vector $\vec{x}$ is said to be Pareto-optimal if $\vec{x}$ is non-dominated in the whole solution space X.

4.2.2. Game theory in MOPs

In order to apply game theory to deal with the MOPs, a matching between MOP and game theory should be designed. Comparing the MOPs with the game theory, the MOP can be described as a game as follows: k objectives in MOP can be described as the k players in game theory; solution space (X) in MOP can be described as the decision space S in game theory; $f_i(x)$ in MOP (i.e., objective function i) can be described as the utility function u_i in game theory; and finally, constraints in MOP can be described as the constraints in game theory. By defining the matching of φ_i: $X \to S_i$ and φ_i:$f_i \to u_i$, as the decision strategies space and set, while $\bigcup_{i=1}^{k} S_i = X$; the game theory model of MOP can be defined as set (24).

$$G = \{S;U\} = \{S_1, S_2, \ldots, S_k; u_1, u_2, \ldots, u_k\} \tag{24}$$

4.2.3. Evolutionary stable strategy

The initial contribution of game theory is the concept of evolutionary stable strategy (ESS) introduced by a well-known biologist called Maynard Smith. The ESS is adopted by all members of a population and cannot be defeated by any invasive mutant strategy under the influence of natural selection (Maynard-Smith, 1982). The ESS is an improvement of the Nash equilibrium that excludes the conventional assumption of agent rationality. Maynard-Smith (1982) show that a game-theoretic equilibrium can be achieved through a process of Darwinian selection (Ficici and Pollack, 2001).

The concept of ESS shows that such equilibrium strategies can be considered as solutions for the multi-objective problems (i.e., the ESS is a non-dominated solution of a game, and multi-objective problems can be embodied by the game). Evolutionary games consist of several factors and need game players, strategy sets, and game matrices (Hart et al., 2008). Each game player having a conflict objective function selects his/her own strategy and game based on the game matrix of every other player. Therefore, the fitness domain is changed by the strategy of opponent player strategy for every game. Consequently, it is certain that the evolutionary game can be implemented by a co-evolutionary algorithm (EA) and embodied by a co-EA based on game theory. For this approach, each population is considered as game players (individuals). The fitness of individuals in the population is assessed from each objective function and rewarded from the game matrix (the fitness function).

4.2.4. Game-based EA and rewarding procedure

In this section, co-EA is briefly described based on the ESS concept designed for obtaining the Pareto front of MOPs. The game can be defined as a tool for optimizing the objective functions of MOPs during an evolutionary algorithm. The competition for this

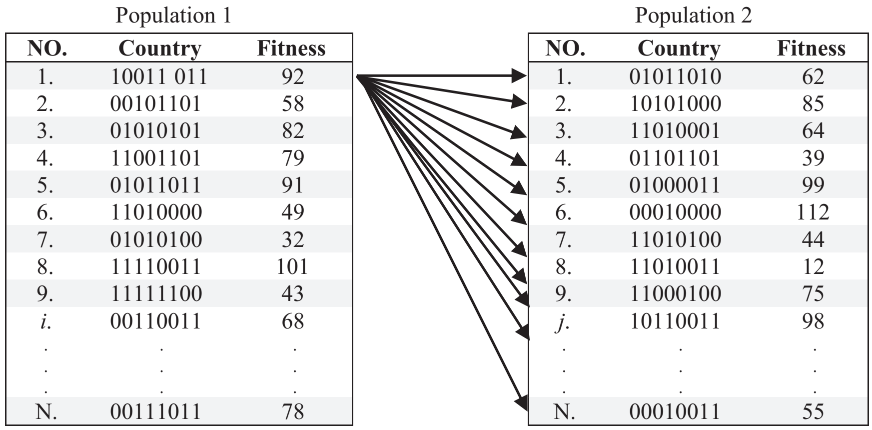


Fig. 1. Population architecture for a game model-based evolutionary algorithm.

game-model is the comparison of the optimized result of the objective function for each weed in the IWO algorithm. A victory in this game is determined based on the degree of how many each solution dominates the other solutions. All the weeds in the population are rewarded as a result of winning or losing the game. Reward can be determined by the percentage of victories during the game. In order to design an evolutionary algorithm, we first establish game players with randomly-generated populations. All the weeds in each population take "fitness" on the basis of the rewarded values. During the game, each weed in the first population plays the game with other weeds in the remaining populations and all weeds are rewarded by a specified value as shown in Fig. 1. In this paper, we defined the reward value and the fitness of each weed as a result of Eqs. (25) and (26), respectively.

$$Reward(i,j) = e^{\left(w_a \frac{f_a(x_j) - f_a(x_i)}{\max f_a} + w_b \frac{f_b(x_j) - f_b(x_i)}{\max f_b}\right)} \tag{25}$$

$$Fitness(i) = \frac{\sum_{j=1}^{N} Reward(i,j)}{N} \tag{26}$$

where f_a, f_b and N are the value of objective functions a and b, and the size of population, respectively. Also, x_i and x_j are the weed i in the first population and weed j in the second population, respectively. In addition, w_a and w_b are the importance coefficients of the objective functions, while $w_a + w_b = 1$.

4.2.5. Invasive weed optimization (IWO)

IWO is a stochastic population-based evolutionary algorithm that mimics the colonizing behavior of weeds. This algorithm was first applied to optimization problems by Mehrabian and Lucas (2006). This evolutionary algorithm is a bio-inspired numerical optimization procedure that simulates natural behavior of weeds in colonizing to find suitable places for growth and reproduction. There are four steps of the algorithm as described below:

(1) *Population initialization*: A population of initial solutions (weeds) is randomly generated over the problem space.
(2) *Reproduction*: Seeds are produced by the members of the population based on their relative fitness in the population.
(3) *Random distribution*: The produced seeds are randomly distributed over the D-dimensional solution space following normal distribution with mean equal to zero and a positive standard deviation. The standard deviation (sd) for the normal distribution variable at each iteration ($iter$) is calculated adaptively according to Eq. (27):

$$sd_{iter} = \left(\frac{iter_{max} - iter}{iter_{max}}\right)^{pow} (sd_{max} - sd_{min}) + sd_{min} \tag{27}$$

where sd_{max} and sd_{min} are the maximum and the minimum sd and pow is the nonlinear modulation index. In addition, $iter_{max}$ is the maximum allow iteration for the algorithm.
(4) *Competitive exclusion*: Once the population size reaches its maximum after a number of iterations, an elimination procedure should be employed. In this procedure, the seeds and their parents are ranked together and those with better fitness survive and proceed with the algorithm.

The pseudo code of IWO is shown as Fig. 2.

In the following, we describe a summary of basic definitions of fuzzy domination to present better description of the MOIWO (Kundu et al., 2011).

Definition 2. (Fuzzy *k*-Dominance by a Solution): The mapping $\mu_k^{dom} = f_k(X) \to [0,1]$, where $k \in \{1,2,..,n\}$ defines a given uniform non-decreasing membership function. A solution $\vec{p} \in X$ is determined as k-dominance solution $\vec{p} \in X$, if $f_k(\vec{x}) < f_k(\vec{p})$. This relationship is represented as $\vec{x} \prec_k^F \vec{p}$. If $\vec{x} \prec_k^F \vec{p}$, the degree of fuzzy k-dominance is equal to $\mu_k^{dom}\left(f_k(\vec{p}) - f_k(\vec{x})\right) \equiv \mu_k^{dom}\left(\vec{x} \prec_k^F \vec{p}\right)$.

Definition 3. (Fuzzy Dominance by a solution): Solution $\vec{x} \in \Psi$ is determined as fuzzy dominate solution $\vec{p} \in \Psi$ if and only if $\forall k \in \{1,2,..,n\}$, $\vec{x} \prec_k^F \vec{p}$. If $\vec{x} \prec_k^F \vec{p}$, the degree of fuzzy dominance $\mu_k^{dom}\left(\vec{x} \prec_k^F \vec{p}\right)$ is calculated by computing the intersection of the fuzzy relationships $\vec{x} \prec_k^F \vec{p}$ for each$k$. Also, the fuzzy intersection operation, denoted with $''\bigcap''$, is performed using a family of functions called t-norms and is calculated as (28)

$$\mu^{dom}\left(\vec{x} \prec_k^F \vec{p}\right) = \bigcap_{k=1}^{n} \mu_k^{dom}\left(\vec{x} \prec_k^F \vec{p}\right) \tag{28}$$

Definition 4. (Fuzzy Dominance in a Population): Let X be a population of solutions. A solution $\vec{p} \in P$ is defined to be fuzzy dominated X if it is fuzzy dominated by any other solution $\vec{x} \in P$. In this case, the degree of fuzzy dominance can be calculated by performing a union operation $\} \cup \}$ over every possible $\mu^{dom}\left(\vec{x} \prec_k^F \vec{p}\right)$ implemented with t-conforms. Hence, the degree of fuzzy dominance of a solution $\vec{p} \in P$ in the set X is given by (29)

$$\mu^{dom}\left(X \prec^F \vec{p}\right) = \bigcup_{\vec{x} \in X} \mu^{dom}\left(\vec{x} \prec^F \vec{p}\right) \tag{29}$$

The Pseudo code of calculating fuzzy dominance is proposed in Fig. 3 (Kundu et al., 2011).

4.2.6. Multi-objective invasive weed optimization (MOIWO)

In the first step of the proposed MOIWO algorithm, a population of random weeds is generated with size Pop_{Size}. The weeds are then ranked by applying the fuzzy dominance sorting as Fig. 4.

```
Set initial parameters (NP_init, S_max, S_min, iter_max)
Randomly generate NP_init number of weeds over a small region of the search space
iter = 0
While (termination=False) Do
        Evaluate the fitness of the population
        Rank the population
        Produce S seeds from each weed S ∈ [S_min, S_max]:
                S_i = S_min + (current population − rank_i) × (S_max − S_min)
        Calculate standard deviation as (27)
        iter = iter + 1
        If iter > iter_max Then
                termination=True
        EndIf
EndWhile
```

Fig. 2. Pseudo code of IWO.

```
For k = 1:n            // Compute Fuzzy dominance of each solution in the population
    μ(k) = 0
    For i = 1:n
        μ(i) = 1
        For j = 1:m            // Compute Fuzzy j-dominance of each solution
            IF y_j(x_i) − y_j(x_k) < 0
                μ_i^dom(x_k ≺_i^F x_i) = 0
            ElseIF y_j(x_i) − y_j(x_k) < p_j
                μ_i^dom(x_k ≺_i^F x_i) = (y_j(x_i) − y_j(x_k))/p_j
            Else
                μ_i^dom(x_k ≺_i^F x_i) = 1
                μ^dom(x_k ≺^F x_i) = μ^dom(x_k ≺^F x_i) * μ_i^dom(x_k ≺_i^F x_i)
            End
        μ(k) = μ(k) + μ^dom(x_k ≺^F x_i) − μ(k) * μ^dom(x_k ≺^F x_i)
        End
    End
End
```

Fig. 3. The pseudo code of calculating fuzzy dominance.

Afterward, each weed generates a number of seeds based on its rank; so that the weeds with the higher rank generate higher number of seeds. Next, a neighborhood search structure (NSS) procedure is performed to distribute the seeds around the parent weeds. The reproduced and distributed seeds are added to the weed population. Indeed, the NSS is a procedure to achieve new solutions by slightly changing the current solution. In this paper, the NSS is applied on the weeds at all iterations by utilizing variable neighborhood search-VNS (Mladenovic and Hansen, 1997).

In recent years, VNS has increasingly gained lots of attention and a large number of successful applications have been reported (Hansen and Mladenovic, 2001). Using two or more neighborhoods, instead of one, in its structure has differentiated the VNS from the most other local search heuristics. Particularly, it is based on the principle of systematic change of neighborhood during the search (Behnamian et al., 2009). In addition, to avoid costing too much computational time, the best number of neighborhoods is often three (Vahdani and Zandieh, 2010; Mohammadi et al., 2013), which is followed by our algorithm. The three neighborhoods employed in our algorithm are defined below:

(I) Swap: In the swap operator, places of two random selected bits are exchanged.
(II) Reversion: In the reversion operator, a random part of a solution is selected and its permutation is reversed.
(III) Inversion: In the inversion operator, one bit is chosen randomly and its value is replaced with new random value.

It should be noted that the VNS is performed on a percent of population (P_E). Furthermore, the number of outer and inner iterations of the proposed VNS are equal to $ItVNS_{max}$ and n_{Repeat}, respectively. This procedure continues until the population size exceeds a predefined upper limit (i.e., Pop_{max}). The population is again ranked in order to select best Pop_{max} weeds. This procedure continues until the stopping criterion is met. The number of function calls-NFC (Črepinšek et al., 2012) has been considered as the stopping criterion in the proposed GVIWO. Finally, the Pseudo code of the proposed GVIWO is provided as Fig. 4.

4.2.7. *Exploration and exploitation*

Any meta-heuristic algorithm must have two capabilities, exploration and exploitation, in order to better search the space and find reasonable solutions (Mousavi and Alfi, 2015; Arab and Alfi, 2015). Exploration refers to the ability of discovering unknown regions in the solution space to find the global optimum and exploitation indicates the process of searching those regions of the solution space within the neighborhood of previously visited solutions (Liu et al., 2009, 2013; Crepinsek et al., 2011, 2013). Interested readers are referred to a comprehensive study of exploration and exploitation prepared by Črepinšek et al. (2013) and references therein.

In the following of this section, we explain how the proposed GVIWO provides efficient exploration and exploitation through searching the solution space. In general, the main motivation of the proposed GVIWO algorithm is to combine the ability of IWO in global search (Hajimirsadeghi and Lucas, 2009) and the power of VNS with efficient operators in local search (Fontaine et al., 2013).

In the proposed GVIWO, *Reproduction* and *Random distribution* operators play a key role in exploration. Through the reproduction, the neighborhood of each weed is globally discovered with high dependency to the value of sd_{max}, wherein, the higher the value of sd_{max}, the more spaces that are explored. In this paper, the value of sd_{max} is considered high enough to better explore the solution space. It is worth mentioning that by Eq. (27) the proposed GVIWO starts with high rate of exploration and during the algorithm, the exploitation rate is continually increased. This mechanism helps the algorithm to initially discover the majority of the solution space then focus on improving the quality of the solutions by local searches.

```
Set the parameters (PopSize, Popmax, itermax, Smax, Smin, pow, sditer, sdmax, sdmin, PE, ItVNSmax, nRepeat, NFCmax)
Set NFC=0
Generate initial Populations (Pop1, Pop2) ← PopSize
Evaluate fitness of each weed based on the Game Theory
Update NFC
Merge two populations (New Pop)
Rank the New Pop using fuzzy dominance sorting method
Archive the solutions
Separate two populations for two players (Pop1, Pop2)
Calculate the number of allowed seeds for each member of PopSize (PopSize, Smax, Smin);
terminate ← False
While (terminate = False) Do for each weed at each population
    Update the sditer (itermax, sdmax, sdmin, pow)
    Produce allowed seeds for each weed based on their rank in each population (Pop12, Pop22);
    Evolve weeds as follow: (Pop13, Pop23);
    For i=1:(PopSize* PE)
    Choose a weed randomly among the population;
        For j=1: ItVNSmax
            For k=1:3
                For r=1:nRepeat
                    If k=1 Then
                        Use inversion operator;
                    ElseIf k=2 Then
                        Use insertion operator;
                    ElseIf k=3 Then
                        Use swap operator;
                    EndIf
                EndFor
            EndFor
        EndFor
    EndFor
    Merge all created population (Pop1, Pop2, Pop12, Pop22, Pop13, Pop23);
    Evaluate fitness of each member of merged population based on the Game Theory;
    Update NFC
    Rank the population using fuzzy dominance sorting method;
    Update Archive ← (Archive = Archive + merged population);
    If number(merged population) > Popmax Then
        Archive = Archive (1: Popmax);
    EndIf
    Archive = Archive (1: Popmax);
    it=it+1;
    If NFC has reached NFCmax Then
        terminate = True;
    EndIf
EndWhile
```

Fig. 4. The pseudo code of the proposed GVIWO.

On the other hand, in order not to avoid from local optimums during the algorithm and efficiently explore the space, the diversity of the population must be kept. Diversity is defined as differences among individuals at genotype or phenotype levels (Črepinšek et al., 2013). Based on McPhee and Hopper (1999), "Progress in evolution depends fundamentally on the existence of variation of population. Unfortunately, a key problem in many Evolutionary Computation (EC) systems is the loss of diversity through premature convergence. This lack of diversity often leads to stagnation, as the system finds itself trapped in local optima, lacking the genetic diversity needed to escape." In this paper, the diversity at the phenotype level is used to calculate the fitness function through the ESS method from game theory. By this method, all the solutions in the both populations are considered to calculate the fitness of each individual and those individuals in the unexplored areas receive better values in terms of fitness function (see Eqs. (25) and (26)).

In order to measure the exploration and exploitation abilities of the proposed GVIWO algorithm, the measures called the "Exploration ratio" and "Exploitation ratio", are computed by using an ancestry tree and calculating the percentage of nodes in the tree for which the distance between parent and children individuals is over a threshold (Liu et al., 2013).

4.2.8. *Solution representation*

A solution encoding in the proposed BS*p*HCMP should determine location of *p* hub nodes and allocation of spokes to the located hubs. In this paper, a continuous solution representation (CSR) is devised which avoids creation of infeasible solutions during the search process and makes the solving process much easier. The proposed CSR includes three parts indicating (1) location of hubs, (2) allocation of spokes to the located hub, (3) assignment of transportation mode between hubs, which are explained in detail hereafter.

4.2.8.1. CSR for the location of hubs. The first matrix corresponds to the location decision presented by a $(1 \times \boldsymbol{N})$ matrix, in which $\boldsymbol{N}$ denotes the number of nodes. This matrix is filled with random

Potential Nodes

1	2	3	4	5	6		1	0	0	1	0	1
0.94	0.45	0.02	0.67	0.11	0.59	→ H=3 →	1	0	0	1	0	1

Fig. 5. Hub location scheme.

Potential Nodes

		1	2	3	4	5	6
Location of Hubs		1	0	0	1	0	1
Hub	1	0.81	0.27	0.95	0.79	0.67	0.70
Spoke	2	0.90	0.54	0.48	**0.95**	0.75	0.03
Spoke	3	**0.72**	0.95	0.80	0.15	0.74	0.27
Hub	4	0.91	0.96	0.14	0.03	0.39	0.04
Spoke	5	0.63	0.15	0.42	0.84	0.65	**0.99**
Hub	6	0.09	0.97	0.91	0.93	0.17	0.82

Fig. 6. Allocation scheme.

numbers belonging to [0, 1]. In this matrix, the first maximum $\boldsymbol{H}$numbers are considered as located hubs as depicted in Fig. 5. In Fig. 5, three hubs must be located among six nodes. Therefore, the first, fourth and sixth nodes are located as hubs.

4.2.8.2. CSR for the allocation of spokes to located hub. This part of solution representation corresponds to the allocation of spokes to the located hub. To this aim, a ($\boldsymbol{N} \times \boldsymbol{N}$) matrix is filled with random numbers belonging to [0, 1]. The maximum random number among the intersection arrays of each row corresponding to the spoke nodes with the columns corresponding to the located hubs denotes the allocation scheme as Fig. 6.

4.2.8.3. CSR for the transportation mode between hubs. In order to show the assignment of transportation modes on the links between hubs, a ($\boldsymbol{N} \times \boldsymbol{N}$) matrix is generated with random numbers belonging to[0, 1]. All bits of the matrix are multiplied by the number of modes and then rounded up. The value of arrays corresponding to the link between hubs denotes the number of modes assigned to that link. For example, consider the above-mentioned example, in which, there are links between hubs 1–4, 1–6 and 4–6 and also three different transportation modes. In Fig. 7, modes number 2, 1 and 3 are assigned to the links between hubs 1–4, 1–6 and 4–6, respectively.

5. Computational experiments

In order to validate the correctness of the proposed BMINLP model and the performance of the proposed solution approach, several numerical experiments are done and the related results are reported in this section. First of all, the performance of the proposed game based MOIWO (GMOIWO) is investigated in comparison with well-known exact epsilon-constraint solution method (Haimes et al., 1971). The performance of the proposed GVIWO has been evaluated in comparison to well-known algorithms in the literature such as imperialist competitive algorithm (ICA, Mohammadi et al., 2013), particle swarm optimization (PSO, Goksal et al., 2013), simulated annealing (SA, Abdinnour-Helm, 2001), non-dominated sorting genetic algorithm (NSGA-II, Niakan et al., 2014), Pareto archive evolutionary strategy (PAES, Mohammadi et al., 2013), scatter search (SS, Martí et al., 2015), tabu search (TS, Jin and Hao, 2015) and differential evolution (DE, Zahiri et al., 2014; Mohammadi and Tavakkoli-Moghaddam, 2015). Due to space limitation, the comparison between two of these algorithms (PAES and NSGA-II) is provided. All the mathematical models were coded in the GAMS optimization software utilizing BARON solver. Moreover, the meta-heuristics were coded in C++ and tested on a Pentium 4 CPU with 3.0 GHz CPU and 6GB of memory.

5.1. Tightness of GVIWO and effect of game theory

This section not only shows the gap between the Pareto solutions of the proposed GVIWO algorithm with optimal solutions obtained from BARON solver, but also illustrates the effect of game theory method on obtaining high-quality Pareto solutions. In this regard, two experiments are done on two randomly-generated numerical instances (i.e., P_1 and P_2). The details of P_1 and P_2 have been shown in Table 2. In order to show the effectiveness of game theory, the classical MOIWO algorithm (Kundu et al., 2011) is applied to find the Pareto solutions. Figs. 8 and 9 illustrate the Pareto frontiers of the GVIWO and MOIWO algorithm as well as GAMS software for both P_1 and P_2, respectively.

To generate the trapezoidal fuzzy parameters according to Jimenez et al. (2007), the four prominent points are first determined. For this aim, consider a trapezoidal parameter $\tilde{A} = [a^1, a^2, a^3, a^4]$. Two values of a^2 and a^3 are first generated randomly by utilizing the uniform distribution functions specified in Table 2, where $a^2 < a^3$. Then, without loss of generality, two random numbers (r_1, r_2) are generated between .2 and .8 using a uniform distribution by which the a^1 and a^4 are then calculated as $a^1 = (1 - r_1)a^2; a^1 = (1 + r_2)a^3$. The value of α-cut is considered equal to .5 in all experiments.

It can be seen from Figs. 8 and 9 that the proposed GVIWO can obtain Pareto solution very close to the optimal Pareto frontier obtained by GAMS. In addition, the effectiveness of the game theory method can be shown by comparing the Pareto frontiers of the GVIWO and MOIWO. The proposed GVIWO, by the competitive between objectives (players in the game theory), the solution space is better searched and high-quality solutions are found, while the classical MOIWO is disable to find solutions in the middle of the Pareto frontier.

5.2. Performance of the proposed GVIWO

In this section, the performance of the proposed GVIWO is compared with NSGA-II and PAES with respect to four different comparison metrics, namely quality (QM), spacing (SM), diversity (DM) and mean ideal distance (MID) metrics (Mohammadi et al., 2013). Before running the algorithms, the tuned parameters of the proposed MOIWO, NSGA-II and PAES, employing well known response surface methodology (RSM) (Mohammadi et al., 2013), for small- (S) and large-sized (L) instances are tabulated as Table 3.

Some special numbers of hubs have been considered for each number of nodes. Also, each problem instance is shown as "*Number of nodes # number of hubs*", for example, 50#8 means 50 nodes and 8 hubs to be located. In order to compare the performance of the proposed GVIWO with NSGA-II and PAES, 67 different test problems varying from 10 to 100 nodes are randomly generated and the results are expressed in Tables 4–9. Table 4 reports the results of QM and SM and Table 5 shows DM and MID for small-sized problems. Besides, Tables 6–9 correspond to large-sized instances. It can be seen that for almost all test problems, the proposed GVIWO outperforms NSGA-II and PAES regarding all comparison metrics, where better results have been bolded.

According to Tables 4–9, some benefits of the proposed GVIWO are as follows:

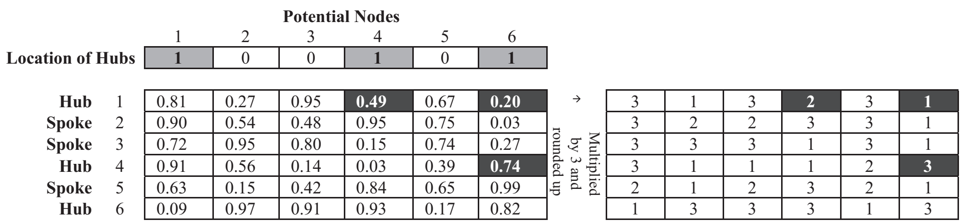


Fig. 7. Structure of hub network.

Table 2
Sources of random generation of the parameters.

Problem no.	Parameters				
P_1	I	H	M	c	f	t
	10	3	2	(100,600)	$(10^5,3\times10^5)$	(100,300)
	s	μ	v	r	σ	δ
	Poison(300)	Poison(600)	Poison(20)	Poison(50)	.90	.80
P_2	I	H	M	c	f	t
	20	5	3	(400,900)	$(10^6,5\times10^6)$	(200,500)
	s	μ	v	r	σ	δ
	Poison(400)	Poison(1000)	Poison(30)	Poison(60)	.80	.70

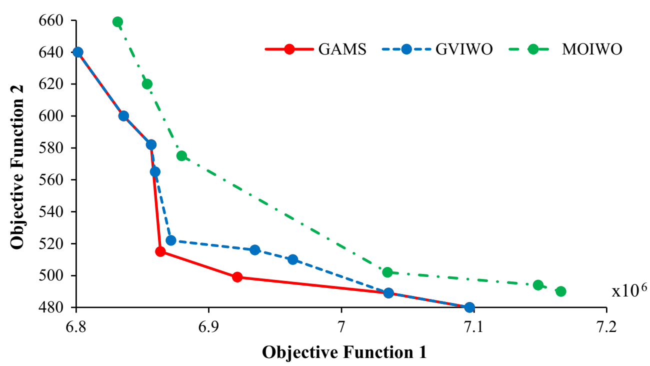


Fig. 8. Pareto frontier of GAMS, GVIWO and MOIWO for P_1.

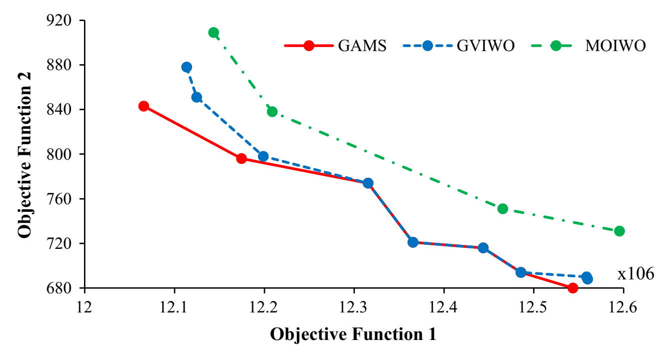


Fig. 9. Pareto frontier of GAMS, GVIWO and MOIWO for P_2.

- Proposed GVIWO can obtain a greater number of Pareto optimal solutions with higher qualities in comparison with NSGA-II and PAES.
- Proposed GVIWO provides Pareto solutions that are more uniformly distributed in the Pareto front with less average values of the spacing metric.
- The average values of the diversity metric in the proposed GVIWO are considerably greater than NSGA-II and PAES (i.e., GVIWO finds Pareto solutions with more diversity).
- In most of the test problems, the values of MID in the proposed GVIWO are smaller than those of NSGA-II and PAES.
- Proposed GVIWO leads to better results in larger problems.

It should be noted that through four comparison metrics, QM is the most important metric that directly corresponds to the quality of the solutions. From Table 4 to 9, it was concluded that the proposed GVIWO algorithm outperforms other algorithm in terms of QM in all the problems. In other metrics, in almost all test problems, GVIWO shows higher performance. In order to see whether the significant difference exists between the performance of the GVIWO, NSGA-II and PAES in terms of SM, DM and MID metric, two analyses are conducted called "paired t test" and "non-parametric Friedman test". In "paired t test", let D_i be equal to difference between the calculated values of two algorithms for test problem i. So the statistics are as (30):

$$t=\frac{\sqrt{n}\times\vec{D}}{S_D}\quad\text{where}\quad\vec{D}=\frac{\sum\vec{D}}{n}\quad\text{and}\quad S_D=\sqrt{\frac{\sum\left(D_i-\vec{D}\right)^2}{n-1}}\tag{30}$$

We conducted both analyses by 67 test problems in the SPSS software. The analytical results for "paired t test" and "non-parametric Friedman test" have been provided in Tables 10 and 11, respectively.

With referencing to the t table, for 66° of freedom the significances (2-tailed) are closed to 0.000. The detailed statistics are shown in Table 10. These tests show that there are statistical significant difference between solutions obtained by GMOIWO and those of the improved NSGA-II and the PAES.

Table 11 reports the test statistic (χ^2) value ("Chi-square"), degrees of freedom ("df") and the significance level ("Asymp. sig."), which is all we need to report the result of the Friedman test. From Table 11, it can be seen that there is an overall statistically significant difference between the mean of the related algorithm in terms of SM, DM and MID metric.

It should be noted that the non-parametric Friedman test is an omnibus analysis and it tells us whether there are overall differences between algorithms, but does not pinpoint which algorithms in particular differ from each other. For this aim, we run "post hoc tests". To do so and examine where the differences actually occur,

Table 3
Result of RSM and tuned parameters.

Algorithm	Parameters and values			
GMOIWO	Pop_{Size} 15(S), 40(L)	Pop_{max} 70(S), 110(L)	$iter_{max}$ 125(S), 250 (L)	S_{max} 7(S), 9(L)
	S_{min} 1(S), 2(L)	sd_{min} .55(S), 0.8(L)	sd_{max} .01(S), 0.02 (L)	pow 1(S), 1(L)
	P_E 0.3(S), 0.65 (L)	$ItVNS_{max}$ 17(S), 20(L)	n_{Repeat} 6(S), 8(L)	NFC 10,000(S), 20,000(L)
NSGA-II	Pop_{Size} 100(S), 250 (L)	$P_{Crossover}$ .7(S), .9(L)	$P_{Mutation}$ .1(S), .3(L)	NFC 10,000(S), 20,000(L)
PAES	Pop_{Size} 100(S), 250 (L)	NFC 10,000(S), 20,000 (L)		

Table 4
QM and SM for small-sized problems.

Problem no.	Quality metric (QM)			Spacing metric (SM)		
	PAES	NSGA-II	GVIWO	PAES	NSGA-II	GVIWO
10#3	0.00	0.20	**0.80**	0.720	0.633	**0.419**
10#4	0.00	0.00	**1**	0.749	0.780	**0.318**
15#3	0.00	0.05	**0.95**	0.735	0.579	**0.498**
15#4	0.00	0.10	**0.90**	0.583	0.512	**0.275**
15#5	0.00	0.00	**1**	0.620	0.798	**0.475**
20#3	0.00	0.15	**0.85**	0.688	0.700	**0.273**
20#4	0.00	0.00	**1**	0.592	0.692	**0.347**
20#5	0.00	0.25	**0.75**	0.838	0.862	**0.454**
20#6	0.00	0.10	**0.90**	0.578	0.744	**0.512**
25#3	0.00	0.00	**1**	0.590	0.747	**0.232**
25#4	0.00	0.00	**1**	**0.568**	0.844	0.572
25#5	0.00	0.03	**0.97**	0.591	0.823	**0.511**
25#6	0.00	0.00	**1**	0.674	0.731	**0.395**
30#3	0.00	0.20	**0.80**	0.624	0.573	**0.374**
30#4	0.00	0.00	**1**	0.870	0.596	**0.379**
30#5	0.00	0.00	**1**	0.672	0.855	**0.322**
30#6	0.00	0.00	**1**	0.574	0.511	**0.243**
30#7	0.00	0.00	**1**	0.862	0.696	**0.312**
30#8	0.00	0.00	**1**	0.892	0.567	**0.452**

Table 5
DM and MID metric for small-sized problems.

Problem no.	Diversity metric (DM)			Mean ideal distance (MID)		
	PAES	NSGA-II	GVIWO	PAES	NSGA-II	GVIWO
10#3	0.802	**1.076**	0.852	0.781	**0.609**	0.674
10#4	0.764	0.880	**1.106**	0.758	0.534	**0.241**
15#3	0.944	1.030	**1.190**	0.819	0.470	**0.252**
15#4	0.883	0.856	**1.012**	0.812	0.483	**0.271**
15#5	**0.948**	0.886	0.845	0.835	0.762	**0.283**
20#3	**1.020**	0.827	0.933	0.686	0.670	**0.511**
20#4	**1.038**	0.973	0.876	0.808	0.587	**0.487**
20#5	0.686	**1.005**	0.940	0.767	0.765	**0.226**
20#6	0.662	0.964	**1.020**	0.719	**0.441**	0.666
25#3	0.914	0.928	**1.064**	0.618	0.698	**0.565**
25#4	0.642	0.993	**1.029**	0.834	0.695	**0.569**
25#5	0.836	0.994	**1.238**	0.701	0.625	**0.231**
25#6	0.839	1.004	**1.059**	0.782	**0.473**	0.631
30#3	0.988	0.991	**1.272**	0.823	**0.639**	0.668
30#4	0.818	1.084	**1.119**	0.631	**0.520**	0.693
30#5	0.777	0.862	**1.279**	0.638	**0.453**	0.630
30#6	0.902	**1.013**	0.920	0.765	**0.485**	0.593
30#7	0.934	0.871	**1.138**	0.746	0.758	**0.457**
30#8	0.834	0.835	**0.944**	0.868	0.428	**0.288**

Table 6
Comparison metrics for 40-sized problems.

Problem no.	PAES	NSGA-II	GVIWO	PAES	NSGA-II	GVIWO
	Quality metric (QM)			Spacing metric (SM)		
40#3	0.00	0.08	**0.92**	0.896	0.558	**0.444**
40#4	0.09	0.18	**0.73**	0.884	0.521	**0.315**
40#5	0.00	0.20	**0.80**	0.803	0.409	**0.385**
40#6	0.00	0.00	**1**	0.897	0.395	**0.373**
40#7	0.00	0.00	**1**	0.830	0.608	**0.359**
40#8	0.00	0.00	**1**	0.701	0.359	**0.282**
40#9	0.00	0.05	**0.95**	0.799	0.608	**0.274**
40#10	0.00	0.00	**1**	0.673	0.595	**0.335**
	Diversity metric (DM)			Mean ideal distance (MID)		
40#3	0.821	1.123	**1.194**	0.816	0.663	**0.379**
40#4	0.816	**1.142**	0.956	0.933	0.545	**0.320**
40#5	0.849	0.829	**1.293**	0.921	0.677	**0.431**
40#6	0.768	1.061	**1.255**	0.829	0.620	**0.352**
40#7	0.856	1.104	1.014	0.908	0.660	**0.307**
40#8	0.754	0.969	**1.074**	0.984	0.834	**0.587**
40#9	1.098	1.098	**1.182**	0.936	0.661	**0.429**
40#10	0.742	1.125	**1.259**	0.912	0.656	**0.589**

Table 7
Comparison metrics for 50-sized problems.

Problem no.	PAES	NSGA-II	GVIWO	PAES	NSGA-II	GVIWO
	Quality metric (QM)			Spacing metric (SM)		
50#3	0.00	0.00	**1**	0.958	0.572	**0.303**
50#4	0.00	0.00	**1**	0.860	0.450	**0.243**
50#5	0.00	0.00	**1**	0.798	0.409	**0.586**
50#6	0.00	0.10	**0.90**	0.716	0.516	**0.303**
50#7	0.00	0.05	**0.95**	0.939	0.527	**0.394**
50#8	0.00	0.00	**1**	0.703	0.662	**0.476**
50#9	0.00	0.00	**1**	0.628	0.783	**0.314**
50#10	0.00	0.00	**1**	0.946	0.775	**0.521**
50#11	0.00	0.00	**1**	0.902	0.583	**0.442**
50#12	0.00	0.00	**1**	0.922	0.496	**0.276**
	Diversity metric (DM)			Mean ideal distance (MID)		
50#3	0.923	0.959	**1.070**	0.827	0.624	**0.219**
50#4	0.871	**0.976**	0.969	0.704	0.599	**0.421**
50#5	0.807	0.862	**1.154**	0.726	0.557	**0.310**
50#6	1.002	0.930	**1.329**	0.646	0.478	**0.296**
50#7	1.060	0.925	**1.092**	0.846	0.689	**0.297**
50#8	0.992	1.158	**1.248**	0.788	0.562	**0.261**
50#9	0.863	0.899	**1.214**	0.722	0.609	**0.483**
50#10	1.076	0.924	**1.125**	0.842	0.609	**0.575**
50#11	0.802	0.963	**1.137**	0.620	0.556	**0.528**
50#12	0.913	1.083	**1.375**	0.886	0.417	**0.492**

we need to run separate "Wilcoxon signed-rank test" on the different combinations of the algorithms. The results of "Wilcoxon signed-rank tests" have been provided in Tables 12–14 for SM, DM and MID metric, respectively.

Referring to Tables 12–14, it can be shown that there are statistically significant differences between each pair of algorithms in terms of SM, DM and MID metric. Consequently, the superiority of the proposed GVIWO is demonstrated by these analyses.

At the final remarks of this section, an experiment is conducted to analyze the performance of the proposed GVIWO in terms of CPU time. To do so, 20 test problems are solved by three algorithms and the CPU time that is needed to reach the best solutions are reported for each algorithm. Size of the test problems are as follows: $I \in [10, 200]$, $H \in [2, 30]$, and $M \in [2, 4]$.

This comparison has been illustrated in Fig. 10. It can be seen that the proposed algorithm also outperforms NSGA-II and PAES in terms of CPU time. In addition, the CPU time of all the algorithms are increased by increasing the size of test problems. Among them, the CPU time of the proposed GVIWO is linearly increased while

Table 8
Comparison metrics for 70-sized problems.

Problem no.	PAES	NSGA-II	GVIWO	PAES	NSGA-II	GVIWO
	Quality metric (QM)			Spacing metric (SM)		
70#3	0.00	0.00	**1**	0.957	0.517	**0.423**
70#4	0.00	0.00	**1**	0.812	0.657	**0.359**
70#5	0.00	0.00	**1**	0.683	0.601	**0.224**
70#6	0.00	0.00	**1**	0.729	0.552	**0.512**
70#7	0.00	0.04	**0.96**	0.631	0.562	**0.335**
70#8	0.00	0.10	**0.90**	0.852	0.772	**0.443**
70#9	0.00	0.00	**1**	0.612	0.703	**0.497**
70#10	0.00	0.00	**1**	0.901	0.641	**0.242**
70#11	0.00	0.05	**0.95**	0.883	0.774	**0.251**
70#12	0.00	0.00	**1**	0.856	0.531	**0.420**
70#13	0.00	0.00	**1**	0.921	0.724	**0.394**
70#14	0.00	0.00	**1**	0.936	0.721	**0.557**
70#15	0.00	0.00	**1**	0.774	0.669	**0.520**
70#16	0.00	0.00	**1**	0.889	0.555	**0.494**
	Diversity metric (DM)			Mean ideal distance (MID)		
70#3	0.929	0.872	**1.379**	0.895	0.655	**0.282**
70#4	0.606	1.065	**1.143**	0.638	0.636	**0.245**
70#5	0.617	0.772	**1.300**	0.669	0.481	**0.319**
70#6	0.667	0.805	**0.971**	0.607	0.468	**0.312**
70#7	0.860	0.758	**1.111**	0.782	0.496	**0.239**
70#8	0.893	0.754	**1.358**	0.633	0.649	**0.330**
70#9	0.859	1.048	**1.296**	0.722	0.647	**0.227**
70#10	0.780	0.932	**1.380**	0.866	0.571	**0.385**
70#11	0.819	0.920	**1.228**	0.764	0.572	**0.203**
70#12	0.718	0.758	**0.917**	0.711	0.486	**0.372**
70#13	0.898	1.042	**1.325**	0.662	0.610	**0.437**
70#14	0.675	0.949	**1.367**	0.732	0.639	**0.270**
70#15	0.875	0.840	**1.240**	0.887	0.532	**0.334**
70#16	0.673	0.905	**1.279**	0.637	0.534	**0.371**

the CPU times for two other algorithms are increased exponentially, particularly for PAES algorithm.

At the end of this section, the performance of the proposed GVIWO is investigated through Fig. 11 to show the behavior of the GVIWO algorithm in exploration and exploitation. For this aim, the values of MID metric that shows the quality of the solutions are reported after each function call during the search algorithm.

Fig. 11 demonstrates that the proposed GVIWO algorithm balances between optimization and convergence, wherein we noticed that the GVIWO's exploration drops drastically before 2000th function call and reaches to stable after 15,000th function call. Accordingly, we conclude that the GVIWO algorithm's exploration has been performed efficiently toward finding the "right" directions quickly after around 2000 function calls. Afterward, then exploitation power dominates to search local optima in a finer manner. This may be the reason that the GVIWO algorithm better balances between optimization and convergence.

5.3. Sensitivity analysis

In order to recognize the most significant parameters of the proposed model, several sensitivity analyses are carried out and the impact of parameters alteration on the objective functions is investigated. To do so, different experiments are performed on the test instance P_2. Figs. 12 and 13, respectively, illustrate the sensitivity of the first and second objective functions vs. number of hubs in the network. As shown in Fig. 12, the value of the first objective function is increased smoothly when $H \leq 6$ and then increased with higher slope for $H > 6$. In Fig. 12, when the number of hubs (H) is increased, the fixed cost is also increased but the spokes can be allocated to more close hubs resulting in lower transportation cost. Since the value of fixed cost is higher than transportation cost, therefore, the first objective function is smoothly increased by increase in H. On the other hand, when the value H becomes greater

Table 9
Comparison metrics for 100-sized problems.

Problem no.	PAES	NSGA-II	GVIWO	PAES	NSGA-II	GVIWO
	Quality metric (QM)			Spacing metric (SM)		
100#3	0.00	0.00	**1**	0.872	0.433	**0.315**
100#4	0.00	0.00	**1**	0.732	0.419	**0.236**
100#5	0.00	0.00	**1**	0.506	0.521	**0.431**
100#6	0.00	0.05	**0.95**	0.548	0.534	**0.474**
100#7	0.00	0.00	**1**	0.845	0.510	**0.419**
100#8	0.00	0.00	**1**	0.694	0.629	**0.370**
100#9	0.00	0.00	**1**	0.838	0.588	**0.458**
100#10	0.00	0.00	**1**	0.683	0.632	**0.359**
100#11	0.00	0.00	**1**	0.721	0.680	**0.472**
100#12	0.00	0.00	**1**	0.752	0.692	**0.454**
100#13	0.00	0.10	**0.9**	0.512	**0.457**	0.579
100#14	0.00	0.00	**1**	0.746	0.441	**0.283**
100#15	0.00	0.00	**1**	0.645	0.609	**0.484**
100#16	0.00	0.00	**1**	0.519	0.428	**0.294**
100#17	0.00	0.00	**1**	0.696	0.858	**0.247**
100#18	0.00	0.00	**1**	0.577	0.559	**0.443**
	Diversity metric (DM)			Mean ideal distance (MID)		
100#3	0.771	1.033	**1.212**	0.862	0.569	**0.255**
100#4	0.793	1.022	**1.240**	0.698	0.673	**0.258**
100#5	0.648	0.724	**1.098**	0.795	0.743	**0.302**
100#6	0.836	0.860	**1.084**	0.893	0.621	**0.480**
100#7	0.690	0.911	**1.394**	0.622	0.797	**0.317**
100#8	0.754	0.867	**0.918**	0.776	0.527	**0.282**
100#9	0.833	0.963	**1.343**	0.724	0.596	**0.245**
100#10	0.700	0.951	**1.357**	0.793	0.653	**0.319**
100#11	0.716	0.817	**1.298**	0.679	0.518	**0.312**
100#12	0.847	0.873	**0.949**	0.828	0.718	**0.239**
100#13	0.706	0.706	**1.031**	0.899	0.667	**0.330**
100#14	0.930	1.094	**1.068**	0.656	0.659	**0.227**
100#15	0.994	0.767	**1.240**	0.835	0.749	**0.385**
100#16	0.892	0.742	**0.968**	0.658	0.758	**0.203**
100#17	0.737	0.849	**1.261**	0.898	0.737	**0.372**
100#18	0.834	0.779	**0.953**	0.841	0.595	**0.437**

than 5, the allocation of the spokes is not changed. Consequently, the increase of the first objective function directly corresponds to the fixed cost increase. Hence, the increase rate of the first objective function for $H > 6$ is higher than a situation when $H \leq 6$.

In Fig. 13, the value of the second objective function is decreased when the number of hubs was increased. Since the number of hubs is low, higher volume of flows are consolidated in the hubs that results to higher congestion and consequently higher waiting time. Hence, the value of the second objective function is increased. By increasing the number of hubs, allocation of spokes to the hubs is getting balanced and consequently the waiting time is decreased. It should be also mentioned that after 7 hubs in the network, the second objective function is not changed when the allocation topology of the hub-and-spoke network remains constant.

In the following, the effect of hub failures and retrieve rates on the structure of the network is investigated. To this end, Figs. 14 and 15 depict the sensitivity of the second objective function under alteration of ν_k and r_k, respectively. It can be seen from Fig. 14 that by increasing the rate of breakdowns in the hubs (ν_k), the value of the second objective function is increased in a polynomial way. In addition, Fig. 15 illustrates that the second objective function is decreased when the retrieve rate in the hubs is increased (i.e., hub are retrieved quickly). In Fig. 15, the failure rate ν is fixed to 20. These two experiments demonstrate the importance of taking uncertainty in hub's operation into account, while this paper not only considers hub failures, but also considers the rate of failures uncertain (i.e., fuzzy number).

Another important experiment is performed to investigate the effect of α-cut in Section 4.1. Higher values of α deal with tighter constraint. In most of problems, α-cut that equals to 1 results in

Table 10
Detailed statistics of paired t test.

Metric	Pair	Paired differences					t	df	Sig. (2-tailed)
		Mean	Std. deviation	Std. error mean	95% Confidence interval of the difference				
					Lower	Upper			
SM	GVIWO NSGA-II	−.22330	.14833	.01812	−.25948	−.18712	−12.322	66	.000
DM		.20752	.18412	.02249	.16261	.25243	9.226	66	.000
MID		−.22188	.18467	.02256	−.26692	−.17684	−9.835	66	.000
SM	GVIWO PAES	−.32184	.17026	.02080	−.36336	−.28031	−15.473	66	.000
DM		.30881	.19944	.02436	.26016	.35745	12.674	66	.000
MID		−.39251	.16623	.02031	−.43306	−.35196	−19.327	66	.000

Table 11
Detailed statistics of non-parametric Friedman test.

Metric	Test statistics			
	N	Chi-square	df	Asymp. sig.
SM	67	99.075	2	.000
DM	67	79.669	2	.000
MID	67	104.021	2	.000

Table 12
Detailed statistics of Wilcoxon signed-rank test for SM.

Statistics	Pair		
	PAES vs. NSGA-II	PAES vs. GVIWO	NSGA-II vs. GVIWO
Z	−4.813	−7.090	−6.859
Asymp. sig.	.000	.000	.000

Table 13
Detailed statistics of Wilcoxon signed-rank test for DM.

Statistics	Pair		
	PAES vs. NSGA-II	PAES vs. GVIWO	NSGA-II vs. GVIWO
Z	−4.617	−6.890	−6.284
Asymp. sig.	.000	.000	.000

Table 14
Detailed statistics of Wilcoxon signed-rank test for MID metric.

Statistics	Pair		
	PAES vs. NSGA-II	PAES vs. GVIWO	NSGA-II vs. GVIWO
Z	−6.625	−7.090	−6.412
Asymp. sig.	.000	.000	.000

infeasible problem. Since there is no constraint with fuzzy numbers in this paper, variation of α-cut only affects the values of the second objective function while the first objective is not influenced by the α-cut value. Fig. 16 illustrates the effect of α-cut increase on the second objective function value. As it can be seen, second objective is increased once the α-cut is increased. It has been found that the second objective function is completely sensitive to the alteration of α-cut in comparison to the first objective. It is noteworthy that by increasing the α-cut the second objective function is increased in a polynomial way with degree 3 (see dashed line in Fig. 16).

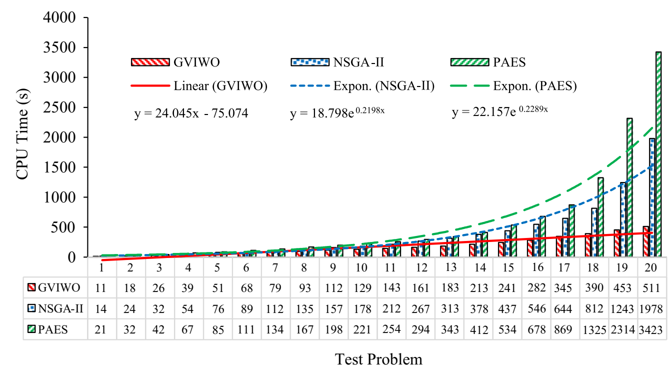


Fig. 10. Comparison of CPU time of the algorithms.

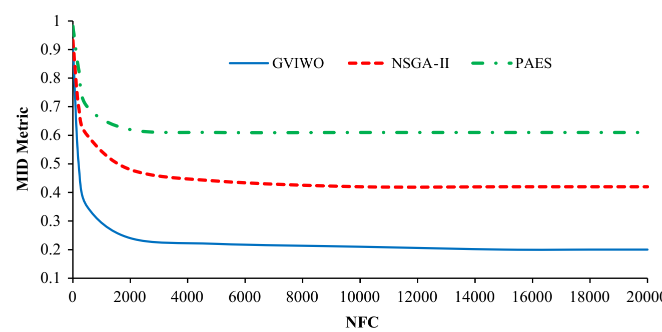


Fig. 11. Better balance between exploration and exploitation for GVIWO algorithm.

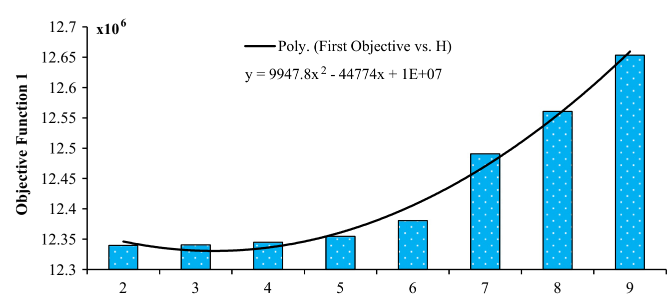


Fig. 12. Objective function 1 vs. number of hubs (H).

As an important managerial insight, companies that notify more the center objective are more vulnerable regarding the uncertainty. Therefore, these companies should try to better control the uncertainty by designing hub network by taking uncertainty into account.

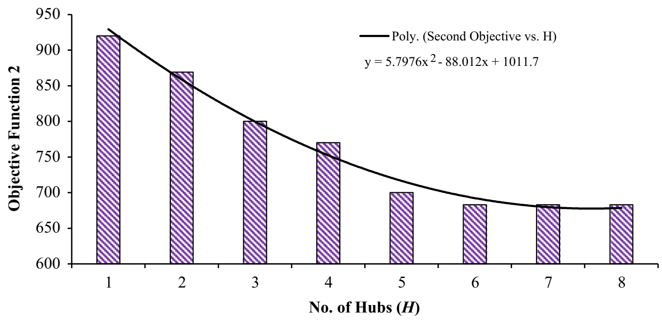


Fig. 13. Objective function 2 vs. number of hubs (H).

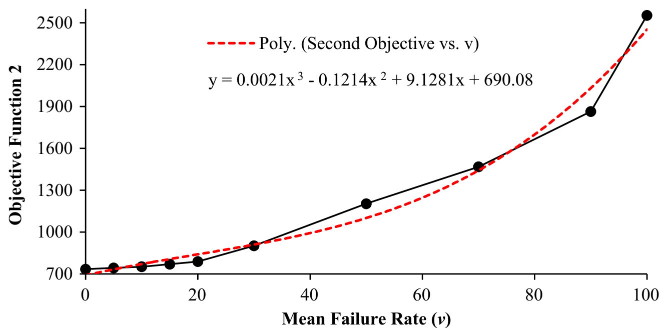


Fig. 14. Objective function 2 vs. failure rate at hubs (v_k).

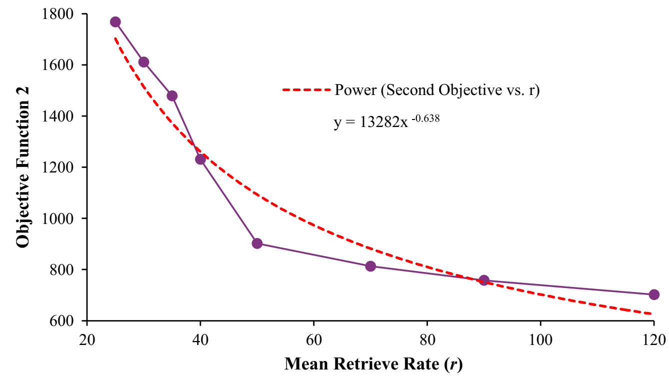


Fig. 15. Objective function 2 vs. retrieve rates at hubs (r_k).

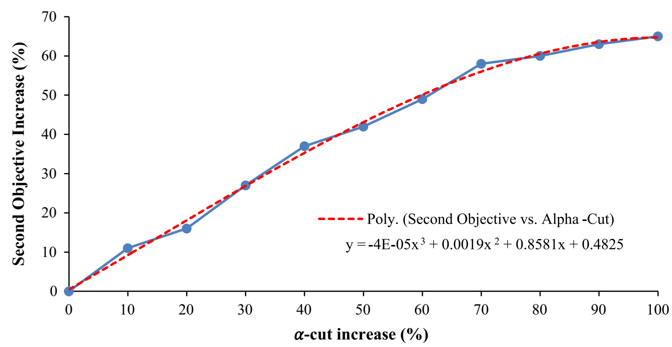


Fig. 16. Increase in objective function 2 vs. increase in α-cut.

This paper has attempted to address this concern in the proposed model.

5.4. Case study

In this section, a real case of passenger transportation in Iran is studied to validate the performance of the proposed model and the solution approach in real-world problems. The real case corresponds to a transportation network in Iran with 37 cities (see Fig. 17). This instance has been prepared by Karimi and Bashiri (2011). Since the flow data is originated from two criteria including tourism and industrial, they calculated the importance of cities by Technique for Order Preference by Similarity to Ideal Solution (TOPSIS). Then, considering the population of each city and its importance, flows data between cities were computed.

In this case, three road, rail and air transportation modes have been considered. The unit transportation cost (c_{ij}^m) and transportation time (t_{ij}^m) for all modes can be calculated based on the distance between cities. The values of σ and δ are considered to be equal to 0.9 and 0.7, respectively. The value of μ_k is calculated as Eq. (31). Failure and retrieve rates, v_k and r_k, are determined according to the report of the Iranian Transportation Ministry and Crisis Management (ITMCM).

In order to generate trapezoidal fuzzy parameters, the four prominent points for each parameter at each level are determined based on the data obtained by Karimi and Bashiri (2011). For instance, consider the fixed cost of opening a hub at node k, f_k. The trapezoidal parameter $\tilde{f}_k$ is generated through Eq. (32).

$$\mu_k = \left(1+\frac{p-1}{2}\right)\frac{\sum_{i,j} s_{ij}}{p} \quad (31)$$

$$\tilde{f}_k = \Big[(1-\epsilon_1)\Big(1-\frac{\epsilon_1+\epsilon_2}{4}\Big)f_k, \Big(1-\frac{\epsilon_1+\epsilon_2}{4}\Big)f_k, \Big(1+\frac{\epsilon_1+\epsilon_2}{4}\Big)f_k, (1+\epsilon_2)\Big(1+\frac{\epsilon_1+\epsilon_2}{4}\Big)f_k\Big] \quad (32)$$

where ε_1 and ε_2 are randomly generated from the interval [0.2, 0.8] using a uniform distribution. After implementing the proposed model and solution approaches on the real dataset by $p = 3$, the results obtained by the proposed GVIWO algorithm are illustrated in Figs. 18 and 19. Fig. 18 shows the Pareto frontier for the real dataset obtained by the proposed GVIWO. In addition, Fig. 19 illustrates the structure of the sample solution from the Pareto frontier shown in Fig. 19.

It can be extracted from Fig. 18 that by increasing the cost only 27%, the maximum traveling time decreases up to 55%.

In Fig. 19, it can be seen that nodes number 10, 15 and 31 have been considered to be as hubs. In addition, all three modes of transportation have been utilized between hubs number 10 and 31, while railway has been constructed from hub 10 to 15, road has been used between hubs 15 and 31, and finally airway is used from hub 15 to hub 31. It was also concluded from Fig. 19 that spokes (cities) are more likely to be allocated to hubs with lower failure rate (υ) as well as higher service rate (μ). In Fig. 19, the longest transportation time belongs to the travel between spokes 9 and 32 with 1120 units of time.

In order to investigate another network to minimize the longest time, Fig. 20 shows the hub network with 4 hubs. Like Fig. 18, a solution from the middle of the Pareto frontier is selected for further information. As it can be seen, the nodes number 31, 12, 2 and 15 have been located as hubs. Having four hubs in the network has led to lower value for the longest transportation time that belongs to the travel between nodes 19 and 32 with 935 units of time.

Several extra experiments were conducted on the real dataset in order to investigate the contribution of each transportation network to transport the shipments. It was extracted that the road, rail and air mode are assigned to the hub-to-hub links with 28%, 46%, and 26%, respectively. Therefore, the railways play a key role in transporting the shipment in the networks with both median and center objectives.

1	Abadan	2	Ahvaz	3	Arak	4	Ardabil	5	Bandar Abbas
6	Birjand	7	Bojnurd	8	Bushehr	9	Chabahar	10	Esfahan
11	Gorgan	12	Hamedan	13	Ilam	14	Iranshahr	15	Kerman
16	Kermanshah	17	Khark	18	Khoramabad	16	Mashhad	20	Nowshahr
21	Ramsar	22	Rasht	23	Sabzevar	24	Sanandaj	25	Sari
26	Shahrekord	27	Shahrud	28	Shiraz	29	Sirjan	30	Tabriz
31	Tehran	32	Urmia	33	Yasooj	34	Yazd	35	Zabol
36	Zahedan	37	Zanjan						

Fig. 17. Iranian cities in the real dataset.

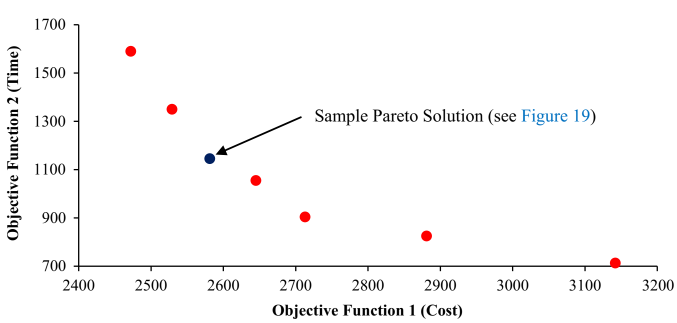
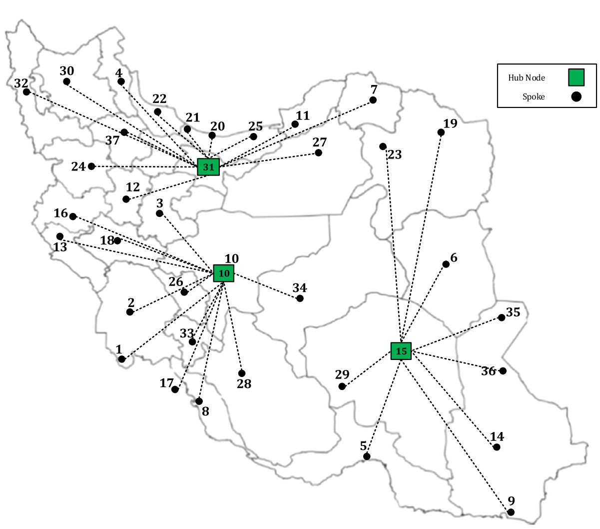
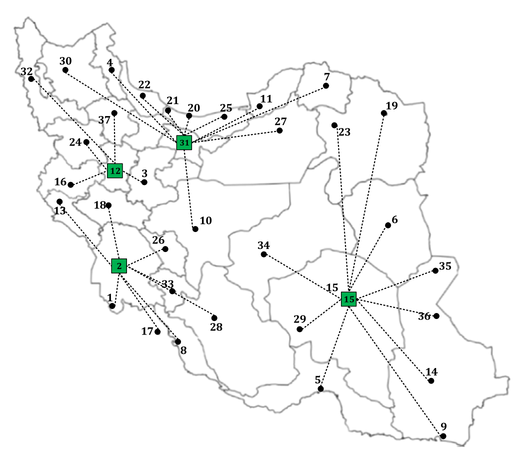


Fig. 18. Pareto frontier of the real dataset.



31→10: Road, Rail, Air; **31→15**: Road, Air; **10→15**: Road, Rail

Fig. 19. Hub network's structure for sample Pareto solution with 3 hubs.



31→12: Road; **31→2**: Road, Rail, Air; **31→15**: Road, Rail; **2→12**: Road; **2→15**: Road, Rail; **12→15**: Road, Rail

Fig. 20. Hub network's structure for sample Pareto solution with 4 hubs.

6. Conclusion

This paper proposed a bi-objective single allocation *p*-hub center-median problem (BS*p*HCMP) with respect to transportation modes over the network. The aim of this model was to locate *p* hubs, allocate the spokes to exactly one of the located hubs and assign transportation modes to the hub-to-hub links in such a way that the total transportation costs as well as maximum transportation time between each pair of origin–destination nodes were simultaneously minimized. The first purpose of this paper was to study the BS*p*HCMP by taking into account the uncertainty in the flows, costs, times and hub operations. We modeled the proposed problem through a bi-objective mixed-integer non-linear programming (BMINLP). To this aim, a fuzzy queuing approach was applied to model the uncertainties in the hub network.

Due to the complexity of the BMINLP model, the second purpose of this paper was to propose an efficient and powerful evolutionary algorithm, based on game theory and invasive weed optimization algorithm (GVIWO) to solve the BS*p*HCMP model and obtain near optimal Pareto solutions. Several experiments were conducted to evaluate performance of the proposed algorithm. Moreover, the superiority of the proposed GVIWO was shown in comparison with two well-known algorithms, NSGA-II and PAES.

Several sensitivity analyses were done showing the effect of uncertainty and the importance of taking uncertainty into account in hub location problems. It was concluded that the hub failure significantly affects the structure as well as the objectives values such that the spokes are more likely to be allocated to the hubs with lower failures.

Finally, a real transportation case has been studied to show the applicability of the proposed model and solution approaches of this paper in real world problems. In addition, the effect of the number of hubs in the network was investigated as well as the contribution of road, rail and air transportation modes in designing the hub network.

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