A Kinematic Approach for 6-DOF Part Positioning

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Abstract. This article proposes a fixturing system consists of a cuboid baseplate located through a 3-2-1 configuration of locators. The locators are mounted on machine table/pallet and posses one axial DOF. The workpiece is mounted on the baseplate and all the elements are assumed to be rigid with zero friction. The positioning error of the workpiece is calculated and the compensation is performed by the axial movement of the locators. The proposed analytical model is verified by the simulation performed in the CAD model.

Keywords: Analytical model, Fixturing system, Part positioning, kinematic model.

1   Introduction

There is a competition in the manufacturing industry to design and deliver a variety of high quality products to their customers in the shortest time. Due to rapid change in production technology and customer demand, the manufacturers need to develop flexible manufacturing practices to achieve a rapid turnaround in product development [1]. Among other factors, the use of feasible fixtures is one of the factors influencing the final part's quality. Fixtures are devices used to support, locate and hold a workpiece at a desired position and orientation in machine's workspace during manufacturing. The final part's quality is influenced by the capability of the fixture to precisely hold and locate it on the machine considering different functional conditions during fabrication. About 10-20% of total manufacturing cost is associated with the fixtures in traditional FMS systems [2]. The design of fixtures is important to precisely hold the workpiece and compensate the errors that the workpiece can encounter during machining or assembling operation, so that higher product's quality can be ensured [3].

The need of high quality production, at lower cost, has accelerated the research efforts in fixture design. To cope with current market demand, Ryll et al. [4] emphasize on the need of "intelligent" fixtures which should be capable of self-configuring; reducing and compensating dimensional errors; providing stability and adapting

clamping forces to guarantee optimum performances. This fixture should be generic and should be able to adapt to different workpiece configurations.

2   Positioning Errors

Dimensional errors of the parts from a part family cause the initial misplacement between the workpiece and machine tool affecting the final product quality. The possible causes of the positioning errors between the machine tool and the workpiece are shown in Figure 1, which are:

- Error due to the placement of locators [5–8]
- Geometric/form defects of the workpiece [9–12]
- Errors due to deformation of locators [13–18]
- Kinematic defects/ machine tool errors [19–26]
- Misc. errors due to tool wear, heat, NC codes, etc…

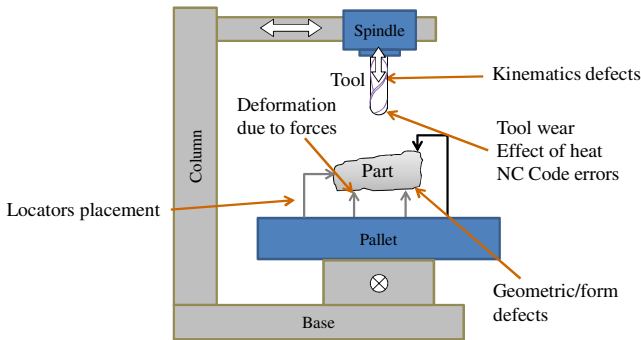
Fig. 1. Errors between the machine tool and the workpiece

Rough workpiece's dimensions are varied from one part to another, so the machining allowances have to be added. Even after the addition of allowances, the rough workpiece may not be completely included in required position, which causes the wastage of the workpiece due to incomplete machining. To avoid the loss of time and material, it is necessary to precisely place each new part relative to machine tool. But this placement needs a mobilization mechanism on the machine. This mechanism should assure the kinematic transformation to place the workpiece at an optimal position by compensating the positioning error between the workpiece and the machine-tool. A high number of degrees of freedom (DOF) machine would be an easy way to perform this compensation.

In an existing serial production environment, the global choice of 5-axis machines in the whole production line is not an economically feasible choice. So a new fixturing system is proposed. This fixturing system is able to perform a 6 DOF workpiece's repositioning on a low DOF production machine through the axial motion of 6 sup-

porting locators placed at 3-2-1 configuration. The initial and final positions of the workpiece are given as the input data and an algorithm calculates the positioning error and the axial displacement of each locator required to compensate this positioning error.

The proposed system can be used on the existing machines as well as on automatic production lines where the number of axis is limited for each station. The proposed system allows better positioning of the workpiece on the fixture and hence limiting the required allowances. It also insures a prepositioning of complex parts for precise machining operations. The necessary geometric and kinematic models of the proposed fixturing system are presented in this article.
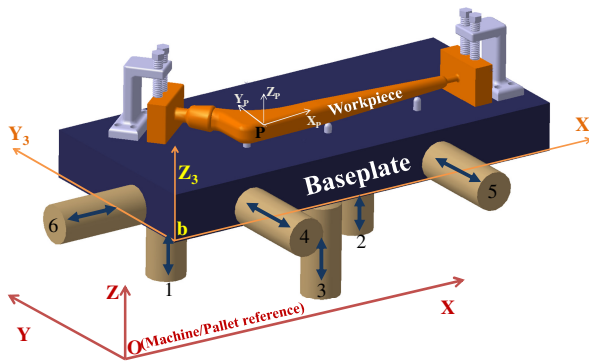

Fig. 2. Proposed fixturing system principle

3   Proposed Fixturing System

This article proposes a fixturing system consists of a set of six locators whose positions and orientations are defined through locating holes of the machine table/pallet, a cuboid baseplate, and a workpiece fixed on the baseplate as shown in Fig. 2. Hip prosthesis is chosen as the demonstrated workpiece because it requires repetitive machining operation on expensive material and the dimensions of part change according to patient need. The baseplate is introduced because when the locators are directly in contact with the rough workpiece surfaces, it is impossible to attain the precise positioning of the workpiece through the axial displacement of 6 locators due to uncertainty of the contacting points caused by the local geometrical defect at rough contacts. The positioning surfaces of the baseplate are considered to be perfectly plane and orthogonal. This assumption causes the surface normals to always remain parallel to the contacts' normals, which enables us to predict the exact location of the workpiece by the locators' positions. Thus the addition of intermediate baseplate avoids this positioning uncertainty: kinematic model will be independent of part geometry.

The locators are assumed to be in a 3-2-1r configuration [27] and possess only one axial DOF. The lateral position of each locator is chosen by considering the constraints of accessibility, stability of the workpiece and manufacturing knowledge. It is

also assumed that the workpiece is mounted rigidly on the baseplate and no additional deformation occurs between workpiece and baseplate except those caused during clamping the workpiece.

3.1   Analytical Formulation

For kinematic analysis, all the elements of the fixturing system are assumed to be rigid. It is assumed that the positioning error of the baseplate is negligible as compared to the positioning error of the workpiece. Also the unknown initial position of the workpiece could imply large displacements (LD) during correction phase; the kinematic model is built using homogeneous transformation matrices (HTM) and LD formulation. The initial position of the workpiece can be measured through CMM while its final position is the position according to which the machine tool is programmed. This position is known by the part program. These positions are compared and if the difference is more than the allowed tolerance, the algorithm calculates the unique relative axial position of each locator to relocate the workpiece at the required position.
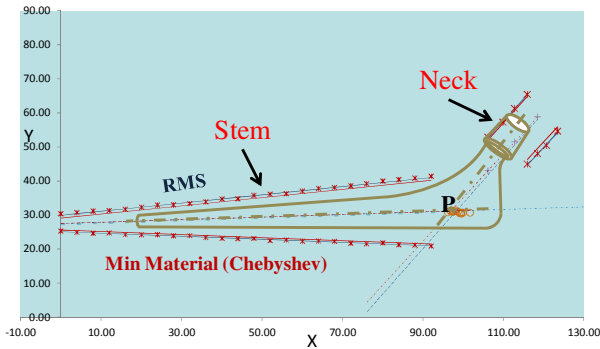

Fig. 3. 2D Demonstration of measurement though CMM

The measurement principle of the hip prosthesis though CMM (in 2D) is shown in Fig. 3. Rough part dimensions are larger than the final product. Random measured points are generated in MS Excel for stem and neck of the hip prosthesis. RMS and Chebyshevs' surface association criteria are presented [28], [29], and theoretical centerlines (for neck and stem) are then deduced. The angle between these centerlines should be under the tolerance range. Point P denotes the intersection of centerlines. In 3D space, the definition of point P, in machine reference, cancels 3 DOF; the definition of the XY plane cancels two more DOFs and the last DOF is canceled by defining the angle of stem axis with XZ plane, completing workpiece placement in the machine space. Some position variations among the parts of the same part family will remain. Random measuring points are generated and the point P is calculated for each set of measuring points. The generated distribution of P is also presented in Fig. 3.

The HTM of cuboid baseplate position is the function of its surface normals calculated from the positions of the six locators [12]. This HTM is shown in Eq. (1) where a, b and c are the unit vector components; 1, 2 and 3 are the unit vectors in Z, Y and X directions while x_b, y_b and z_b are the coordinates of baseplate origin.

$$[P_{Ob}] = \begin{bmatrix} a_3 & a_2 & a_1 & x_b \\ b_3 & b_2 & b_1 & y_b \\ c_3 & c_2 & c_1 & z_b \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Similarly, the HTM of the workpiece position in machine coordinates is defined considering YPR transformation as shown in Eq. (2) with α, β and γ being the rotations along Z, X and Y axes respectively.

$$[P_{OP}] = \begin{bmatrix} \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma & -\sin\alpha\cos\beta & \cos\alpha\sin\gamma + \sin\alpha\sin\beta\cos\gamma & x_P \\ \sin\alpha\cos\gamma + \cos\alpha\sin\beta\sin\gamma & \cos\alpha\cos\beta & \sin\alpha\sin\gamma - \cos\alpha\sin\beta\cos\gamma & y_P \\ -\cos\beta\sin\gamma & \sin\beta & \cos\beta\cos\gamma & z_P \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Positioning transformation scheme of the proposed fixturing system is shown in Fig. 4 where X_i represents the position vector of reference i while $[P_{ij}]$ represents the transformation matrix from position i to j.   The HTM of the baseplate with respect to machine reference ($[P_{Ob}]$) is calculated from the locators' initial positions. The transformation of the workpiece relative to the machine ($[P_{OP}]$) can be measured through CMM. Thus the required transformation of workpiece with respect to baseplate ($[P_{bP}]$) is deduced and HTM of the error compensation ($[P_{Ob'}]$) is calculated as shown in Eq. (**3**). Final absolute positions of all the six locators, required to compensate the workpiece positioning error, are shown in Eq. (4).
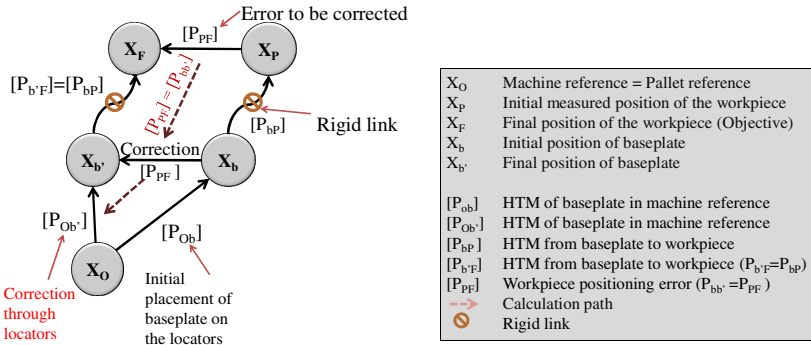

Fig. 4. Fixturing system reference transformation

$$[P_{bP}] = [P_{Ob}]^{-1}[P_{OP}]$$
$$[P_{Ob'}] = [P_{OF}][P_{bP}]^{-1} \tag{3}$$

$$[P_{Ob'}] = \begin{bmatrix} a'_3 & a'_2 & a'_1 & x'_b \\ b'_3 & b'_2 & b'_1 & y'_b \\ c'_3 & c'_2 & c'_1 & z'_b \\ 0 & 0 & 0 & 1 \end{bmatrix} = [P_{OF}]([P_{Ob}]^{-1}[P_{Ob'}])^{-1} \tag{4}$$

The resolution of the above equations give the positions of locators which are imposs-ible to attain because the contacting points of locators on the baseplate change as a result of rigid body motion of the baseplate on locators. This is shown with a 2D ex-ample in Fig. 5, where the final calculated positions of the arc centers of locators are shown by 1* and 2*. Due to the constraint of uniaxial motion, the locators cannot be advanced to these positions. To overcome this mathematical issue, a line is drawn between the points 1* and 2* (plane in our case of 3D), and the points of intersections of this line with the locators' axes are calculated. Moving the locators at these calcu-lated positions will enable us to perform the required workpiece transformation. In the same manner, axial advancements of all the six locators are calculated through the contacting points of all three contacting surfaces. The final axial position of locator 1 is shown in Eq. (5) with a'_1, b'_1 and c'_1 being the unit vector components of the ba-seplate surface. The advancements of the rest of the locators are deduced similarly.

$$z'_1 = \frac{D_1 - a'_1 x_1 - b'_1 y_1}{c'_1} \tag{5}$$

(a) drawing line and calculating new axial positions (b) perform advancement
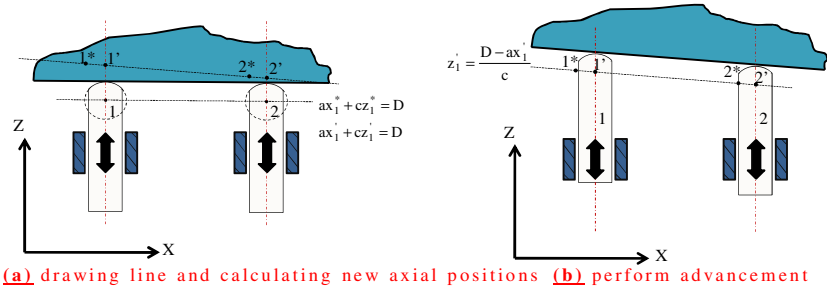
Fig. 5. Calculating the axial advancements of locators

3.2   Case Study

In order to validate the kinematic model, a case study is performed on a hip prosthesis repositioning through CATIA® simulation.   A CPT® 12/14 Hip Prosthesis by Zimmer [30] is chosen as a demonstrative workpiece. The part is created in CATIA® with slightly larger dimensions and supports are added. It is supposed that this workpiece is clamped rigidly on the baseplate which is further located through six rigid locators. An inverse impression of the workpiece (like a half die) is created with the original hip prosthesis dimensions and is placed on a fixed position with reference to the ma-chine origin. This position represents the tool path on the machine as the tool moves with reference to machine and not with reference to workpiece. Boolean operation is

performed to simulate the machining operation by subtracting the common material from the workpiece. Two slots are made in the supports during machining of the first half part which will help to place the workpiece on two well positioned blocks after inverting.

The analytical model is implemented in a worksheet directly linked to CATIA® model which furnishes the initial position ([P_{OP}]) of the workpiece as shown in table 1(a). This position should be obtained by CMM in real environment. The initial position of the baseplate ([P_{Ob}]) is a function of locators' positions shown in Table 1(b). The machining performed on this initially roughly placed workpiece is shown in Fig. 6. The workpiece should be repositioned at the required position ([P_{OF}]) to perform a precise machining operation. This final position is known by the part program and is shown in the Table 2.

Table 1. Initial positions of locators and the workpiece

(a) Initial locators' positions (Axial positions are highlighted)

Locator no	x (mm)	y (mm)	z (mm)
1	70	100	**15.00**
2	180	100	**15.00**
3	120	40	**15.00**
4	70	**10.00**	40
5	180	**10.00**	40
6	**8.00**	60	40

(b) Initial workpiece position

Plane Angle	Degree	Point P	mm
α_i	0.75	x_P	102.62
β_i	-0.06	y_P	57.23
γ_i	0.45	z_P	70.19

Table 2. Required position of the workpiece (Objective)

Plane Angle	Degree	Point P	mm
α_f	0	x_F	100
β_f	0	y_F	60
γ_f	0	z_F	70

Gray: Machined surface
Orange: Rough surface
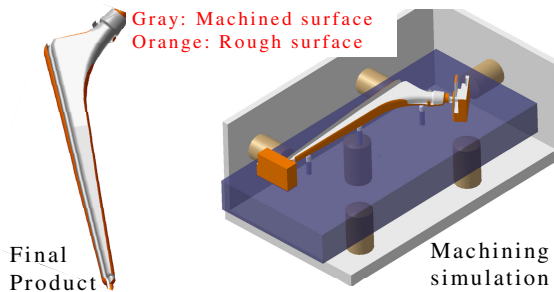
Final Product

Machining simulation

Fig. 6. Machining simulation on the workpiece at initial position

The algorithm calculates the final axial positions of all the six locators (Table 3) to compensate the workpiece positioning error. The locators are moved to these new

positions and the machining simulation is re-performed. This time the material removal was uniform throughout the workpiece as shown in Figure 7.
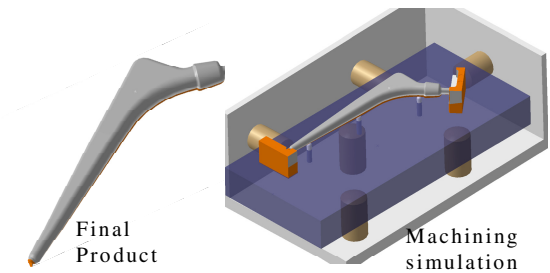

Final
Product

Machining
simulation

Fig. 7. Machining simulation on the workpiece after repositioning

Table 3. Calculated final position of the six locators (Axial positions are highlighted)

Locator no	x (mm)	y (mm)	z (mm)
1	70	100	14.62
2	180	100	15.48
3	120	40	14.95
4	70	13.20	40
5	180	11.75	40
6	5.61	60	40

Simple investigation reveals that the workpiece was not at the exact required position. The 6 DOF repositioning error of the workpiece is shown in Table 4(a) while the same for the second side is shown in Table 4(b). This positioning uncertainty is due to the limited advancement precision (10 μm) of locators. This positioning uncertainty can be expressed as robustness of the proposed model.

Table 4. Workpiece positioning error due to locators' precision

(a)   First side of the workpiece

Plane Angle	Degree	Point P	mm
α_i	0.002	x_P	0.006
β_i	0.008	y_P	-0.004
γ_i	-0.002	z_P	0.001

(a)   Second side of the workpiece

Plane Angle	Degree	Point P	mm
α_i	0.00	x_P	0.002
β_i	0.00	y_P	0.00
γ_i	0.00	z_P	0.00

3.3   Robustness of the Model

The workpiece position uncertainty is calculated from the Plucker coordinates[31] as the function of precision of locators' advancements. In our case, using the locators' input positions (Table 1. Initial positions of locators and the workpiece(a)), the uncertainty at reference point P (Table 2) is deduced as a function of six advancements,

$$\left\{\begin{array}{c} \delta x_P \\ \delta y_P \\ \delta z_P \\ \delta\alpha \\ \delta\beta \\ \delta\gamma \end{array}\right\} = \left\{\begin{array}{c} dx_6 - \frac{6dy_4}{11} + \frac{6dy_5}{11} - \frac{4dz_1}{11} + \frac{4dz_2}{11} \\ \frac{18dy_4}{11} - \frac{7dy_5}{11} + \frac{4dz_1}{11} + \frac{10dz_2}{33} - \frac{2dz_3}{3} \\ \frac{8dz_1}{11} - \frac{46dz_2}{33} + \frac{5dz_3}{3} \\ \frac{dz_1}{110} + \frac{dz_2}{132} - \frac{dz_3}{60} \\ \frac{dz_1}{110} - \frac{dz_2}{110} \\ -\frac{dy_4}{110} + \frac{dy_5}{110} \end{array}\right\} = \left\{\begin{array}{c} 31/11 \\ 128/33 \\ 125/33 \\ 1/30 \\ 1/55 \\ 1/55 \end{array}\right\} \times \xi \qquad (6)$$

where, dz_1, dz_2, dz_3, dy_4, dy_5 and dx_6 are uncertainties of the locators' advancements. In order to calculate the maximum positioning error, all the term are arranged so that their effect is added to the positioning error. The right most vector in Eq. (6) is the maximum positioning error as the function of precision of locators' advancements ξ, in our case, assumed to 10μm.

4   Conclusion

A fixturing system has been proposed which is capable of performing the compensation of the positioning error of the workpiece through the advancement of six locators. To allow a repetitive repositioning of irregular parts, a baseplate has been placed in between the machine table and the workpiece. The baseplate has been located through a 3-2-1 locating configuration and all the fixturing elements were considered to be rigid. The kinematic model calculated the locators' advancements which enabled us to relocate the workpiece indirectly by baseplate relocation. The kinematic model has been simulated in CATIA and the results verified the analytical model.

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