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Nonlinear structural damage detection based on cascade of Hammerstein models

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Abstract

Structural damages can result in nonlinear dynamical signatures that can significantly enhance their detection. An original nonlinear damage detection approach is proposed that is based on a cascade of Hammerstein models modelisation of the structure. This model is estimated by means of the Exponential Sine Sweep Method from only one measurement. On the basis of this estimated model, the linear and nonlinear parts of the output are estimated, and two damage indexes (DIs) are proposed. The first DI is built as the ratio of the energy contained in the nonlinear part of an output versus the energy contained in its linear part. The second DI is the angle between the subspaces described by the nonlinear parts of two set of outputs after a principal component analysis. The sensitivity of the proposed DIs to the pres-

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ence of damages as well as their robustness to noise are assessed numerically on spring-mass-damper structures and experimentally on actual composite plates with surface-mounted PZT-elements. Results demonstrate the effectiveness of the proposed method to detect nonlinear damage in nonlinear structures and in the presence of noise.

Keywords:

Structural health monitoring, non-linear system identification, damage detection.

1 1. Introduction

The process of implementing a damage detection strategy for aerospace, 2 civil, and mechanical engineering is referred to as structural health monitoring (SHM). In many cases, damages that appear on complex structures (such 4 as cracks, impacts, or delaminations) can result in nonlinear dynamical re-5 sponses that may be used for damage detection [1–4]. Furthermore, complex 6 structures often exhibit a nonlinear behavior even in their healthy states. A 7 robust and reliable SHM system must then be able to deal with nonlinear 8 damages, and to distinguish between their effects and inherent nonlinearities 9 in healthy structures. Several limitations of existing methods that are fac-10 ing these issues have been recently identified in a report by Farrar *et al.* [1]. 11 The first problem to be addressed is that "nonlinear behavior does not gen-12 eralize". This implies that the nonlinear models already in use are never 13 general enough to encompass all the structure encountered in real life. The 14 second problem is that "nonlinear approaches are computationally cumber-15 some, expensive, and requires too many parameters to be defined". Currently 16

¹⁷ developed nonlinear models are thus not adequate for practical use of SHM
¹⁸ systems. The work presented here attempts to face these two problems on
¹⁹ the basis of a simple, but rather general, nonlinear model identified by means
²⁰ of a simple signal processing procedure.

In order to build a damage index (DI) that is sensitive to nonlinearities 21 different approaches have already been proposed [1, 2]. Some DIs are based 22 on a physical modeling of the damaged structure whereas some are computed 23 without any physical assumption (black box models). Among these black-box 24 approaches, some assume a parametric underlying signal processing model, 25 whereas some are fully non-parametric. To feed these models, random inputs 26 as well as deterministic broadband or narrowband inputs are used. In this pa-27 per, the focus is put on nonlinear damage detection approaches based on DIs 28 built using a non-parametric black box model estimated using a deterministic 29 broadband signal. There have been relatively few works in that direction. In 30 a linear framework, some authors [5, 6] have shown that a nonlinear damage 31 will impact the transmissibility functions (i.e. the frequency domain ratio 32 between two different outputs of the system) and they used such information 33 to detect and locate the damage. Extending the notion of transmissibility 34 functions to nonlinear systems that can be described by Volterra series, Lang 35 et. al [7, 8] were able to quantify the decrease of linearity generated by a 36 nonlinear damage and thus to effectively detect and locate it. However, as 37 such approaches are focusing on the loss of linearity, they do not seem to be 38 able to deal with systems that are nonlinear in their healthy states, a fact 39 that is quite common in real life. To overcome this drawback, several authors 40 attempted to fit a nonlinear model to the nonlinear structure under study 41

⁴² and to compare the actual and predicted outputs, or directly the model co-⁴³ efficients, under different damage conditions [9–12]. By doing so, they were ⁴⁴ able to detect numerically and experimentally a nonlinear damage even in ⁴⁵ an initially nonlinear structure. However, the models they used where para-⁴⁶ metric (mainly frequency domain ARX models) and thus were not easy to ⁴⁷ manipulate and neither able to model, without any *a priori* on it, a general ⁴⁸ nonlinear structure.

We propose here an original approach devoted to nonlinear damage de-49 tection in possibly nonlinear structures based on a simple, but rather general, 50 nonlinear model estimated by means of standard signal processing tools. This 51 approach is based on the assumption that the structure under study can be 52 modeled as a cascade of Hammerstein models [13], made of N branches in 53 parallel composed of an elevation to the n^{th} power followed by a linear fil-54 ter called the n^{th} order kernel, see Fig. 1(a). The Exponential Sine Sweep 55 Method [14, 15], previously developed and validated by the authors for dif-56 ferent purposes, is then used to estimate the different kernels of the model. 57 Exponential sine sweeps are a class of sine sweeps that allow estimating a sys-58 tem's N first kernels in a wide frequency band from only one measurement. 50 Two damage indexes are then build on the basis of this estimated model. 60 The first one reflects the ratio of the energy contained in the nonlinear part 61 of the output versus the energy contained in its linear part and is specially 62 suited for single-input single-output (SISO) systems. The second one is the 63 angle between the subspaces described by the nonlinear parts of two set of 64 outputs after a principal component analysis. This one is specially suited for 65 single-input multi-output (SIMO) systems. As a first step toward the use 66

of this method for SHM, the sensitivity of the proposed DIs to the presence
of damages as well as their robustness to noise are assessed numerically on
SISO and SIMO systems and experimentally on two actual composite plates
with surface-mounted PZT-elements (one healthy and one damaged).

The cascade of Hammerstein models as well as the mathematics behind it are first described in Sec. 2. The two proposed DIs are then defined in Sec. 3. Their sensitivity to the presence of damages as well as their robustness to noise are assessed numerically in Sec 4 and experimentally in Sec. 5. A general conclusion is finally drawn in Sec. 6.

⁷⁶ 2. Cascade of Hammerstein models estimation using the exponen⁷⁷ tial sine sweep method

78 2.1. Cascade of Hammerstein models

A possible approach to non-linear system identification is to assume that systems have a given block-structure. Following the "sandwich" approach [13], a non-linear system can be represented as N parallel branches composed of three elements in series: a static non-linear part sandwiched between two linear parts. Such systems are a subclass of Volterra systems and it can be shown that any continuous non-linear system can be approximated by such a model [16].

Cascade of Hammerstein models are a simplification of this "sandwich" approach. In a cascade of Hammerstein system [13], each branch is composed of one nonlinear static polynomial element followed by a linear one $h_n(t)$ as shown in Fig. 1(a). The relation between the input e(t) and the output s(t)of such a system is given by Eq. (1) where "(*)" denotes the convolution



Figure 1: (a) Cascade of Hammerstein model and (b) temporal separation after deconvolution.

91 operator.

$$s(t) = \sum_{n=1}^{N} (h_n * e^n) (t)$$
 (1)

It can easily be shown from Eq. (1) that cascades of Hammerstein mod-92 els correspond to Volterra models having diagonal Kernels in the temporal 93 domain [15]. Thus, cascades of Hammerstein models represent a subclass 94 of all the nonlinear "analytical" systems described by Volterra models, and 95 are thus rather general nonlinear models. Furthermore, any cascade of Ham-96 merstein models is fully represented by its kernels $\{h_n(t)\}_{n \in \{1...N\}}$, which are 97 only a set of linear filters. This model is thus also quite simple to use and 98 intuitive to understand. 99

100 2.2. Exponential sine sweeps

Estimating each kernel $h_n(t)$ of a cascade of Hammerstein models is not a straightforward task. An simple estimation method that has been proposed ¹⁰³ previously by the authors [15] for this purpose and that is the basis of the
¹⁰⁴ damage detection procedure is briefly recalled here.

To experimentally cover the frequency range over which the system under study has to be identified, cosines with time-varying frequencies are commonly used. Indeed, if $e(t) = cos[\phi(t)]$ is the input of the cascade of Hammerstein models, the output of the nonlinear block $e^n(t)$, see Fig. 1(a), can be rewritten using Chebyshev polynomials as in Eq. (2). Details of the computation of the Chebyshev matrix $C = \{c_{n,k}\}$ are provided in [15].

$$\forall n \in [1..N] \quad e^n(t) = \cos^n\left[\phi(t)\right] = \sum_{k=0}^n c_{n,k} \cos\left[k\phi(t)\right] \tag{2}$$

When the instantaneous frequency of e(t) is increasing exponentially from f_1 to f_2 in a time interval T, this signal is called an "Exponential Sine Sweep". It can be shown in [14, 15], that by choosing $T_m = \left(2m - \frac{1}{2}\right) \frac{\ln(f_2/f_1)}{2f_1}$ with $m \in \mathbb{N}^*$, one obtains:

$$\forall k \in \mathbb{N}^* \quad \cos\left[k\phi(t)\right] = \cos\left[\phi(t + \Delta t_k)\right] \quad \text{with} \quad \Delta t_k = \frac{T_m \ln(k)}{\ln(f_2/f_1)} \tag{3}$$

Eq. (3) is another expression of the k^{th} term in the linearization presented in Eq. (2). In summary, for any exponential sine sweep of duration T_m , multiplying the phase by a factor k yields to the same signal, advanced in time by Δt_k .

119 2.3. Kernel recovery in the temporal domain

If an exponential sine sweep is presented at the input of a cascade of Hammerstein models, we obtain by combining Eq. (3) and Eq. (1) the following relation:

$$s(t) = \sum_{n=1}^{N} (\gamma_n * e)(t + \Delta t_n) \quad \text{with} \quad \gamma_n(t) = \sum_{k=1}^{n} C(k, n) h_k(t) \tag{4}$$

where $\gamma_n(t)$ corresponds to the contribution of the different kernels to the nth harmonic.

In order to separately identify each kernel $h_n(t)$ of the cascade of Hammerstein models, a signal y(t) operating as the inverse of the input signal e(t) in the convolution sense, is needed. The Fourier transform Y(f) of the inverse filter y(t) can be built by means of Eq. (5):

$$Y(f) = \frac{1}{E(f)} \simeq \frac{\overline{E}(f)}{|E(f)|^2 + \epsilon(f)}$$
(5)

where E(f) and $\overline{E}(f)$ are respectively the Fourier transform of e(t) and its complex conjugate, and $\epsilon(f)$ is a frequency-dependent real parameter chosen to be 0 in the bandwidth of interest and to have a large value outside, with a continuous transition between the two domains, see [15].

After convolving the output of the cascade of Hammerstein models s(t)is given in Eq. (4) with y(t), one obtains Eq. (6), also illustrated in Fig. 1(b):

$$(y*s)(t) = \sum_{n=1}^{N} \gamma_n(t + \Delta t_n)$$
(6)

Because $\Delta t_n \propto ln(n)$ and $f_2 > f_1$, the higher the order of non-linearity n, the more advanced is the corresponding $\gamma_n(t)$, see Fig. 1(b). Thus, if T_m is chosen long enough, the different $\gamma_n(t)$ do not overlap in time and can be separated by simply windowing them in the time domain. Using Eq. (7), the family $\{h_n(t)\}_{n\in[1,N]}$ of the kernels of the cascade of Hammerstein models under study can then be fully extracted.

$$\begin{pmatrix} h_1(t) \\ \vdots \\ h_N(t) \end{pmatrix} = \tilde{C}^T \begin{pmatrix} \gamma_1(t) \\ \vdots \\ \gamma_N(t) \end{pmatrix}$$
(7)

In Eq. (7), C^T stands for the transpose of the Chebyshev matrix C, and \tilde{C} represents C, from which the first column and the first row have been removed.

It can be noticed here that the proposed method is not fully nonparametric. Indeed, one parameter, N the order of nonlinearity up to which the nonlinear model has to be estimated, is still to be chosen. Its choice mainly depends on the noise conditions and on the length of the input exponential sine sweep [15].

¹⁴⁹ 3. Novelty damage indexes

In the case of a structure with distributed actuators and sensors, we can 150 consider several configurations to perform damage monitoring. As here an 151 active SHM approach has been retained, measurements of one sensor can be 152 used by defining a path over the structure that leads to a single-input single-153 output (SISO) system. However, the measurements of all sensors can also be 154 used simultaneously, thus defining in that case a single-input multi-output 155 (SIMO) system. The two novelty damage indexes proposed in the following 156 correspond to these two configurations. 157

158 3.1. Decomposition of the output signal into linear and nonlinear parts

By rephrasing Eq. (1) which expresses the output of the cascade of Hammerstein models s(t) as a function of the input signal e(t) and of the Hammerstein kernels $\{h_n(t)\}_{n\in[1,N]}$, it is possible to decompose the output of the cascade of Hammerstein models as follows:

$$s(t) = (h_1 * e)(t) + \sum_{n=2}^{N} (h_n * e^n)(t) = s^{\mathrm{L}}(t) + s^{\mathrm{NL}}(t)$$
(8)

where $s^{L}(t) = (h_1 * e)(t)$ stand for the linear and $s^{NL}(t) = \sum_{n=2}^{N} (h_n * e^n)(t)$ the nonlinear parts of the output signal s(t).

As the input signal e(t) is known and as the Hammerstein kernels $\{h_n(t)\}_{n \in [1,N]}$ have been estimated previously, those linear and nonlinear parts of the output signal are then easily evaluated and can be used to build damage indexes.

168 3.2. DI_1 : Ratio of the nonlinear energy to the linear energy

In the single-input, single-output case (SISO), there is only one input e(t) and one output s(t). Taking advantage of Eq. (8), we propose a damage index (DI) that is defined as the ratio of the energy contained in the nonlinear part of the output of the cascade of Hammerstein models versus the energy contained in the linear part of the output of the cascade of Hammerstein models. By denoting $S^{L}(f)$ and $S^{NL}(f)$ the Fourier transform of $s^{L}(t)$ and $s^{NL}(t)$, we propose a damage index defined as follow:

$$DI_{1} = \frac{\int_{f_{1}}^{f_{2}} |S^{NL}(f)|^{2} df}{\int_{f_{1}}^{f_{2}} |S^{L}(f)|^{2} df}$$
(9)

where f_1 and f_2 have been defined earlier in Sec. 2.2.

In a given composite structure, as the nonlinear damage (impact, delamination, or crack) becomes more severe, it is expected to contribute more and more to the nonlinear part of the output $s^{\text{NL}}(t)$. As a consequence, DI₁ is expected to be sensitive to the presence of the damage, but also to its extent. This will be demonstrated numerically in Sec. 4.3 and experimentally in Sec. 5.

183 3.3. DI₂ : Angle between nonlinear subspaces

In the single-input, multiple-output framework (SIMO), there is still one input e(t) but now J outputs $\{s_j(t)\}_{j\in[1..J]}$. Taking advantage of Eq. (8), it is still possible to decompose each output $s_j(t)$ into its linear $s_j^{\rm L}(t)$ and nonlinear $s_j^{\rm NL}(t)$ parts. Following previous work by the authors [17], the idea is then to monitor the subspaces spanned by the nonlinear parts of each outputs set.

In a discrete-time matrix form, let $\mathbf{s}^{NL} \in \mathbb{R}^{U \times J}$ be the nonlinear parts of the *J* output signals having each a length of *U* samples. Let $\mathbf{A}^{NL} \in \mathbb{R}^{U \times J}$ be the separating matrix of \mathbf{s}^{NL} . This matrix is obtained from a principal component analysis technique [18] and is defined as follows:

$$\boldsymbol{A}^{\mathrm{NL}} = \boldsymbol{\Lambda}_{\boldsymbol{s}^{\mathrm{NL}}}^{-\frac{1}{2}} \times (\boldsymbol{P}^{\mathrm{NL}})^{\mathrm{T}}$$
(10)

where $\boldsymbol{P}^{\text{NL}} = \left[\underline{\boldsymbol{p}}_{1}^{\text{NL}}, \dots, \underline{\boldsymbol{p}}_{J}^{\text{NL}}\right]$ is the matrix of eigenvectors of $\boldsymbol{s}^{\text{NL}}$ and $\Lambda_{\boldsymbol{s}^{\text{NL}}}$ is the diagonal matrix of eigenvalues of $\boldsymbol{s}^{\text{NL}}$. If the reduction using singular value decomposition (SVD) is possible [19], the separating matrix can then be rewritten as follows:

$$\boldsymbol{A}^{\mathrm{NL}} = \boldsymbol{I}_{J \times J} \times \boldsymbol{\Gamma}^{\mathrm{NL}} \times (\boldsymbol{V}^{\mathrm{NL}})^{\mathrm{T}}$$

$$= \begin{bmatrix} \boldsymbol{I}_{J \times J_{p}} & \boldsymbol{I}_{J \times (J - J_{p})} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Gamma}_{1}^{\mathrm{NL}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Gamma}_{2}^{\mathrm{NL}} \end{bmatrix} \begin{bmatrix} \boldsymbol{V}_{1}^{\mathrm{NL}} & \boldsymbol{V}_{2}^{\mathrm{NL}} \end{bmatrix}^{\mathrm{T}}$$

$$= \boldsymbol{A}_{1}^{\mathrm{NL}} + \boldsymbol{A}_{2}^{\mathrm{NL}}$$

$$(11)$$

where $\Gamma_{1}^{\text{NL}} = diag(\sigma_{1}, \ldots, \sigma_{J_{p}}), V_{1}^{\text{NL}} = \left[\underline{\boldsymbol{v}}_{1_{1}}^{\text{NL}}, \ldots, \underline{\boldsymbol{v}}_{1_{J_{p}}}^{\text{NL}}\right] \in \mathbb{R}^{U \times J_{p}}$ and $A_{1}^{\text{NL}} \in \mathbb{R}^{U \times J_{p}}$ are respectively the matrix of singular values, the matrix of right singular vectors, and the separating matrix associated to the principal subspace of $\boldsymbol{s}^{\text{NL}}$. $\Gamma_{2}^{\text{L}} = diag(\sigma_{J_{p}+1}, \ldots, \sigma_{J}), V_{2}^{\text{NL}} = \left[\underline{\boldsymbol{v}}_{1_{J_{p}+1}}^{\text{NL}}, \ldots, \underline{\boldsymbol{v}}_{1_{J}}^{\text{NL}}\right] \in$ $\mathbb{R}^{U \times (J-J_{p})}$ and $A_{2}^{\text{NL}} \in \mathbb{R}^{U \times (J-J_{p})}$ are respectively the matrix of singular values, the matrix of right singular vectors, and the separating matrix associated to the residual subspace of $\boldsymbol{s}^{\text{NL}}$.

Let $A_1^{
m NL}$ and $\tilde{A}_1^{
m NL}$ be two matrices built as described previously from 205 measurements in a healthy state and in an unknown state. Let's $R\{(A_1^{\rm NL})^{\rm T}\}$ 206 and $R\{(\tilde{A}_1^{\mathrm{NL}})^{\mathrm{T}}\}$ be the range subspaces of matrices $(A_1^{\mathrm{NL}})^{\mathrm{T}}$ and $(\tilde{A}_1^{\mathrm{NL}})^{\mathrm{T}}$, 207 and $\boldsymbol{P}_{R\{(\boldsymbol{A}_1^{\mathrm{NL}})^{\mathrm{T}}\}}$ and $\boldsymbol{P}_{R\{(\boldsymbol{\tilde{A}}_1^{\mathrm{L}})^{\mathrm{T}}\}}$ the orthogonal projections on these range 208 subspaces obtained though SVD (see [17] for details). We then denote 209 $\underline{\phi}\left[R\{(\boldsymbol{A}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}, R\{(\boldsymbol{\tilde{A}}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}\right]$ the principal angle vectors between the range 210 subspaces $R\{(\boldsymbol{A}_1^{\mathrm{NL}})^{\mathrm{T}}\}$ and $R\{(\boldsymbol{\tilde{A}}_1^{\mathrm{L}})^{\mathrm{T}}\}$. Using the SVD tool, the Euclidean 211 norm of the sinus of this angle is defined as follow [20]: 212

$$\|\sin\left(\underline{\phi}\left[R\{(\boldsymbol{A}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}, R\{(\boldsymbol{\tilde{A}}_{1}^{\mathrm{L}})^{\mathrm{T}}\}\right]\right)\|_{2} = \|\boldsymbol{P}_{R\{(\boldsymbol{A}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}^{\perp}} \times \boldsymbol{P}_{R\{(\boldsymbol{\tilde{A}}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}}\|_{2} \quad (12)$$
$$= \|(\boldsymbol{I}_{J_{p}\times J_{p}} - \boldsymbol{P}_{R\{(\boldsymbol{A}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}}) \times \boldsymbol{P}_{R\{(\boldsymbol{\tilde{A}}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}}\|_{2}$$

²¹³ We then propose to define a damage index as:

$$\mathrm{DI}_{2} = \frac{\|\sin\left(\underline{\phi}\left[R\{(\boldsymbol{A}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}, R\{(\boldsymbol{\tilde{A}}_{1}^{\mathrm{NL}})^{\mathrm{T}}\}\right]\right)\|_{2}}{J_{p}}$$
(13)

where J_p is the number of principal components retained in the principal subspaces. This damage index can be interpreted as the angle between the subspaces described by the nonlinear parts of the outputs in the healthy state and in the unknown state.

In a given composite structure, as the nonlinear damage becomes more severe, it is expected to contribute more and more to the nonlinear parts of the different outputs $\{s_j(t)\}_{j\in[1..J]}$ and then to increase the angle between the associated principal subspaces. As a consequence, DI₂ is expected to be sensitive to the presence of the damage, but also to its extent. This will be demonstrated numerically in Sec. 4.4 and experimentally in Sec. 5.

4. Simulation results

225 4.1. Simulated systems

In order to validate the proposed approach and the associated novelty damage indexes, numerical simulations have been carried out for single-input, single-output (SISO) and single-input, multi-output (SIMO) systems. The systems that have been chosen are simple one degree of freedom and five degrees of freedom spring-mass-damper (SMD) systems and are shown in Fig. 2.

The damage in those systems has been introduced by means of a bilinear stiffness k[x(t)] as a very easy way to simulate a breathing crack. Such cracks







(b)

Figure 2: (a) Simulated single degree of freedom SISO system and (b) simulated five degrees of freedom SIMO system.

have a lower stiffness when the crack is open than when the crack is closed.
Thus, the bilinear stiffness is defined as follows:

$$k[x(t)] = \begin{cases} k^{I} & \text{if } x(t) < 0\\ (1-\alpha)k^{I} & \text{if } x(t) > 0 \end{cases}$$
(14)

In this definition, k^{I} denotes the linear stiffness of the original undamaged system and the *damage-parameter* is the coefficient α . If $\alpha = 0$, the stiffness is fully linear and the system is healthy. If $\alpha = 1$, the stiffness when the crack is open is null and thus, the system if fully damaged.

The chosen SISO system is a SMD system where the input is the force f(t) applied to the mass M and the output is the displacement x(t) of the mass M, as shown in Fig. 2(a). For this system M = 1 kg, b = 2 Ns/m and ²⁴³ $k^I = 20000 \text{ N/m}.$

The chosen SIMO system is a serie of five SMD systems where the input is the force f(t) applied to the mass M_1 and the outputs are the displacements $\{x_1(t), \ldots, x_5(t)\}$ of the masses $\{M_1, \ldots, M_5\}$ as shown in Fig. 2(b). The damage is introduced by means of the bilinear stiffness $k_4[x(t)]$ as defined by Eq. (14). For this system $M_1 = M_2 = M_3 = M_4 = M_5 = 1$ kg, $b_1 = b_2 = b_3 =$ $b_4 = b_5 = 2$ Ns/m, $k_1 = k_2 = k_3 = k_5 = 20000$ N/m, and $k_4^I = 20000$ N/m.

250 4.2. Input signal

In order to estimate the linear and nonlinear parts in the ouptut of this 251 system, an input signal has been designed as described in Sec. 2.2. The start 252 and stop frequencies have been chosen as $f_1 = 2.25$ Hz and $f_2 = 225$ Hz, 253 knowing that $f_r = 25$ Hz is the resonance frequency of both undamaged 254 systems. The sweep duration has been chosen as T = 8.86 seconds for the 255 SISO system and T = 88.6 seconds for the SIMO system both with an input 256 amplitude of E = 0.1 N. The response of this system to this input signal has 257 been simulated using $Simulink^{\text{TM}}$ with a fixed-step Runge-Kutta algorithm 258 running at $f_s = 563$ Hz. A zero-mean Gaussian white noise has been added 259 to the input of the simulation in order to simulate environmental noise. It is 260 assumed that environmental noise is larger than measurement noise, and thus 261 no noise has been added to the output of the simulation. To illustrate the 262 robustness of the proposed DIs to noise, the noise variance has been chosen 263 as a function of the root-mean-square power of the input signal in order to 264 have a signal to noise ratio (SNR) of 60 or 30 dB (i.e. a noise with a standard 265 deviation of 6×10^{-5} N or 2×10^{-3} N). Simulation have been carried out 266 for values of α ranging from 0 (healthy state) to 0.45 (half-damaged state) 267

by steps of 0.025. For each α value, the simulations have been repeated 30 times in order to compute the mean and standard deviation of both DIs when subjected to noise.



 $_{271}$ 4.3. Damage detection using DI_1 for the SISO system

Figure 3: (a) Input sweep without noise and (b) added noise for a SNR of 30 dB. Estimated (c) linear and (d) nonlinear parts of the output signal x(t).

The noise-free input signal, the added noise, and the estimated linear $x^{I}(t)$ and nonlinear $x^{NL}(t)$ parts of the output signal x(t) of the system of Fig. 2(a) are shown on Fig. 3. From this figure, it can seen that a SNR of 30 dB implies the addition of a relatively large amount of noise to the input signal. Furthermore, by analyzing the estimated linear part of the ouptut signal $x^{L}(t)$, it can be seen that, as expected, the chosen nonlinear system basically acts as a resonant filter. Finally, it can be seen that the system under study is effectively nonlinear as a non-null nonlinear part $x^{NL}(t)$ in the output signal is being estimated by the previously described procedure.



Figure 4: Average and standard deviation of the DI_1 values for different values of the damage parameter α with (a) SNR= 60 dB and (b) SNR= 30 dB.

In Fig. 4 the averages and standard deviations over the 30 trials of the 281 DI_1 values for the different values of the damage parameter α and with 282 SNR= 60 dB and SNR= 30 dB are shown. First of all, it can be seen 283 that the damage index DI_1 increases in both cases almost monotically with 284 the damage parameter α . Moreover, even when the noise power is relatively 285 large (see the curve for SNR=30 dB) the standard deviations remain small 286 around the average values. Finally, for a value of the damage parameter 287 $\alpha = 0$ (*i.e.* in the linear case), the DI₁ value should be zero and is found dif-288 ferent from zero. This thus means that a part of the noise is here interpreted 289 by the estimation process as a nonlinear part of the output. We can thus 290

conclude from that simulation that the damage index DI_1 is able to detect and to quantify the amount of damage in the nonlinear system with a high robustness to noise.

²⁹⁴ 4.4. Damage detection using DI₂ for the SIMO system



Figure 5: (a) Input signal f(t) (top) and output signal $x_4(t)$ for $\alpha = 0.45$ (bottom). (b) Average and standard deviation of the DI₂ values for different values of the damage parameter α and for a SNR of 30 dB.

In Fig. 5(a) the noisy input signal f(t) as well as one of the five output 295 signals, $x_4(t)$, of the system of Fig. 2(b) are shown. It can be seen that 296 as previously the nonlinear system of Fig. 2(b) filters the input signal and 297 possesses a clear resonant frequency in the bandwidth under study. Further-298 more, the fact that this system is nonlinear can be easily seen as the output 290 signal is not symmetrical with respect to the horizontal axis. In Fig. 5(b)300 the averages and standard deviations over the 30 trials of the DI_2 values 301 computed by retaining $J_p = 5$ principal components for the different values 302

of the damage parameter α and for a SNR of 30 dB are shown. First of all, 303 it can be seen that again the damage index DI_2 increases almost monotically 304 with the damage parameter α . Moreover, even if the noise power is relatively 305 large (SNR = 30 dB) the standard deviations still remain very small around 306 the average values. The curves for a SNR of 60 dB are not shown here as 307 they are very similar to the one for a SNR of 30 dB but with lower standard 308 deviations. We can thus conclude from that simulation that the damage in-309 dex DI_2 is here also able to detect and to quantify the amount of damage in 310 the nonlinear system with a high robustness to noise. 311

312 5. Experimental results

313 5.1. Plate specimens

The two composite plates employed in this study consist of a piece of 314 aircraft composite fuselage. The dimensions of these structures are $(400 \times$ 315 $300 \times 2 \text{ mm}^3$). They are both made up of 16 layers Carbone epoxy material. 316 The layer sequences are: $(0^{\circ}, 45^{\circ}, -45^{\circ}, 90^{\circ}, 90^{\circ}, -45^{\circ}, 45^{\circ}, 0^{\circ})$. An optimal 317 placement of ten PZ29 piezoceramic patches with dimensions $(30 \times 20 \times$ 318 0.2 mm^3) has been achieved on these two structures using the controllability 319 and observability gramians [21]. The composite plate shown in Fig. 6(a)320 was used as the baseline for damage detection. Fig. 6(b) shows the second 321 composite plate, manufactured from the same material and layer sequences, 322 having the same dimensions and PZT number and placement as the first one. 323 However, in this plate, an calibrated impact damage with a 5 mm diameter 324 was produced by projecting on the center of the plate a steel ball at a high 325 and controlled velocity. This composite plate will be used as a damaged plate 326

327 example.

328 5.2. Data acquisition and Hammerstein Kernels estimation

The input excitation and the data acquisition were performed using a 329 voltage amplifier (TREK MODEL 601C) and charge amplifiers (type 5011B). 330 This excitation was applied sequentially to nine PZT elements and consists of 331 an exponential sine sweep signal with $f_1 = 100$ Hz, $f_2 = 30$ kHz, T = 3.2588 s 332 and an amplitude of 10 V (see Sec. 2). Using a real time prototype system 333 dSPACE, temporal signals were acquired with a sampling frequency $f_s =$ 334 100 kHz from nine channels: one corresponding to the excitation applied to a 335 given PZT actuator, and the eight others corresponding to the measurements 336 collected by the PZT sensors. Under those conditions, the SNR is found to 337 be approximately of 60 dB. 338

A first database has been built, by collecting 10 times in the healthy and 339 damaged states the signals for all the paths starting from the PZT element 340 number 7, located near the center of the plate, close to the damage. The 341 aim of this database is to quantify the environmental variability existing 342 for a given path by computing the damage index DI_1 mean and standard 343 deviation over the 10 trials and to infer a detection threshold for DI_2 defined 344 by Eq. (13). Another database has been built by collecting in each state 345 (healthy or damaged) the signals for the $9 \times 8 = 72$ paths existing between 346 all pairs of PZT elements. The objective of this database is to illustrate the 347 ability of both DIs to detect damages among the different paths and actuators 348 that are considered. For both databases, the Hammerstein Kernels have been 349 estimated using the method described in Sec. 2 up to an order of nonlinearity 350 N = 8. This choice has been done with respects to noise conditions and to 351

the length of the input exponential sine sweep [15].

³⁵³ 5.3. Sensibility to environmental noise and to damage for one actuator

To assess the sensibility of the DI_1 defined in Eq.(9) to the presence of 354 environmental noise, DI_1 values have been computed for all the repetitions 355 for the healthy and damaged plates, as described in Sec. 5.2. In Fig. 7(a), 356 the mean and standard deviation of the DI_1 values computed over the 10 357 repetitions for each path are shown for both states. From this figure, we can 358 see that even in the healthy state, the DI_1 values are around 0.77. This means 359 that there is a non-negligible part of the energy in the nonlinear part of the 360 output and thus that the system under study is nonlinear in its healthy state. 361 This illustrates the fact that the proposed method can handle systems that 362 are nonlinear in their healthy state. From that figure, it can also be seen that 363 the variations caused by environmental noise on DI_1 values remains relatively 364 low and that the DI_1 values for the damaged case are well above the DI_1 values 365 for the healthy state. As such, we can conclude that experimentally the 366 proposed DI_1 is not much sensitive to environmental noise and is effectively 367 sensitive to the presence of the damage. 368

The damage index DI_2 defined by Eq. (13) is comparing the nonlinear 369 subspaces spanned by a reference state and by an unknown state. As so, 370 this DI is relative by nature and a decision threshold needs to be defined in 371 order to decide whether or not there is presence of a damage. To do so, we 372 decided here to proceed experimentally by using the 10 repetitions for PZT 373 7 in the healthy state. The first repetition has been chosen as the reference 374 state, and the nine others as unknown (but healthy) states. The DI_2 values 375 obtained by comparing these unknown (but healthy) states to the reference 376

one are plotted on Fig. 7(b). It can be seen that the environmental noise 377 does not cause large changes to these DI_2 values. On the basis of these 378 values, a decision threshold has then been defined as ten times the maximum 379 value obtained previously. The factor ten is arbitrarily chosen but is thought 380 here to be sufficiently large for reasonable decision making. This decision 381 threshold is also plotted on Fig. 7(b). Now, the impact of environmental 382 noise in the damaged case can be assessed by comparing, for each trial the 383 DI_2 value obtained by comparing the healthy and damaged states. As shown 384 in Fig. 7(b), for each trial, the obtained DI_2 values are not so influenced by 385 noise and are always above the decision threshold value. Thus, we can say 386 that the DI_2 defined by Eq. (13) is not very sensitive to noise and appears 387 to be sensitive to the presence of the damage for this actuator. 388

³⁸⁹ 5.4. Sensibility to the presence of the damage for all the actuators

To assess the sensibility of the DI_1 defined in Eq. (9) to the presence 390 of the damage for different actuators and paths over the plate, DI_1 values 391 have been computed for each of the 72 paths measured on the healthy and 392 damaged plates (as described in Sec. 5.2). For sake of brevity, DI_1 values 393 are presented here in a synthetic manner actuator by actuator. Fig. 8(a) 394 depicts the mean and standard deviation of the DI_1 values computed for all 395 the paths starting from each of the nine PZT elements used as actuators for 396 both the healthy and damaged states. From Fig. 8(a), it is clear that the 397 damage introduced in the plate generates nonlinearities and that the DI_1 as 398 defined in Eq. (9) is sensitive to the presence of this damage. Indeed, for all 390 the actuators, the mean DI_1 values computed for all the paths starting from 400 a given actuator are higher for the damaged state than for the healthy one. 401

To assess the sensibility of the DI_2 defined in Eq. (13) to the presence 402 of the damage for different actuators and paths over the plate, DI_2 values 403 have been computed for each of the 9 actuators using measurements from 404 the healthy and damaged plates (as described in Sec. 5.2). These damage 405 index values are compared in Fig. 8(b) to the detection threshold defined in 406 the previous section. From Fig. 8(b), it is clear that the damage introduced 407 in the plate generates nonlinearities and that the DI_2 as defined in Eq. (13) 408 is sensitive to the presence of this damage. Indeed, for all the actuators, 409 the obtained DI_2 values are higher for the damaged state than the chosen 410 decision threshold. 411

412 6. Conclusion

In many cases, damages that appear on complex structures (such as 413 cracks, impacts, or delaminations) can result in nonlinear dynamical re-414 sponses that may be used for damage detection. Furthermore, complex 415 structures often exhibit a nonlinear behavior even in their healthy states. 416 A robust and reliable SHM system must then be able to deal with nonlinear 417 damages, and to distinguish between their effects and inherent nonlinearities 418 in healthy structures. The first problem to be addressed is that the nonlinear 419 models already in use are never general enough to encompass all the structure 420 encountered in real life. The second problem is that the currently developed 421 nonlinear models are not adequate for practical use of SHM systems. The 422 work presented here attempts to face these two problems on the basis of a 423 simple, but rather general, nonlinear model estimated by means of standard 424 signal processing tools. This approach is based on the assumption that the 425

structure under study can be modeled as a cascade of Hammerstein models. 426 The Exponential Sine Sweep Method, previously developed and validated 427 by the authors for different purposes, is then used to estimate the different 428 kernels of the model. Exponential sine sweeps are a class of sine sweeps that 429 allow estimating a model in a wide frequency band from only one measure-430 ment. Two damage indexes are then build on the basis of this estimated 431 model. The first one reflects the ratio of the energy contained in the non-432 linear part of the output versus the energy contained in its linear part and 433 is specially suited for single-input single-output (SISO) systems. The second 434 one is the angle between the subspaces described by the nonlinear parts of 435 two set of outputs after a principal component analysis. This one is spe-436 cially suited for single-input multi-output (SIMO) systems. As a first step 437 toward the use of this method for SHM, the sensitivity of the proposed DIs 438 to the presence of damages as well as their robustness to noise are assessed 439 numerically on SISO and SIMO systems and experimentally on two actual 440 composite plates with surface-mounted PZT-elements (one healthy and one 441 damaged). 442

The work presented here is however only a first step toward a larger use of 443 this method in SHM. Indeed, it has be shown here that the proposed DIs are 444 effectively sensitive to the presence of a non-linear damage and that they can 445 potentially be helpful to quantify its extent. However, this approach can also 446 be cast in the context of a statistical pattern recognition problem. Then, the 447 DIs defined here, or other defined on the basis of the estimated model, can 448 be used to train expert systems that are able to distinguish between different 449 kind of damages [22]. 450

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Figure 6: (a) Healthy and (b) damaged composite plates with a zoom on the impact damage. (c) Schematic representation of the plates under study (circle denotes damage position and rectangles stand for PZTs).



Figure 7: (a) Averages and standard deviations of DI_1 values for the different paths starting from actuator 7. (b) DI_2 values for the different repetitions for actuator 7 and definition of the detection threshold.



Figure 8: (a) Averages and standard deviations of DI_1 values for the different actuators. (b) DI_2 values for the different actuators in comparison to the detection threshold.