Instantaneous frequency estimation of FM signals by $\Psi_{\mathtt{B}}$-Energy Operator

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Abstract

$\Psi_{\mathtt{B}}$ energy operator is an extension of the cross Teager-Kaiser energy operator which is an non-linear energy tracking operator to deal with complex signals and its usefulness for non-stationary signals analysis has been demonstrated. In this letter two new properties of $\Psi_{\mathtt{B}}$ are established. The first property is the link between $\Psi_{\mathtt{B}}$ and the dynamic signal which is a generalization of the Instantaneous Frequency (IF). The second property obtained for frequency modulated signals is a simple way to estimate the IF. These properties confirm the interest of $\Psi_{\mathtt{B}}$ operator to track the non-stationary of a signal. Results of IF estimation in noisy environment of a non-linear FM signal are presented and comparison to Wigner-Ville distribution and Hilbert transform-based method is provided.

1 Introduction

$\Psi_{\mathtt{B}}$-energy operator is an extension of the cross Teager-Kaiser operator which is a non-linear tracking energy operator [1] to deal with complex signals [2]. Based on the Lie bracket, $\Psi_{\mathtt{B}}$ is well suited to measure the instantaneous interaction between two complex signals [3]. The output of $\Psi_{\mathtt{B}}$ is related to the instantaneous Cross Correlation (CC) of complex signals $x(t)$ and $y(t)$, $R_{xy}(t,\tau)$, as follows [2]:

$$\Psi_{\mathtt{B}}(x(t),y(t)) = -\left.\frac{\partial^2 R_{xy}(t,\tau)}{\partial\tau^2}\right|_{\tau=0} - \left.\frac{\partial^2 R^*_{xy}(t,\tau)}{\partial\tau^2}\right|_{\tau=0} \tag{1}$$

Relation (1) shows that $\Psi_{\mathtt{B}}$ is a cross-energy function of two signals and thus links to transforms using the concept of instantaneous CC, such as Time-Frequency Representations (TFRs) can be found. In fact, relationships between $\Psi_{\mathtt{B}}$ operator and some

TFRs such as cross ambiguity function or cross-Wigner-Ville Distribution (WVD) show that Ψ_{B} is well suited to study non-stationary signals [2]-[4]. Ψ_{B} has found applications in both signal and image processing such as time series analysis, gene time series expression data clustering, transient detection or time delay estimation [5]-[8]. Ψ_{B} operator is a symmetric bilinear form defined as follows [2]:

$$\Psi_{\mathrm{B}}(x,y) = 0.5[\Psi_{\mathbb{C}}(x,y) + \Psi_{\mathbb{C}}(y,x)] \tag{2}$$

where the associated quadratic form $\Psi_{\mathbb{C}}(x,x) = \Psi_{\mathrm{B}}(x,x)$ is given by

$$\Psi_{\mathbb{C}}(x,y) = 0.5[\dot{x}^*\dot{y} + \dot{x}\dot{y}^*] - 0.5[x\ddot{y}^* + x^*\ddot{y}] \tag{3}$$

In this letter new properties of Ψ_{B} are established. We show the link between Ψ_{B} and the dynamic signal which describes the rate of the log-magnitude and phase [9]. The second relationship is a simple way to estimate the IF of a Frequency Modulated (FM) signal. Thus, it is natural to use Ψ_{B} a local operator, based of the signal time derivatives, as a basis for tracking instantaneous features such as IF function. In the following, for $x(t) = y(t)$ the notation $\Psi_{\mathrm{B}}(x(t), y(t)) \equiv \Psi_{\mathrm{B}}(x(t))$ is used.

2 Dynamic signal and Ψ_{B} operator

The dynamic signal is similar to the definition of complex cepstrum in the frequency domain and can be viewed as a generalization of the IF of a signal with varying magnitude [9]. Consider an AM-FM signal

$$x(t) = a(t)\mathrm{e}^{\mathrm{j}\phi(t)} \tag{4}$$

$a(t)$ is instantaneous amplitude and $\phi(t)$ instantaneous phase. The IF is given by

$$f_i(t) = \frac{1}{2\pi}\dot{\phi}(t) \tag{5}$$

The dynamic signal is defined by [9]

$$\beta(t) = \frac{d}{dt}\log x(t) = \frac{\dot{a}(t)}{a(t)} + \mathrm{j}\dot{\phi}(t) \tag{6}$$

and the instantaneous bandwidth is given by

$$\mathrm{ib}(t) = \left|\frac{\dot{a}(t)}{a(t)}\right| \tag{7}$$

If one substitutes $x(t)$ (Eq. 4) in equation (2), then one obtains

$$\Psi_{\mathrm{B}}(x(t)) = -\frac{\mid x(t) \mid^2}{2}\left[(\beta^*(t) - \beta(t))^2 + (\dot{\beta}^*(t) + \dot{\beta}(t))\right] \tag{8}$$

which shows that $\Psi_{\mathtt{B}}$ can be written in terms of dynamic signal. Furthermore, equation (8) can also be written as

$$\Psi_{\mathtt{B}}(x(t)) = | x(t) |^2 \left[8\pi^2 f_i^2(t) + \mathrm{i}\mathrm{b}^2(t) - \frac{\ddot{a}(t)}{a(t)} \right] \tag{9}$$

where there is relationship between $\Psi_{\mathtt{B}}$, the IF and the instantaneous bandwidth. Finally, links given by relations (8) and (9) show that $\Psi_{\mathtt{B}}$ conveys precious local information about the instantaneous behavior, over the time, of a signal.
For a FM signal ($a(t) = A$), using equation (9) the IF is given by

$$f_i(t) = \frac{\sqrt{\Psi_{\mathtt{B}}(x(t))}}{2\pi A\sqrt{2}} \tag{10}$$

where A is a constant. Relation (10) is a simple way to estimate the IF of a FM signal. This IF estimation is obtained without involving integral transform and with no a priori knowledge about the phase $\phi(t)$ of the signal. An advantage of $\Psi_{\mathtt{B}}$ operator is its localization property. This localization is the consequence of the differentiation based method.

3 Results

Consider a noisy Polynomial Phase Signal (PPS):

$$\begin{aligned} y(t) &= z(t) + n(t) = A\mathrm{e}^{\mathrm{j}\phi(t)} + n(t) \\ &= A\mathrm{e}^{\mathrm{j}\sum_{k=0}^{P} a_k t^k} + n(t), \;\; t \in [0, T] \end{aligned} \tag{11}$$

where $z(t)$ is a noise-free signal, $n(t)$ is a complex white Gaussian noise, $\phi(t)$ is signal phase, P is order of polynomial and a_k are corresponding polynomial coefficients. T is signal duration. A fourth-order PPS, with parameters $A = 2$ and ($a_0 = 1, a_1 = 10, a_2 = 10, a_3 = 2, a_4 = -5$), is selected and Monte-Carlo simulations implemented to show the effectiveness of $\Psi_{\mathtt{B}}$ (Eq. 10) as an IF estimator. Estimation is evaluated for input Signal to Noise Ratio (SNR) ranging from -6dB to 40dB with an interval of 2dB. Hundred realizations are performed for each SNR and Mean Square Error (MSE) between the estimate value and desired value chosen as performance criteria. Since both $\Psi_{\mathtt{B}}$ (Eq. 10) and unwrapped phase of the analytical signal (Hilbert Transform (HT)) (Eq. 5) involves derivatives, numerical differentiations. For very noisy signals, traditional Euler derivatives are useless. A better option is to use derivatives filters such as Savitzky-Golay differentiation filter [11]. This filter uses a polynomial fit across a moving window that preserves higher order moments of the signal. Derivatives are calculated with 4th order SG filter and 33 point window. Figure 1 shows the results of IF tracking using HT, WVD and $\Psi_{\mathtt{B}}$-based methods for SNR=30dB. Compared to HT and VWD, the best extraction is given by $\Psi_{\mathtt{B}}$ approach. Figure 1(b) shows an agreement of VWD estimate with true IF but with fluctuations of high magnitude due essentially to cross-terms corruption in presence of noise [10]. HT shows a good match but with ripples (modulations errors) (Fig. 1(c)) compared to $\Psi_{\mathtt{B}}$ (Fig. 1(d)). In figure 2, we show the MSE as function of input SNR for HT, WVD and $\Psi_{\mathtt{B}}$-based method. This result shows the ability of $\Psi_{\mathtt{B}}$ to resolve a non-linear FM component. As seen in Fig. 2, across of different SNRs, $\Psi_{\mathtt{B}}$ approach provides a significant performance improvement over HT and VWD approaches. Of particular interest are the results at low SNRs for which WVD approach performs less better than $\Psi_{\mathtt{B}}$ and HT. This is expected, as pointed out in [12]. As the SNR decreases WVD produces biased representation of signal energy as well spurious components (artifacts). Except for higher SNRs ($\geq$ 30dB), HT outperforms WVD approach mainly due to the use of an integral transform which does implicit smoothing (low pass filtering) [10]. For SNR $\geq$ 36 dB, $\Psi_{\mathtt{B}}$ and DWV have similar performances. It is important to mention that $\Psi_{\mathtt{B}}$ like most local approaches or those based on signal derivatives, is sensitive in very noisy environment. For moderately noisy data, classical Euler discretization schemes are robust to noise. For low SNRs, the use of derivatives filters such as SG is necessary for efficient tracking of the IF by $\Psi_{\mathtt{B}}$ or HT. It was found that for different polynomial orders ≥ 3 the obtained results are similar but the precision of estimates both by $\Psi_{\mathtt{B}}$ and HT are dependent on the size of the SG sliding window used.

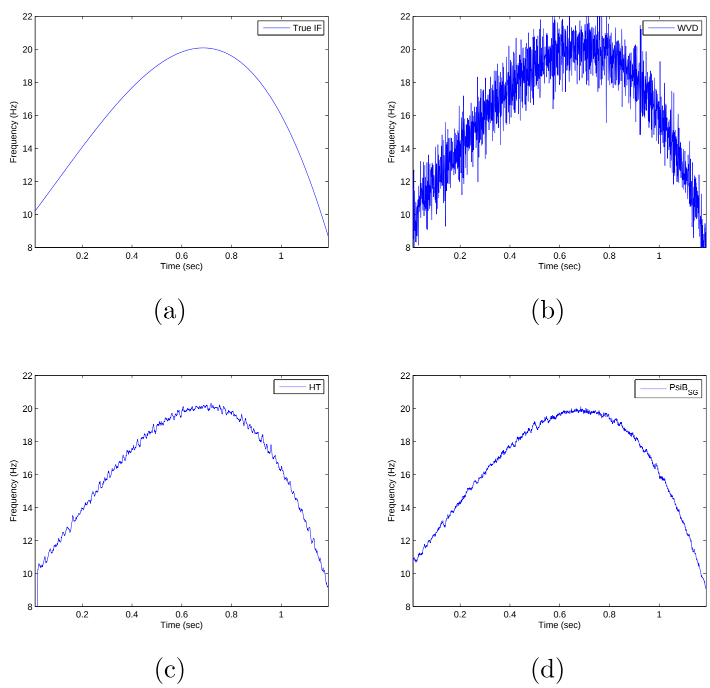

Figure 1: IF estimate for fourth-order PPS. (a) True. (b) WVD. (c) HT. (d) $\Psi_{\mathtt{B}}$ operator.

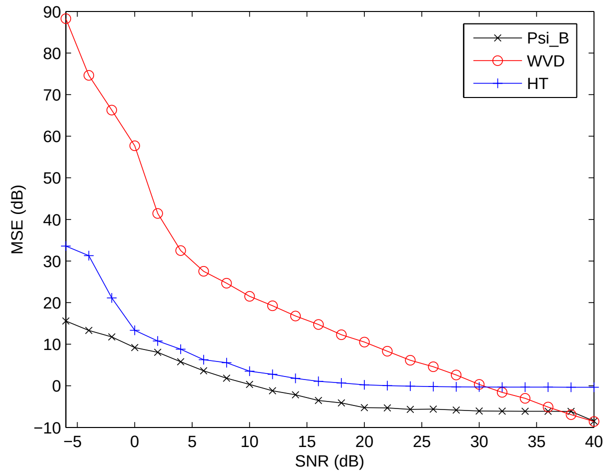

Figure 2: MSE in polynomial IF estimation for HT, WVD and Ψ_B operator.

4 Conclusion

In this letter two new properties of Ψ_B energy operator are presented. The link between Ψ_B and the dynamic signal shows that Ψ_B conveys precious local information about the instantaneous behavior, over the time, of a signal. The second property obtained for FM signals is a simple way to estimate the IF with no a priori knowledge about the phase of the signal. Preliminary results show that Ψ_B is effective for estimating the IF of a non-linear FM signal compared to HT and WVD based approaches. Further, the computational cost of Ψ_B is much lower. As future work we plan to apply Ψ_B to a wide range of synthetic and real signals to confirm the obtained results and to explore new properties of Ψ_B.

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