Using polyhedral models to automatically sketch idealized geometry for structural analysis

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ABSTRACT

Simplification of polyhedral models, which may incorporate large numbers of faces and nodes, is often required to reduce their amount of data, to allow their efficient manipulation and to speed up computation. Such a simplification process must be adapted to the use of the resulting polyhedral model. Several applications require simplified shapes which have the same topology as the original model (e.g. reverse engineering, medical applications, etc). Nevertheless, in the fields of structural analysis and computer visualization for example, several adaptations and idealizations of the initial geometry are often necessary. To this end, within this paper a new approach is proposed to simplify an initial manifold or non-manifold polyhedral model with respect to bounded errors specified by the user or set up, for example, from a preliminary F.E. analysis. The topological changes which may occur during a simplification because of the bounded error (or tolerance) values specified are performed using specific curvature and topological criteria and operators. Moreover, topological changes, whether they kept or not the manifold of the object are managed simultaneously with the geometric operations of the simplification process.

Key-words : polyhedral simplification, conformity, vertex removal, geometry preservation, topologic changes, non-manifold models, visualization, structural analysis, geometry adaptation and idealization, dimensional reduction.

1. INTRODUCTION

Today's, polyhedral models are widely used for many applications such as computer visualization and animation, robotics, tool path generation for numerically controlled machine tools and also for structural analysis. Such models can be produced from a digitized set of points using a triangulation process or generated by various geometric modelers. The main advantages of polyhedral models with respect to other types, based on parametric or implicit representations of surfaces, are found in their ability to describe general object shapes and to be computationally efficient and robust. Moreover, the triangulation process which produces a polyhedral model, is often more efficient and less tedious than a parametric surface reconstruction process. This last process is still hard to automate because of the difficulty to identify patch boundaries, to parametrize patches over the surface and to create continuity conditions between patches, since these processes are not robust enough.

In many cases, when the object geometry is complex, the polyhedral model produced exhibits a large number of faces and vertices. The direct use of such models leads to lengthy computations and produces inefficient manipulations. Therefore, a simplification process is required to produce a simplified geometry adapted to specific applications.

During the structural analysis of mechanical parts, the geometry used to perform computations is tightly linked to the method used (the Finite Element Method for example). Moreover, several analyses are required during the design process of a part to determine its dimensions or, more often, to validate technical solutions. Each analysis is characterized by a set of hypotheses which allow the physical problem to be expressed as a finite element one. Therefore, the geometric model required to form the basis of an analysis must be in agreement with the geometric manifold of the F.E. used (beam, plate and shell, solid). Unfortunately, during the design process of a part, the geometric models generated are often too detailed and hence inadequate with respect to the mechanical analysis process requirements. Thus, geometric adaptation and idealization phases are necessary to create geometric models for the F.E.A. also called idealized or abstracted geometries. Currently, such processes are not automated yet. In many cases, the analyst creates the idealized geometry entirely from the initial geometric data. Few approaches have been developed to generate an idealized geometry from initial geometric models. Some of them are based on the use of expert systems [1], [2]. However, these approaches cannot be applied to a general context and they do not provide enough flexibility. Others, based on the Medial Axis Transform method, have been developed to produce mid-surface abstractions [3], [4], [5], [6], [7]. Nevertheless, their results widely depend on the geometry of the initial shape. Moreover, the robustness of the idealization process to idealize general shapes, which include free-form surfaces, has not yet been demonstrated. Another category of approaches uses the history of the construction of the initial geometric model (i.e. the C.S.G. tree in the present cases) [8], [9]. Such approaches are limited to the idealization of shapes designed through specific modeling techniques. Moreover, the idealization depends on the nature of the primitive used to build the C.S.G. model of the object and on the sequence of operations executed to construct the model (i.e. the same object geometry can be obtained through different C.S.G. trees).

For the computer visualization of complex scenes, polyhedral simplification processes are also widely used to produce successive levels of details of an object. Recently, various simplification techniques have been developed (see references [10], [11] for general overviews). Most of them allow the simplification of an initial polyhedron (i.e. a two-manifold model) without topological changes. Among this set of techniques, those which allow the restoration of the initial geometry according to a geometric criterion (references [12], [13] for example) can be distinguished from those which do not explicitly use such a criterion (references [14], [15], [16], [17], [18] for example). Previous works have also led us to develop a simplification approach [19], [20] with specific shape preservation criteria. Such an approach, which preserves the initial topology of the object and restores its initial geometry according to a distance criterion, is well suited to the situation when the simplified model is used to generate tool path trajectories or to visualize scientific data (i.e. medical data or results of complex transient finite element analyses for examples) where good realism and high accuracy are required.

On the contrary, the approach proposed here allows simplification of either two-manifold or non-manifold polyhedral models, under topological changes controlled in accordance with the specified error values. Rossignac and Borrel [17] have also proposed such a simplification technique but topological changes are not controlled in accordance with a bounded error criterion. Such simplified models are not useful for structural analysis where the topology of a model can be critical with respect to the mechanical behavior modeled. For visualization purposes, the use of these models often significantly alters the realism of the graphic scene.

Conversely, the approach proposed by He et al. [18] allows restricted topological changes. Indeed, only closed two-manifold polyhedral models can be simplified using this method and the resulting models are also closed two-manifold ones.

In the approach proposed here, the simplification of polyhedral models is carried out with respect to one or more bounded error values assigned to distinct areas of the object. Given these values, which characterize the geometric deviation accepted between the initial and the simplified models, the topological modifications which may occur are managed and controlled to produce an acceptable adapted and idealized geometry. As a result, a sketch of the object geometry for structural analysis purposes is obtained. This sketch helps to extract data about the locations of connections between sub domains defining beams, plates, shells, … Such a geometry can be further processed using mechanical data, i.e. inertia, Young modulus, …, to generate effective mechanical models. To this end, connectivity based criteria are used to identify and classify the local configuration of each edge and vertex of the polyhedral model. Such a classification is required to apply the appropriate vertex removal operator developed for each class of vertex. The geometric and topological coherence are checked in accordance with the error values using a geometry restoration criterion and criteria based on topological configurations identified from the classification of the edges and vertices. The combination of geometric and topological criteria allows the integrity of the object (i.e. mainly its connectivity property) to be maintained.

2. OVERVIEW OF THE ALGORITHM

The approach proposed is based on an iterative vertex removal algorithm. First of all, the edges and vertices of the initial polyhedral model are classified in accordance with their local topological configuration (see Section 3.). This classification is required to apply the appropriate selection criterion and vertex removal operators to each class of vertices. Then, the simplification treatment is initialized. A spherical error zone is assigned to each vertex of the initial model. The radius of these spheres can be set up using values either specified by the user and attached to different areas of the object or be automatically assigned within a specific application. As an example, these values can be produced after an *a posteriori* error evaluation of a finite element analysis [23]. In this case, the sizes of the error zones reflect the size of the finite elements required for a specific analysis.

At each face an inheritance process of the error zones is initialized to monitor the geometry restoration during the simplification process. The restoration criterion used is based on the measure of a geometric deviation between the initial and simplified models (see Section 4). Afterwards, the simplification process starts and a loop is executed until no more candidate vertices can be removed. Different criteria based on discrete curvature approximations are used to select the candidate vertex which has the best probability of removal. Then an operator adapted to the classification of the candidate vertex is applied to create a new geometry from the contour polygon of this vertex, i.e. its star-polygon. To this end, different meshing techniques of 3D contour polygons, which take into account an approximation of the principal directions of curvature, are used in accordance with the local geometric configuration around the vertex to be removed. The geometry restoration criterion is then applied to determine whether the vertex can be removed or not. If the geometry of the initial model is correctly restored, the current model is updated using the previously created mesh of the 3D contour polygons and the possible topologic changes involved by this vertex removal are identified and managed. Figure 1 illustrates the main steps of this decimation algorithm.

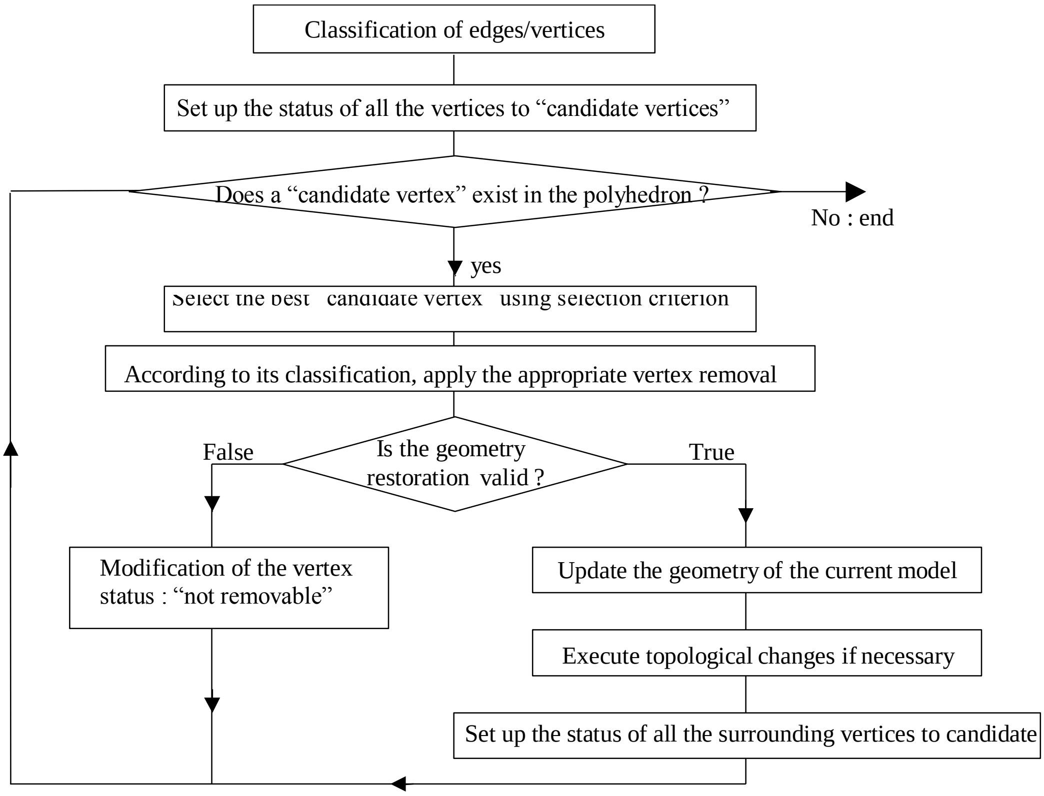

Figure 1 : The main steps of the simplification loop algorithm.

The algorithm proposed here is dedicated to operate topological and manifold changes during the decimation. Nevertheless, before executing such a process, the initial polyhedral model is simplified using the previously developed method [19], [20] without topological changes to significantly reduce the number of faces and vertices of the model. Consequently, the dimensional reduction process described in this paper is applied on polyhedral models which are not characterized by a large number of faces and vertices. Such a pre-processing simplification justifies that the complexity of the dimensional reduction algorithm proposed is not a significant criterion for its complexity. The criteria which allow the management of topological changes and checking of the shape restoration of the object when it becomes non-manifold or when it increase its non-manifold areas are described in Section 4. Vertex removal operators dedicated to each class of vertices are described in Section 5. Then, additional operators allowing specific topologic and manifold changes are described in Section 6. Finally, some results are presented to illustrate the capabilities of the proposed approach in structural analysis and visualization applications.

3. NON-MANIFOLD CLASSIFICATION OF EDGES AND VERTICES

First of all, to carry out the idealization process, a classification of the edges and vertices of the initial polyhedral geometry is required to apply criteria and operators specific to each class of entity. To this end, four kinds of edges and vertices have been identified in accordance with local topologic configurations. This classification process is performed on the input polyhedron to initialize the simplification process.

3.1. Classification of the edges

The edges are classified using the following rules :

- an edge which does not take part in the description of any face is classified as an *isolated* edge,
- an edge which takes part in the description of one face only is classified as a *boundary* edge. This edge holds for the boundary of a domain topologically equivalent to half a disc,
- an edge which takes part in the description of two faces only is classified as a *surface* edge. This edge is located in a domain topologically equivalent to a disc,
- an edge which takes part in the description of more than two faces is classified as a *contact* edge. Such an edge is the common boundary of more than two domains topologically equivalent to half discs.

3.2. Classification of the vertices

The classification of the vertices using criteria based on their connectivity like the criteria used to classify the edges does not uniquely characterize all their possible configurations when the geometric models are non-manifold. Therefore, additional criteria must be set up and lead to the following classification :

- a vertex connected to surface edges only is classified as a *surface* vertex if all the faces meeting at that vertex define one surface only (i.e. each vertex is associated with one and only one contour polygon) (see Figure 2a). Otherwise, this vertex is classified as an *isolated* vertex (see Figure 2d),
- a vertex connected to two boundary edges and to surface edges is classified as a *boundary* vertex if all the faces meeting at that vertex define one surface only (see Figure 2b), otherwise it is an *isolated* vertex (see Figure 2e),
- a vertex connected to at least one contact edge is classified as a *contact* vertex (see Figure 2c) except when it exists a face meeting at that vertex which owns two boundary edges meeting at that node. In this last case the vertex is classified as *isolated* (see Figure 2f),
- finally, when a vertex is connected to more than two boundary edges or at least to one isolated edge, it is classified as an *isolated* vertex (see Figure 2g).

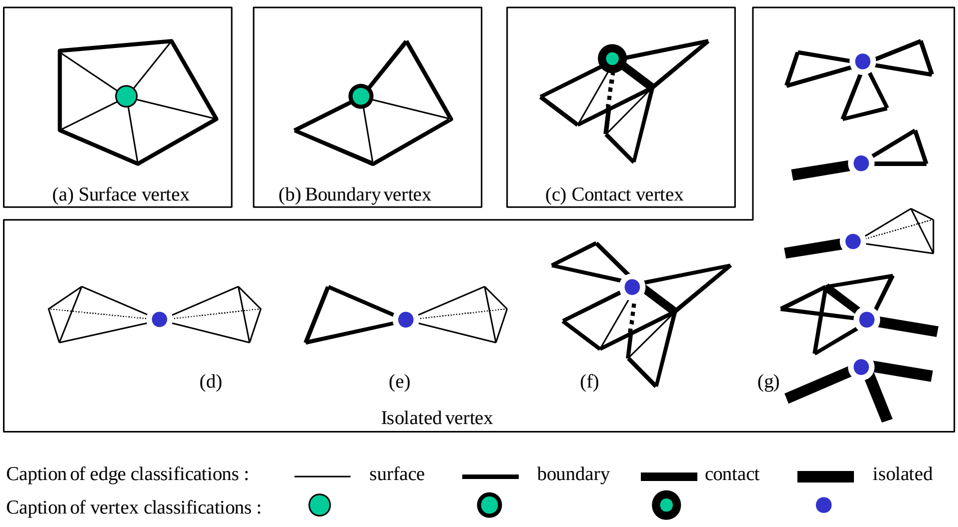

Figure 2 : Illustration of different shape configurations and the associated vertex classifications.

Figure 2 shows the different shape configurations and the associated classification of the vertices. Clearly, the correct identification of each vertex status plays an important role during the idealization process. The identification of isolated vertices allows the preservation of the overall connectivity of the geometric model and thus avoids the creation of disconnected sets of geometric entities. Moreover, if topologic changes occur after each vertex has been removed using the geometric operators described in sections 5 and 6, the classification of the edges and vertices surrounding that vertex must be updated using the classification criteria previously described in order to preserve the coherence of the non-manifold geometric model.

4. CONTROL OF THE GEOMETRIC RESTORATION PROCESS AND TOPOLOGIC CHANGES

The geometric restoration process is based on the error zones assigned to the vertices and on an inheritance mechanism of these error zones attached to faces. At first, spherical error zones centered on each vertex of the input polyhedral model are generated. The radius of each sphere locally defines the maximum deviation accepted between the initial and simplified models according to an application dependent criterion. This model can be either a two-manifold model or a non-manifold one.

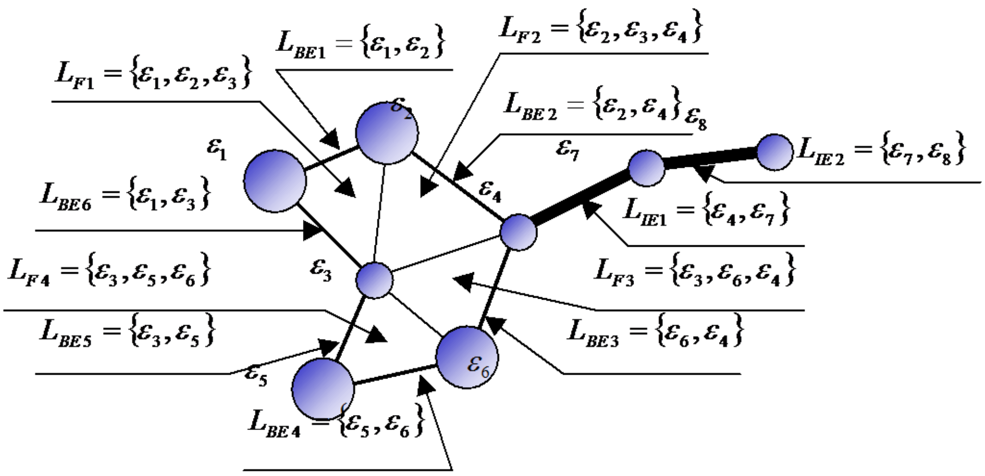

Figure 3 : Error zones ε_i associated with the initial vertices and dependency lists assigned to faces L_{Fi}, boundary edges L_{BEj} and isolated edges L_{IEk}.

Then, an inheritance process is used to monitor the geometric restoration of the object shape during the simplification process. This process is initialized using the input data. A dependency list of error zones, containing all the error zones participating to the local restitution of the object geometry, is assigned to each face of the model. For each face, its dependency list is initialized with the error zones attached to the vertices describing that entity. To complete the geometric restoration control, the same concept of dependency lists is also applied to the boundary and isolated edges of the model. Figure 3 illustrate the dependency list initializations on a simple initial model containing eight error zones ε_i .At the initialization stage, the error zones assigned to the dependency lists of each entity (face or edge) are solely composed of the zones located at their boundary vertices.

During each vertex removal procedure, the geometric restoration criterion is checked and the potential topological changes are managed using the following criteria and rules. The first step of the geometric restoration test merges the lists of the error zones of the faces and edges connected to the candidate vertex removal. This list L is created from the error dependency lists associated to the faces L_{Fi}, the boundary edges

L_{BEj} and the isolated edges L_{IEk} meeting at the candidate vertex. The second step effectively tests the newly created re-meshing scheme to ensure that it intersects the error zones of all vertices, even the removed ones which are kept by the inheritance process during the simplification. The criterion used checks that each error zone ε_l of the list L previously created intersects at least either one face F_i or one boundary edge E_{Bj} or one isolated edge E_{Ik} of the newly created geometry, i.e. $\forall \varepsilon_l \in L$, $\exists$ F_i such that $F_i \cap E_l \neq \{\phi\}$ or $\exists$ E_{Bj} such that $E_{Bj} \cap E_l \neq \{\phi\}$ or $\exists$ E_{Ik} such that $E_{Ik} \cap E_l \neq \{\phi\}$. Hence, this criterion is based on sphere-triangle or sphere-edge intersection tests.

If the shape restoration test is successful, the model geometry is updated. To this end, the newly created re-meshing scheme is locally inserted into the current model. Such a process can involve topological modifications and some non-conformities like quasi-overlapping areas (see section 6.3) which may create self-intersecting polyhedra. Such non-conformities are managed using specific operators described in Section 6. Because of the possible topological changes carried out, the classification of the edges and vertices of the simplified area must be updated using the rules defined in Section 3. Moreover, the dependency list of error zones assigned to each newly created (by the meshing process) face, boundary edge and isolated edge must be updated too. This updating process is carried out from the list L of error zones previously defined. Each error zone ε_i of this list is added to the dependency list L_{Fj} of the newly created face if it intersects this face F_j. In the same way, the error zone ε_i is added to the dependency list L_{BEk} (respectively L_{IEl}) of the newly created boundary edge BE_k (resp. isolated edge IE_l) if it intersects this edge.

5. VERTEX REMOVAL OPERATORS

Each vertex removal is performed using a simplification operator depending on its vertex classification status. The operators which produce the simplification of surface and boundary vertices have been already set up through a previous approach [19], [20] to simplify two-manifold polyhedra. New operators, based on the same concepts have been developed to remove contact and isolated vertices. These operators are able to take into account the transformations associated with these two categories of vertices. Figure 4 illustrate the main phases of the algorithm for all the vertex removal operators.

When a surface vertex is processed, only one closed contour polygon is extracted. Similarly, for a boundary vertex, only one open contour polygon is extracted. The removal of an isolated vertex is the most simple case because no contour polygon surrounds the vertex processed. On the contrary, when contact vertices are processed, several contour polygons surrounding the candidate vertex are extracted. These contours can be either closed or open or degenerated (Figure 5). A degenerated contour is identified when it owns one edge only. When an open contour polygon is identified, a new edge is created to close it. Thus, it becomes a closed contour which can be re-meshed. No re-meshing process is required with degenerated contours.

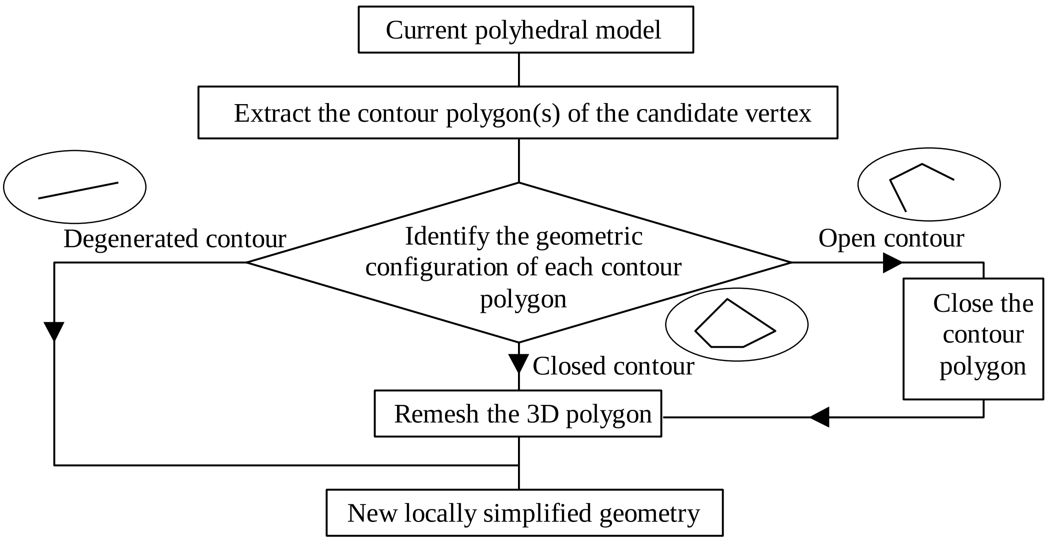

Figure 4 : General algorithm of all the vertex removal operators.

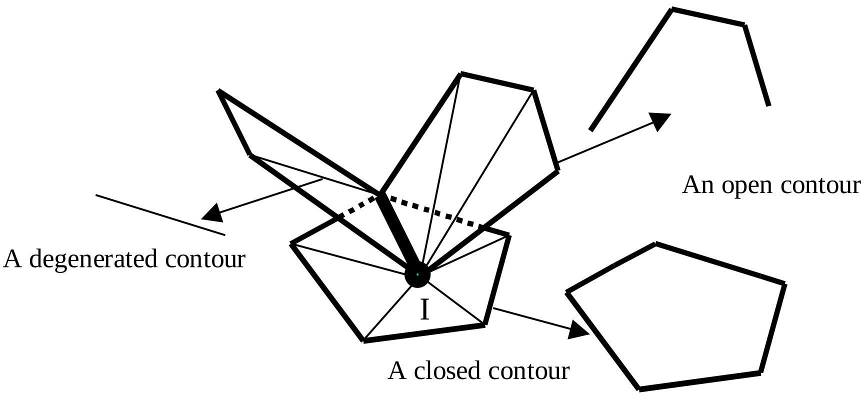

Figure 5 : The different types of contour polygons extracted when a contact vertex is processed.

Two different meshing techniques of the 3D contour polygons extracted are applied in accordance with the discrete curvature distribution of the surface (computed using a discrete gaussian curvature criterion [20], [21]). This criterion helps the creation of a new local geometry which has a high probability of correctly restoring the shape of the object when the candidate vertex is removed. The locally simplified geometry produced is then used to check the restoration of the shape of the object. The two meshing techniques are detailed in [20]. The first one generates triangles as equilateral as possible and the second one generates edges and triangles as close as possible to the candidate vertex. During the dimensional reduction process, the meshing strategy used is based on a mixed use of the two meshing techniques and can be stated as follows. A contour polygon is meshed using the method based on an equilaterality criterion. If the geometry is accurately restored, i.e. the geometry restoration criterion is satisfied., this new mesh is the solution kept. Otherwise, the contour polygon is re-meshed using the second method. The newly created mesh is evaluated using the geometry restoration criterion to determine whether the candidate vertex can be removed or not. Such a strategy leads to an increase in the quality of the simplified geometry produced and improves the efficiency and the robustness of the simplification process when complex geometric configurations are processed.

Surface and boundary vertices are removed according to the operators previously developed in [20]. New operator have been developed for contact and isolated vertices.

5.1. Contact vertex removal operator

The removal of a contact vertex is processed as described in Figure 4. The different contour polygons are extracted and re-meshed according to their types, i.e. closed, open or degenerated polygons. Figure 5 illustrates the different configurations of contour polygons extracted. After re-meshing, the restoration of the object shape is checked. To this end, the geometric restoration criterion is applied to verify that all the error zones associated with the faces, edges and vertices surrounding the contact vertex removed intersect with at least one of these geometric elements. When this test is satisfied, the initial geometry is replaced by all the locally created meshes. Finally, the classifications of all the entities surrounding the removed vertex are updated.

To preserve the global object shape, candidate contact vertices for removal must be selected. To this end, a candidate contact vertex can be selected when it is connected to one or two contact edges only (Figure 6a). The selection criterion used to sort the candidate vertices connected to two contact edges is the same curvature based criterion that which is used to sort boundary vertices (i.e. vertices are processed by decreasing value of their curvature evaluation). Candidate contact vertices connected to two contact edges are processed before those which are connected to one contact edge only. This decimation criterion provides first a simplification of a contact line (set of connected contact edges) prior to the modification of its length. Simplifying contact vertices connected to one contact edge only expresses the fact that contact lines are evaluated to check of their length can be reduced up to zero, i.e. one contact vertex, eventually.

The simplification of a contact vertex connected to more than two contact edges is forbidden to avoid a significant degradation of the overall object shape (Figure 6b). In the case of the Figure 6b, the contact vertex removed conveys more significance with respect to the object shape than a strict distance criterion related to the error zones.

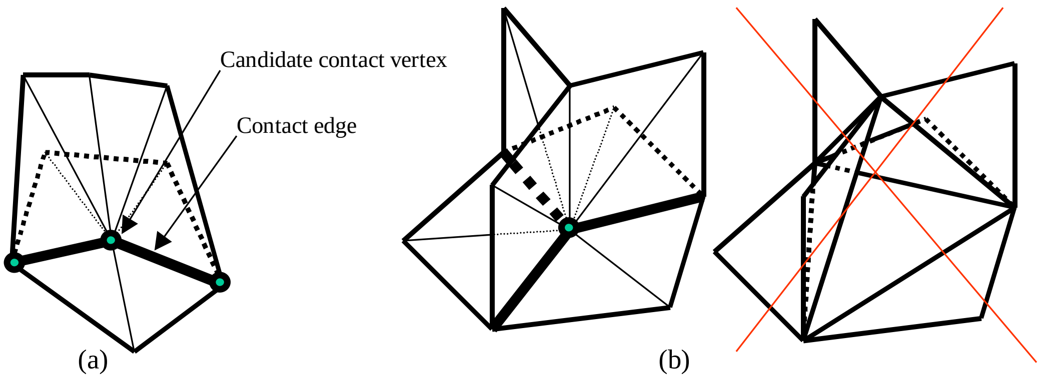

Figure 6 : Selection of the contact vertices for removal : (a) candidate vertices must be connected to one or two contact edges only, (b) forbidden vertex removal configuration to preserve the shape of the object.

5.2. Isolated vertex removal operator

The decimation process of isolated vertices is the most simple case. Like for contact vertices, the simplification of isolated ones is restricted to vertices connected only to two isolated edges. For similar reasons, a criterion allows the preservation of the overall object shape. When an isolated vertex is removed, the shape restoration is checked using the same geometric criterion as is used for the other classes of vertices. The inheritance process of the error zones described in section 4 is applied to monitor the geometric restoration of the object shape during the simplification process.

6. SPECIFIC ADDITIONAL OPERATORS

In addition to the vertex removal operators, three other operators are used to process specific topological changes allowing manifold transformations like surface to line or volume to surface transformation and thus produce a final idealized geometry. The first and the second ones are respectively based on an edge collapse and a face removal operator. The third specific operator allows overlapping or quasi-overlapping areas to be processed.

6.1. Edge collapse operator

Such an operator is used when no more vertices can be removed with the vertex removal operators and performs a surface to line reduction. Figure 7 illustrates via a simple example its necessity to produce the final idealized geometry.

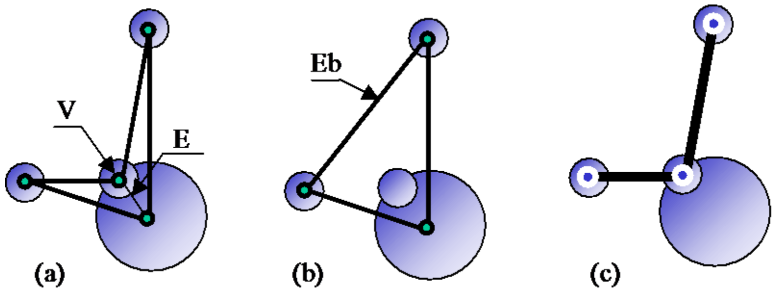

Figure 7 : Necessity of the edge collapse operator : (a) initial configuration, (b) result of the removal of the vertex *V*: the geometric restoration criterion is not satisfied for the boundary edge *Eb*, (c) result of the collapse process of the edge *E* : topological changes have occurred and the distance criterion is satisfied.

This operator is iteratively applied on all the candidate edges of the model until no more edges can be collapsed and the restoration of the initial geometry is verified in accordance with the error zones specified. The candidate edges for the collapse process are identified using the following rules :

- the edge to collapse is a boundary edge *Eb* which owns at least one isolated vertex at one of its extremities (Figure 8a),
- the edge to collapse is a surface edge *Es* whose associated vertices belong to the boundary, isolated or contact types (Figure 8b).

The collapsing process of contact or isolated edges is not performed because this operator would be redundant given the vertex removal one and would not provide new polyhedron configurations.

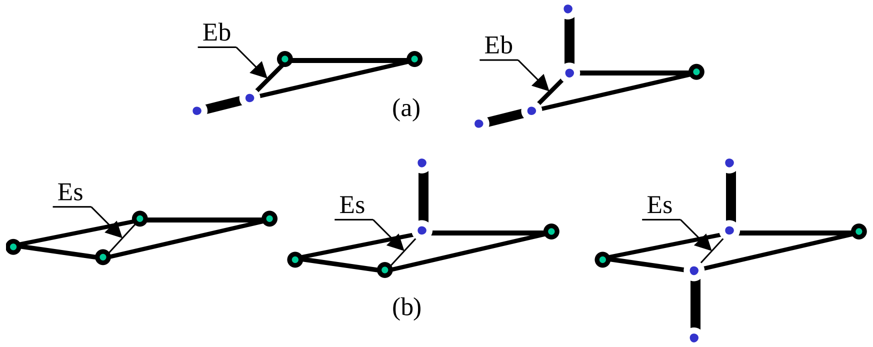

Figure 8 : Valid configurations for the edge collapse process : (a) of a boundary edge *Eb*,
(b) of a surface edge *Es*.

Then, the edge collapse process is accepted if all the error zones associated through the inheritance process to the edges and faces surrounding the edge *E* collapsed, intersect at least one of the geometric entities (Figure 9). Given, the two possible solutions to collapse the edge *E* into one of these extremity vertices, the first one which satisfies the geometry restoration criterion is chosen. When an edge is collapsed, the classifications of the edges and vertices surrounding the topologically modified area are updated using the rules previously defined in Section 3 (i.e. the error zones must intersect the newly created geometry). To carry on the inheritance process, the dependency lists of the modified area entities are updated.

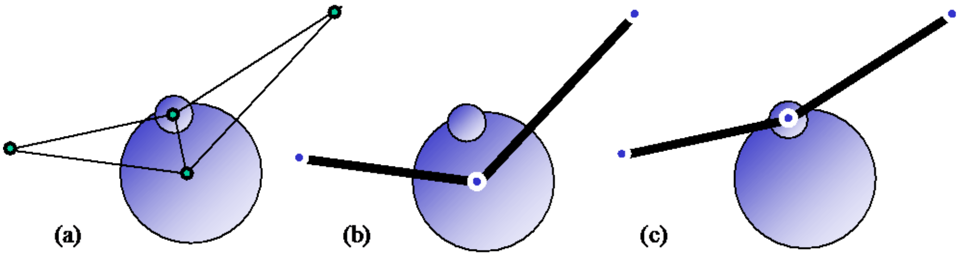

Figure 9 : Restoration of the geometry : (a) initial geometry, (b) geometry restoration criterion not satisfied after the edge collapse process, (c) valid restoration of the initial geometry.

6.2. Face removal operator

This operator is also used when no more vertices can be removed using the vertex removal operators. It is necessary to process specific configurations where surface to line reduction can take place. Candidate faces for removal are identified using the following rules :

- the face is defined by three boundary edges (Figure 10a),
- the face owns two boundary edges only and one of these is the edge of maximum length describing the face (Figure 10b).

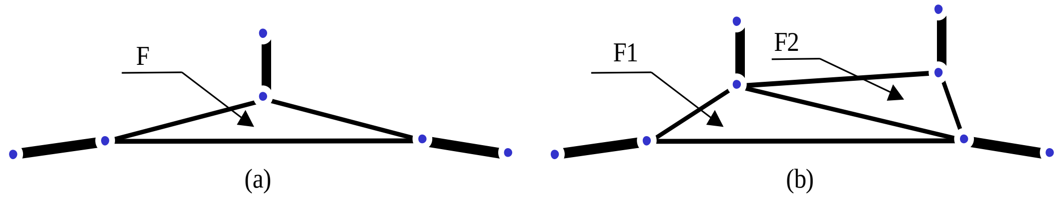

Figure 10 : Valid configurations for the face removal operator : (a) *F* is a candidate face, (b) face *F1* is a candidate face but not *F2*.

Such rules avoids the removal of faces which could lead to significant alterations of the overall object shape. Here again the geometry restoration criterion is checked. All the error zones associated with the face to be removed and with its boundary edges and vertices must keep an intersection with the newly created simplified geometry (Figure 11). Like the previous one, this operator is applied iteratively on the candidate faces. Figure 12 illustrate the successive application of the face removal operator to produce a final idealized geometry.

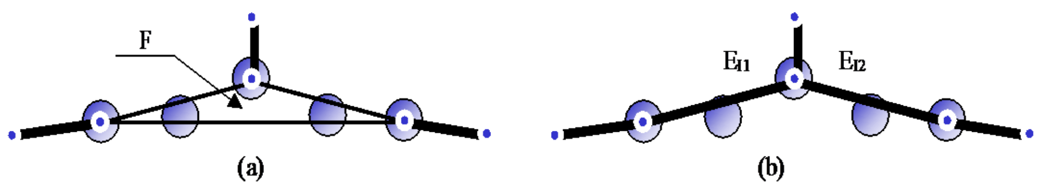

Figure 11 : Restoration of the initial geometry : all the error zones participating with the restoration of the object shape intersect the newly created geometry E_{I1}, E_{I2}

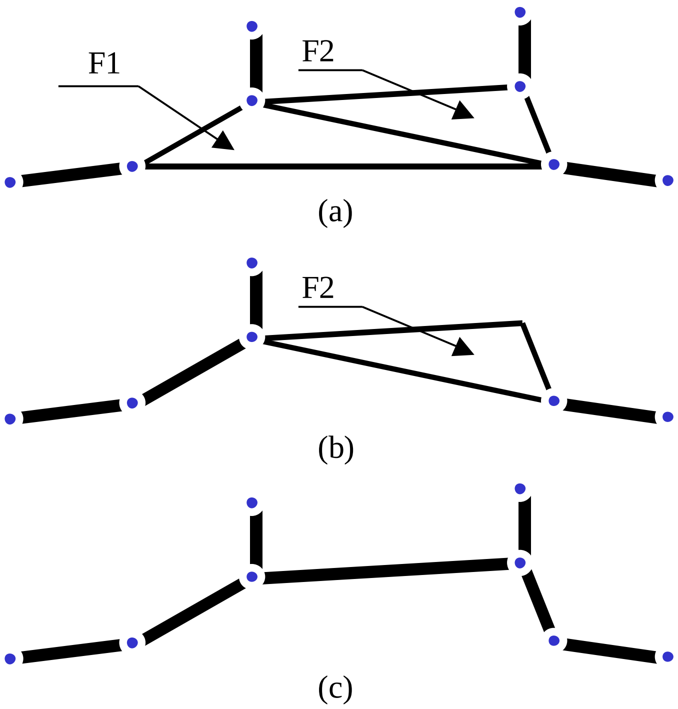

Figure 12 : Repeated use of the face removal operator to produce the final idealized geometry. In the configuration (b) the face *F2* became candidate for removal.

6.3. Processing overlapping areas

This operator is dedicated to the dimensional reduction of volumes (subsets of two manifold and closed face sets) into surfaces (two manifold and opened face sets). Overlapping or quasi-overlapping areas are identified when the same approximated geometric surface can be described by two different polyhedral parts. A quasi-overlapping area is identified when two distinct polyhedral parts lie inside the same envelope defined by the error zones assigned to the vertices of the model. Thus, thin shells can be identified and idealized. Figure 13 illustrate the definition of overlapping or quasi-overlapping areas.

Assuming $V : v_i, i \in \{1,\cdots,n_v\}$, $E : e_j, j \in \{1,\cdots,n_e\}$, $F : f_k, k \in \{1,\cdots,n_f\}$ are respectively the sets of vertices, edges and faces defining a two manifold and closed domain representing the boundary of polyhedral volume, overlapping or quasi-overlapping polyhedral areas are defined by two subsets :

$$V_1 : v_{i1}, i \in \{1,\cdots,n_{v1}\},\ E_1 : e_{j1}, j \in \{1,\cdots,n_{e1}\},\ F_1 : f_{k1}, k \in \{1,\cdots,n_{f1}\} \text{ and}$$

$$V_2 : v_{i2}, i \in \{1,\cdots,n_{v2}\},\ E_2 : e_{j2}, j \in \{1,\cdots,n_{e2}\},\ F_2 : f_{k2}, k \in \{1,\cdots,n_{f2}\} \text{ such that}$$

$$V_1 \cup V_2 = V,\ E_1 \cup E_2 = E,\ F_1 \cup F_2 = F \text{ and } F_1 \cap F_2 = \{\varnothing\},\ E_1 \cap E_2 = E_B,\ V_1 \cap V_2 = V_B$$

E_B is a subset of edges defining the common boundary of the two face sets F_1 and F_2. Similarly, V_B is a subset of vertices defining E_B. From a geometric point of view, if the two face sets F_1 and F_2 describe exactly the same geometry, i.e. : all the vertices v_{i1} lie on the faces f_{k2}, the two face sets are overlapping each other. If L_{F1} and L_{F2} are the dependency lists of error zones associated respectively with the face set F_1 and F_2

through the inheritance process and if F_2 intersects all the error zones of L_{F1}, and F_1 intersects all the error zones of L_{F2}, the two face sets F_1 and F_2 are quasi-overlapping each other.

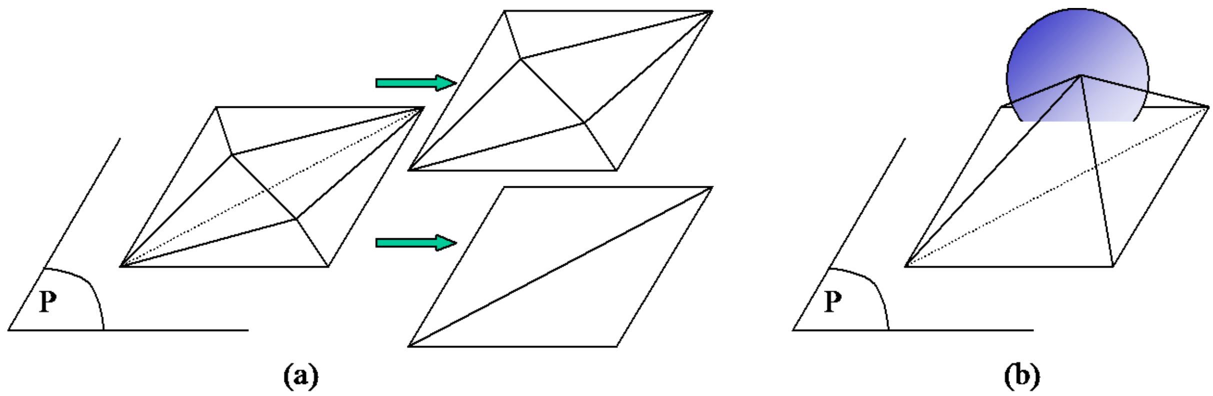

Figure 13 : Illustration of : (a) an overlapping area, (b) a quasi-overlapping area when taking into account the error zones.

The developed operator allows the identification and removal of these duplicated geometric areas. It identifies the closed surfaces of the model and opens each of them when it is compatible with the restoration of the object geometry. The closed surfaces on a non-manifold polyhedral model are identified by searching all the subsets of faces which define a closed two manifold surface. This identification step widely uses the classification of the edges which define the different faces. Then, a contour polygon is extracted from each closed surface using a curvature criterion. For each edge of the closed surface, the angle between its two adjacent faces (i.e. a surface or contact edge, as only two adjacent faces belonging to the closed surface considered) is computed (Figure 14). This angle defines the discrete approximation of the mean curvature of the surface along the edge considered [21]. The contour polygon is extracted using a propagation process initiated from the edge which has the smallest angle. Such a process leads to the identification of the set of connected edges defining a closed contour polygon which has minimal angles. Two open half-surfaces are thus defined. Afterwards, the shape restoration test is applied to check that all the error zones associated with the first half-surface intersect at least one face of the second half-surface. When this test is successful, the closed surface is opened by removing one of the two half-surfaces according to a selection criterion based on the connectivity configuration of each half-surface. This criterion identifies the half-surface which is not connected to any other surface area through contact edges or to isolated edges through isolated vertices. If one of the two half-surfaces satisfies this criterion, it is selected for removal. Nevertheless, geometric configurations exist where the previous criterion fails because both half-surfaces are connected to other geometric elements. In such cases, the contact edges and isolated vertices connecting each half-surface to other geometric elements are extracted. Then, the two half-surfaces are removed and a constrained re-meshing process of the contour polygon takes place with respect to the contact edges and the isolated vertices extracted. Figure 15 illustrates such a constrained re-meshing process when both half-surfaces are connected to other geometric elements. Actually, it allows plane or quasi-plane configurations, which are the most repeated cases, to be processed. Nevertheless, new developments must be made to process cases where the two half-surfaces describe curved geometries such as a cylindrical shell for example.

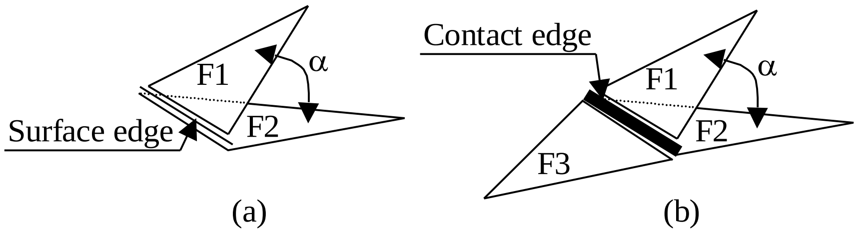

Figure 14 : Curvature criterion used to extract a significant contour polygon : (a) for a surface edge, (b) for a contact edge : only the faces *F1* and *F2* belong to the closed surface processed.

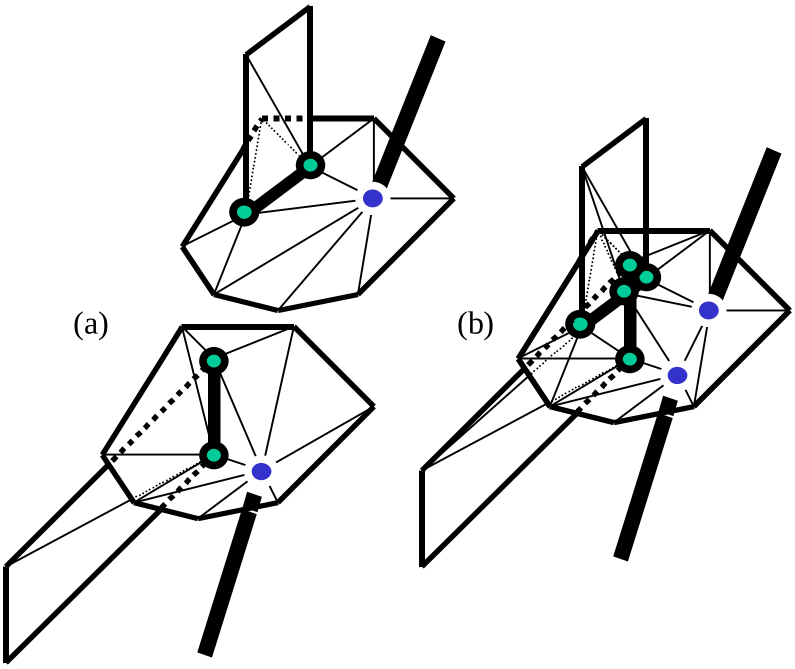

Figure 15 : Constrained re-meshing process applied when both half-surfaces are connected to other geometric elements : (a) the two half-surfaces, (b) result of the constrained re-meshing process.

7. RESULTS

The previous operators can be combined together to produce coherent topological modifications of an input polyhedron. Figure 16 describes the combination strategy used to produce sketched idealized models. Their completeness has not yet been proved. Future works will focus on this point. The simplification of manifold or non-manifold polyhedral models with control topological changes and preservation of the object shape has two main application : the visualization of complex scenes with the creation of level of details and the adaption and idealization processing of geometric models for finite element analysis requirements. The next part of this section illustrates the approach and the operators applied on three examples.

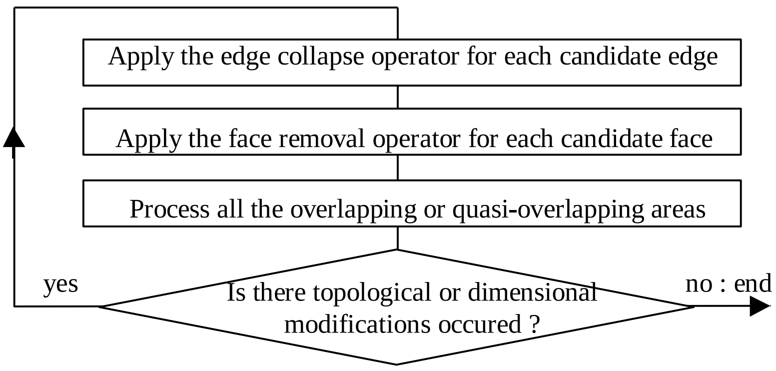

Figure 16 : Combined use of the topological changes and dimensional reduction operators.

The first one illustrates the adaptation process of a 2D polyhedral model describing a plate with its boundary conditions and loads (Figure 17a). The error zones associated with the vertices of this model result in an *a priori* estimation of the element size required to produce a finite element solution which satisfies a user accuracy specification [22]. An *a posteriori* estimation of the finite element sizes could also be considered [23]. The element sizes estimated are directly used as error zone values to monitor the simplification process and thus produce the adapted geometry (Figure 17b). In any case, the size of the error zones reflects the structural behavior of the object since the size of a finite element expresses the amount of strain energy located in the same area of the structure. Hence, the simplification process which takes place allows the removal of geometric areas which put stronger constraints on the mesh than the mesh itself requires, i.e. : geometric areas which require smaller elements than the size set by the *a priori* or *a posteriori* estimators are subjected to large geometric changes whereas areas describing high level of strains are assigned small sizes of error zones and are therefore preserved. Figure 17c shows the result of the simplification process applied. Topological modifications have occurred in accordance with the error zone values assigned to the vertices (i.e. the hole has been removed). The simplified geometry forms the input geometry for the mesh generation phase used to produce the F.E. model.
Figure 18 shows the finite element solution (i.e. map of the Von-Mises stress criterion) obtained on the initial geometric model (Figure 18a) and on the adapted one (Figure 18b). The same finite element discretisation and calculation parameters have been used in Figure 18a and 18b allowing the stress values to be compared. The previously estimated element sizes have not been taken into account. The solution produced using the adapted model gives the same information results as that using the initial one. The element mesh size has been set to a small value to get an accurate numerical solution for each contour and to allow an objective comparison of the solutions. Here, the simplification process clearly shows the difference between the input contour and the result obtained and illustrates the principle of feature removal approaches based on a polyhedral boundary representation of an object. Hence, there is no need to rely on feature data to perform a geometry adaption process of a mechanical structure. Such an approach reduces calculation times and decreases the complexity of the meshing process. Indeed, all the areas of the model which are not significant for the case studied are removed.

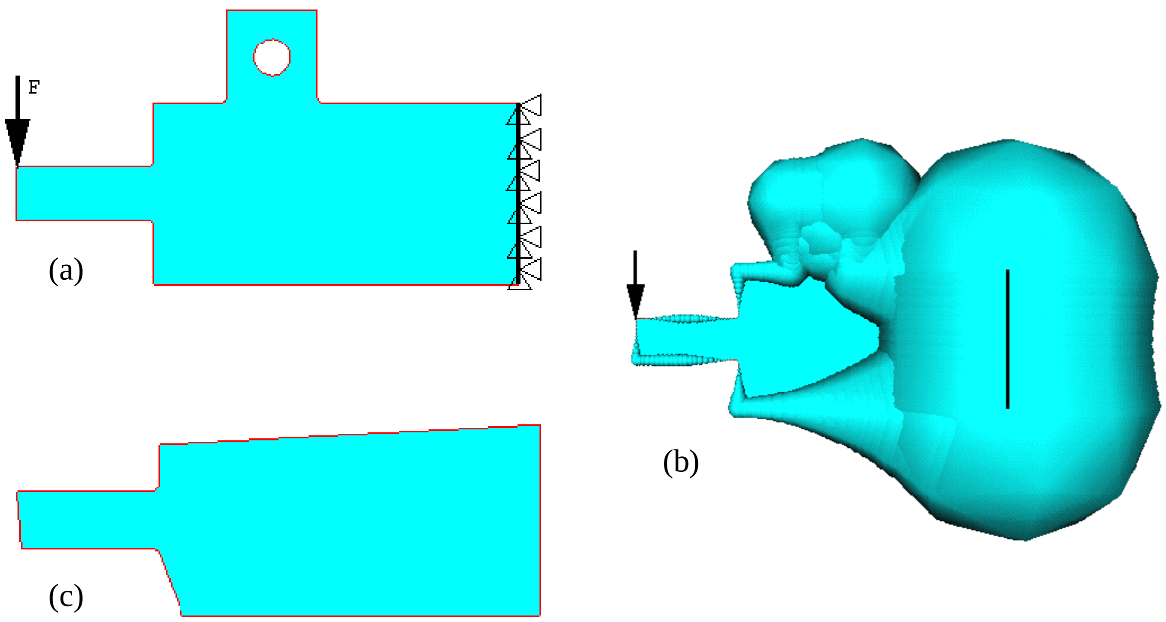

Figure 17 : Adaption process of a 2D plate polyhedral model for finite element analysis : (a) initial geometry, (b) error zones values assigned from an a priori estimation of the element sizes, (c) simplified geometry produced with topological changes.

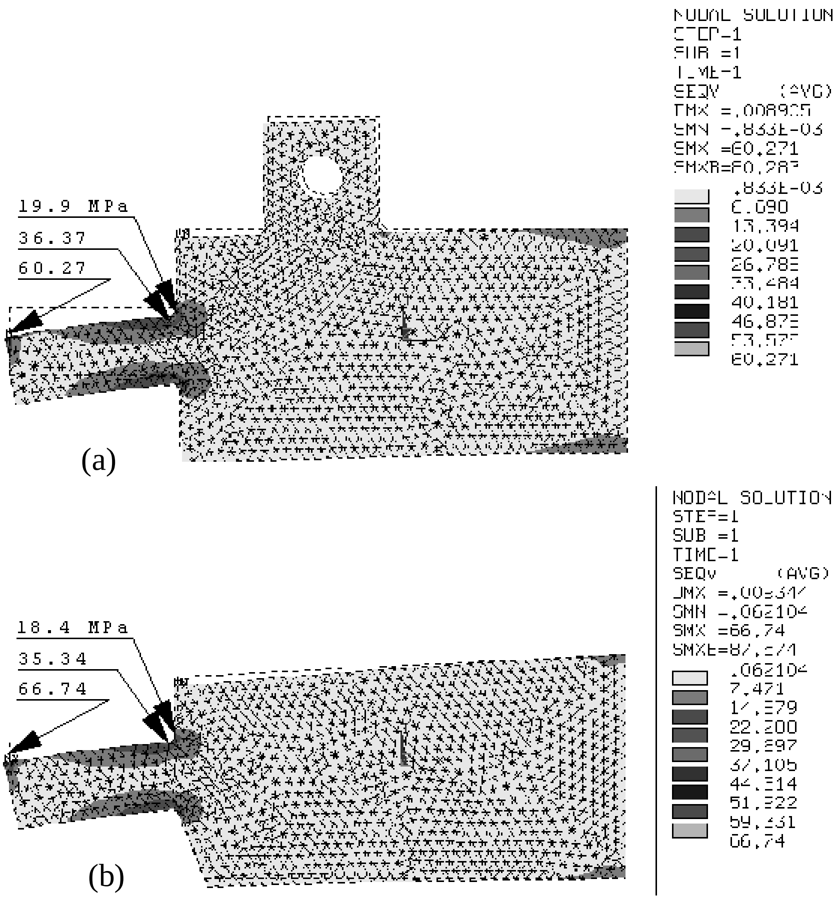

Figure 18 : Map of the Von-Mises stress criterion : (a) on the initial geometry, (b) on the simplified one, using the same calculation parameters in both cases.

The second example illustrates the first step of the idealization process of a polyhedral model of a pipe (Figure 19). The different simplified geometries can be used both for visualization or for finite element analysis purposes. The initial polyhedron is shown in Figure 19a. A first simplification level is produced (Figure 19b) using the approach previously developed [20] which simplifies the model without topological changes using shape preserving criteria. This step reflects the result of the first part of the overall simplification process. To this end, the error zone value for all the vertices of the initial polyhedron is set up to ±1mm. Then, the error zone value is set up to ±10mm which corresponds to the pipe diameter. The idealized geometry produced using the vertex removal operators only is shown in Figure 19c. Finally, the idealized geometry produced using all the topological change operators proposed is shown in Figure 19d. To generate the final idealized model the error zone value remains equal to ±10mm. In this example, the use of the edge collapse operator is required. However, the idealized geometry thus obtained is only a sketch of the neutral fiber needed for structural analysis. Indeed, this sketch is close enough to the neutral fiber to be modified according to cross sectional inertia though this treatment has not been studied yet.

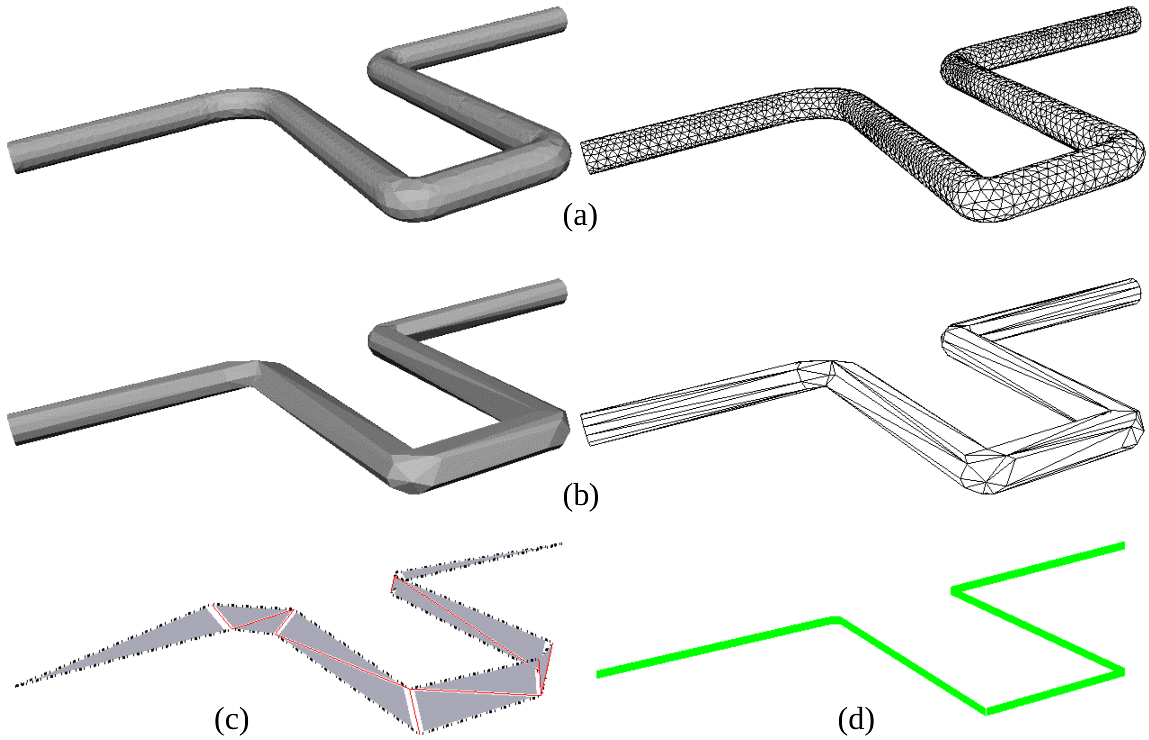

Figure 19 : Idealization process of a pipe : (a) initial polyhedron, (b) simplified model at ±0.01mm and without topological changes, (c) idealized geometry produce using only the vertex removal operators with topological changes, (d) final idealized geometry produced using the complementary operators proposed (i.e. the edge collapse operator in this example).

The last example also illustrates the idealization process of the polyhedral model of a part described by two plates and a cylinder (Figure 20a). As with the previous examples, a first simplification level is produced without topological changes in order to significantly decrease the size of vertices and faces describing the geometry (Figure 20b). To this end, the error zone value used is small : ±0.01mm. Then, the error zones values are increased to produce the successive levels of idealization according to an application criterion which can be based on the engineer's know-how in the case of structural analysis. These values are set up to the thickness of the plates for all the vertices located on them and to the diameter of the cylinder for the vertices located on it.

Figure 20c shows the idealized model produced using the approach proposed. Finally, the error zone value of the vertices located on the plates are set up equal to the width of the plates. Figure 20d shows the result obtained before the face removal operator is applied. Figure 20e shows the final idealized model produced using all the operators proposed. Figure 20 illustrates the various idealizations which could take place according to specific mechanical analyses.

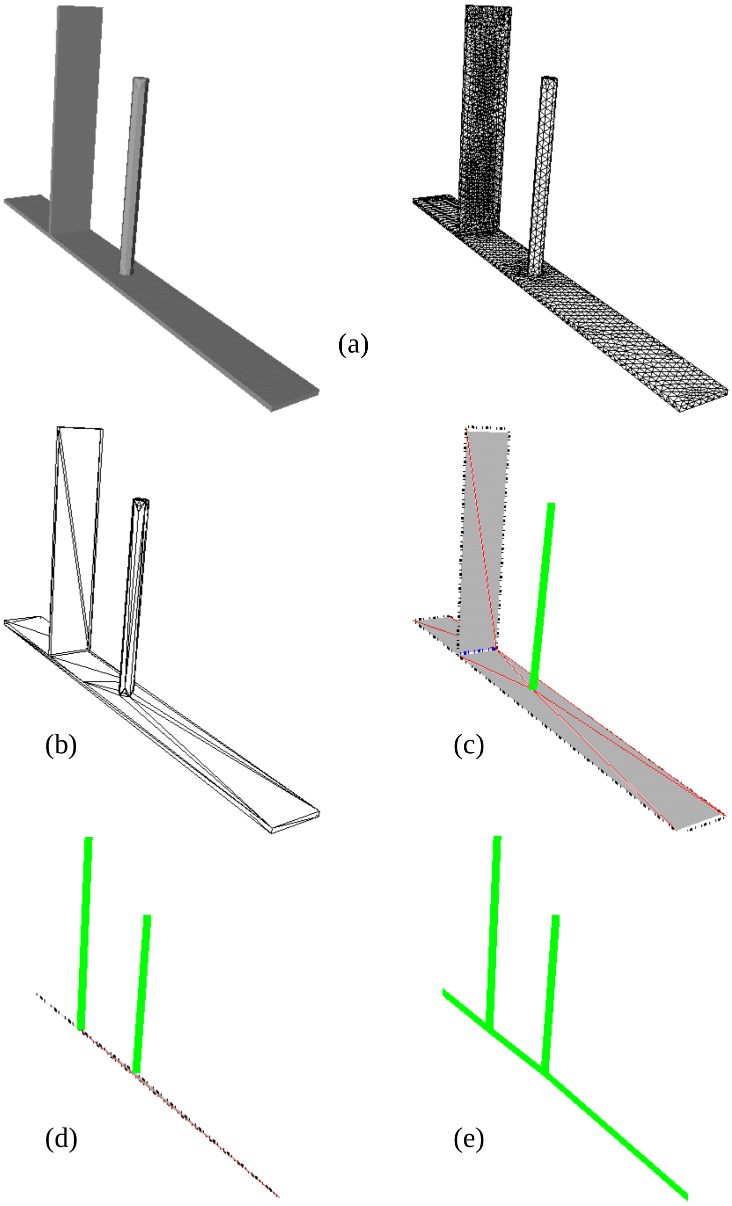

Figure 20 : Idealization of a part described by two plates and a cylinder : (a) initial polyhedron, (b) simplified one at ±0.01mm without topological changes, (c) first idealization level with topological changes, (d) second idealization level produced by increasing error zone values, (e) final idealized model.

8. CONCLUSION AND FUTURE WORK

The treatments presented can be applied to non manifold polyhedra to produce topological transformations and/or manifold changes on an input polyhedron. New status have been set up to describe non manifold configurations of edges and vertices.

In order to process the various configurations of edges and nodes, new operators have been created and the geometry restoration criterion has been extended to provide a satisfactory geometry of the output polyhedron.
The resulting polyhedron preserves the overall shape of the object though it highlights a significantly simplified polyhedron. This polyhedron can be efficient for visualization purposes as well as geometry adaptation or geometry idealization as can be required to create finite element models. In this latter application however, the geometry produced is only an approximation of the geometry required and further treatments are necessary to obtain an effective model.

Future work will focus on the completeness of these operators and on their combination strategies. Complementary treatments will be developed to lead to the real geometric representation used in a FEM model. To this end, additional mechanical data will be taken into account.

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