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Multiscale Modeling Approaches for Composites

George Chatzigeorgiou, Fodil Meraghni, Nicolas Charalambakis

Multiscale Modeling Approaches for Composites outlines the fundamentals of common multiscale modeling techniques and provides detailed guidance for putting them into practice. Various homogenization methods are presented in a simple, didactic manner, with an array of numerical examples.

The book starts by covering the theoretical underpinnings of tensors and continuum mechanics concepts, then progresses to actual micromechanic techniques for composite media and laminate plates. In the last chapters the book covers advanced topics in homogenization, including Green's tensor, Hashin-Shtrikman bounds, and special types of problems. All chapters feature comprehensive analytical and numerical examples (Python and ABAQUS scripts) to better illustrate the theory.

Key features:

- Bridges theory and practice, providing step-by-step instructions for implementing multiscale modeling approaches for composites and the theoretical concepts behind them
- Covers boundary conditions, data-exchange between scales, the Hill-Mandel principle, average stress and strain theorems, and more
- Discusses how to obtain composite properties using various full-field and mean-field approaches
- Includes access to a companion site featuring the numerical examples, Python, and ABAQUS codes

About the Authors

George Chatzigeorgiou is a Research Scientist at CNRS (CR-HDR) based at the Arts et Métiers Institute of Technology, France, and a member of the LEM3-UMR CNRS 7239 laboratory in France. His research is devoted to constitutive modeling of multifunctional materials and homogenization theories of composites.

Fodil Meraghni is a Distinguished Professor at the Arts et Métiers Institute of Technology, France. He is the head of the Composites and Smart Materials Research Group at LEM3-UMR CNRS 7239 in France, working on multiscale modeling using micromechanical approaches for homogenization of polymer composites and shape memory alloys.

Nicolas Charalambakis is a Professor Emeritus in the Department of Civil Engineering at Aristotle University of Thessaloniki, Greece, and a member of the Center for Research and Development of Advanced Materials AUTH and Texas A&M. His research focuses on homogenization and material instabilities.

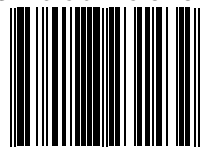
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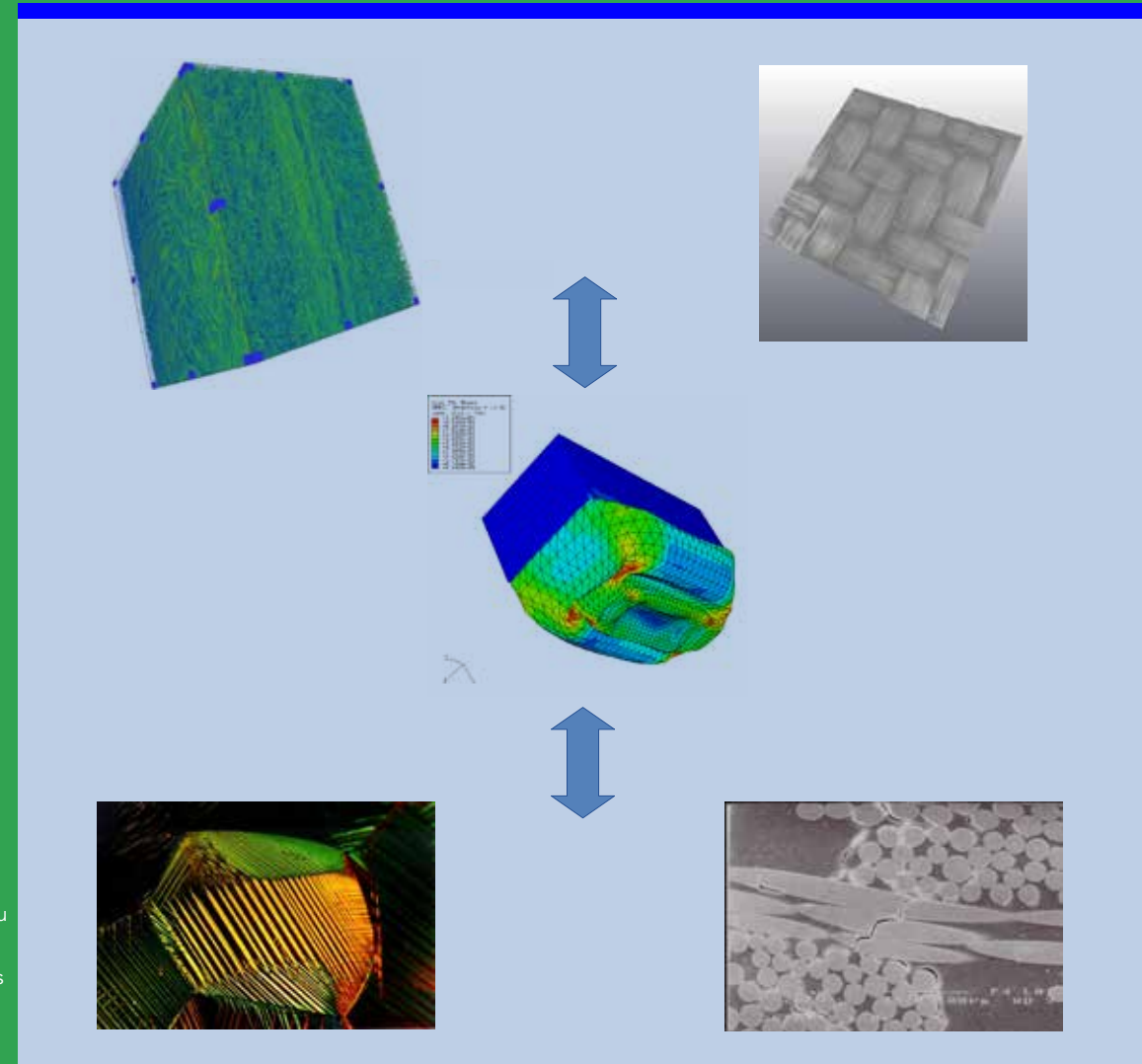
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Chatzigeorgiou
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Charalambakis

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