NUMERICAL MODELING OF UNDERWATER PARAMETRIC PROPAGATION TO DETECT BURIED OBJECTS

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Abstract: *In underwater acoustics, detection of buried objects in sediments (cables, mines,…) is a complex problem. One reason is that acoustic attenuation in these sediments increases with frequency. To ensure sufficient penetration depth in marine sediments, low frequencies have to be used, implying a low resolution. A solution proposed to solve this problem is the parametric emission based on the nonlinear properties of the propagation medium. This method can generate a low frequency wave from two directional high frequencies beams. The parametric propagation is simulated in seawater and marine sediments. The model developed is based on the fractional-step numerical method introduced by Christopher and Parker [1]. In this method, the normal particle velocity is calculated plane by plane from the surface of the transducer to a specified distance. The effects of nonlinearity, attenuation and diffraction are calculated independently for each spatial step. Moreover, to reduce the number of spatial steps, a second order operator splitting scheme is used. The diffraction computation is based on a method of angular spectrum in the frequency domain where the field across a source plane is described by a spatial frequency distribution. To improve code stability, the effects of nonlinearity and attenuation are calculated and associated in shorter propagation substeps. At the interface between water and marine sediments, the transmission conditions are applied. Several tests have been carried out in different configurations (changing the primary frequencies, the parametric frequency, the source geometry, the inclination of the source with the interface, the focal distance,…). The 3D velocity field is calculated in each case, thereby allowing to know the directivity of the source, the velocity amplitude in sediments and the performance.*

Keywords: *nonlinear acoustics, parametric emission, buried objects, fractional-step, 3D field*

1. INTRODUCTION

In order to detect buried objects in marine sediments, low frequency has to be used, implying a low resolution. A solution to solve this problem is the parametric emission. The aim of this work is to present a numerical model to simulate a parametric propagation in seawater and marine sediments. This model is based on the fractional-step numerical method introduced by Christopher and Parker [1]. In this method, the normal particle velocity is calculated plane by plane from the source to a specified distance. The effects of nonlinearity, attenuation and diffraction are calculated independently for each spatial step. Moreover, to reduce the number of spatial steps a second order operator splitting scheme is used. The present paper is organized as follows: first, the principle of the model is described, then, its application with the parametric emission is presented and last, few results are showed.

2. NUMERICAL MODELING

Numerical modeling proposed for nonlinear propagation is a frequency domain approach based on the Burgers equation and the angular spectrum method. This method is a numerical solution of Burgers equation taking into account the effects of nonlinearity, diffraction and attenuation with a split-step operator. Over sufficiently small steps, these effects can be treated independently. Knowing the normal velocity field for a plane z, the particle velocity profile v at position $z+\Delta z$ is [2] :

$$v(z+\Delta z) = v(z) + \left(\hat{L}_A \bullet v + \hat{L}_N \bullet v + \hat{L}_D \bullet v\right)\Delta z \,, \tag{1}$$

where $\hat{L}_A, \hat{L}_N$ and $\hat{L}_D$ are respectively the attenuation operator, the nonlinear operator and the diffraction operator.

The propagation modeling is divided into three steps [2]: diffraction over substep $\frac{\Delta z}{2}$, nonlinearity and attenuation over Δz, and second diffraction over substep $\frac{\Delta z}{2}$.

Moreover, to improve the stability of the code, the calculation of the nonlinearity and attenuation on the distance Δz is divided into several sub-steps.

2.1. Nonlinear and attenuation operators

To model nonlinear propagation, the wave is decomposed into different harmonic components :

$$v(x,y,z,t) = \frac{1}{2}\sum_{n=-\infty}^{+\infty} v_n(x,y,z)e^{j\omega_n t} \,, \tag{2}$$

where $v_n(x,y,z) = |v_n| e^{j\phi_n(x,y,z)}$ and $\phi_n(x,y,z)$ are respectively the complex amplitude and the phase of the nth harmonic $(\omega_n = n\omega)$ at the point (x,y,z).

Thereafter, to simplify notations, it is implied that v_n and ϕ_n systematically depend in x, y and z.

Integration of expressions (1) and (2) in the Burgers equation gives [3] :

$$v_n(z+\Delta z) = \left(\hat{L}_N + \hat{L}_A\right) \bullet v_n = v_n(z) + \left[\frac{j\beta\omega}{2c_0^2} \left(\sum_{k=1}^{n-1} k v_k v_{n-k} + \sum_{k=n+1}^{+\infty} n v_k v_{k-n}^* \right) - \alpha_n v_n \right] \Delta z \quad , \qquad (3)$$

with v_n^* the complex conjugate of v_n, β and c_0 respectively the nonlinearity coefficient and the celerity of the medium, and α_n the attenuation coefficient (in Np.m^{-1}) defined by the François-Garrison model [4] for the frequency f_n.

2.2. Diffraction operator

The diffraction algorithm is based on the angular spectrum method which is a frequency approach where the field of a source plane is described by a spatial frequencies' distribution. The 2D Fourier transform of the velocity field received at depth z is $V_n(k_x, k_y, z, n) = \mathcal{F}_{2D}(v_n(x, y, z, n))$.

At the depth $z + \Delta z$, the velocity field from this source can be calculated by [5] :

$$v_n(x, y, z+\Delta z, n) = \hat{L}_D \bullet v_n = \mathcal{F}_{2D}^{-1}[V_n(k_x, k_y, z, n) \times H(k_x, k_y, \Delta z, n)] \quad , \qquad (4)$$

where H is the transfer function :

$$H(k_x, k_y, \Delta z, n) = e^{j\Delta z\sqrt{k_n^2 - k_x^2 - k_y^2}} \text{ if } k_x^2 + k_y^2 \le k_n^2 \text{ or } e^{-\Delta z\sqrt{k_x^2 + k_y^2 - k_n^2}} \text{ else,} \qquad (5)$$

where $k_n = \frac{\omega_n}{c_0}$ is the wave number at the frequency nf, k_x and k_y are the wave number components in the (x, y) plane.

2.3. Sediment influence

Sediments are considered as an homogeneous fluid with characteristics different from those of seawater. At the water/sediment interface, the algorithm automatically handles the refraction between the two media, but the velocity transmission coefficient T_v depending on the incidence angle at the interface has to be included. It is applied in the spatial frequency domain : $v_n(x, y, z_s^+, n) = \mathcal{F}_{2D}^{-1}[V_n(k_x, k_y, z_s^-, n) \times T_v(k_x, k_y, n)]$,

$$\text{with } T_v(k_x, k_y, n) = \frac{2\rho_0\sqrt{c_0^2 k_n^2 - c_s^2\left(k_x^2 + k_y^2\right)}}{\rho_s c_s\sqrt{k_n^2 - k_x^2 - k_y^2} + \rho_0\sqrt{c_0^2 k_n^2 - c_s^2\left(k_x^2 + k_y^2\right)}} \quad , \qquad (6)$$

where ρ_s and c_s are respectively the sediment's density and celerity, and ρ_0 is the seawater's density.

Beyond the interface, the nonlinear propagation algorithm in sediments is the same as in seawater with specific characteristic parameters β_s, c_s, Δz_s and α_{ns} instead of β, c_0, Δz and α_n respectively.

3. PARAMETRIC EMISSION

In the case of a parametric emission, two high frequency waves f_{h_1} and f_{h_2} are emitted in order to generate a wave at beating frequency $f_l = f_{h_2} - f_{h_1}$ (with $f_{h_2} > f_{h_1}$) by nonlinear interactions.

The parametric ratio p is defined as the ratio between the average of primary frequencies over parametric frequency :

$$p = \frac{f_h}{f_l} \text{ with } f_h = \frac{f_{h_1} + f_{h_2}}{2}. \tag{7}$$

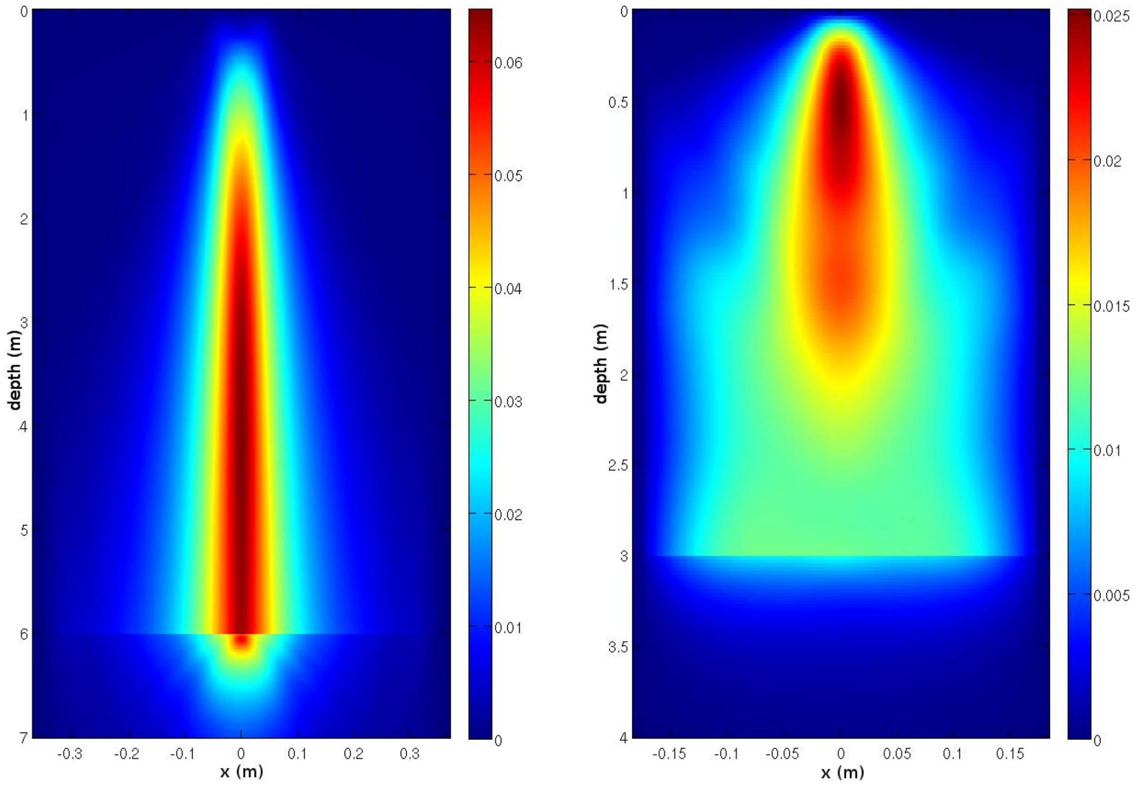
In order to avoid creating even lower frequencies than f_l by interactions between harmonics, parametric ratio is a half-integer : $p = n + \frac{1}{2}$ with n as a non-zero integer [6]. In this case, f_{h_1} and f_{h_2} are multiples of the parametric frequency : $f_{h_1} = (p - \frac{1}{2}) f_l$ and $f_{h_2} = (p + \frac{1}{2}) f_l$. To numerically model a parametric emission, the previously described code is used. At the source's surface, the velocity profile is initialized :

$$v(z = 0, t) = v^0_{p-\frac{1}{2}} e^{j\omega_{p-\frac{1}{2}} t} + v^0_{p+\frac{1}{2}} e^{j\omega_{p+\frac{1}{2}} t}, \tag{8}$$

with $v^0_{p-\frac{1}{2}}(x, y, z = 0) = \left|v^0_{p-\frac{1}{2}}\right| e^{\phi^0_{p-\frac{1}{2}}}$ and $v^0_{p+\frac{1}{2}}(x, y, z = 0) = \left|v^0_{p+\frac{1}{2}}\right| e^{\phi^0_{p+\frac{1}{2}}}$. If needed, a phase shift $\phi^0_n(x, y, z = 0) = k_n\left(\sqrt{x^2 + y^2 + d_f^2} - x \sin\theta_0\right)$ can be introduced to take into account both a focalization at a distance d_f and an inclination of the source with an incidence angle θ_0.

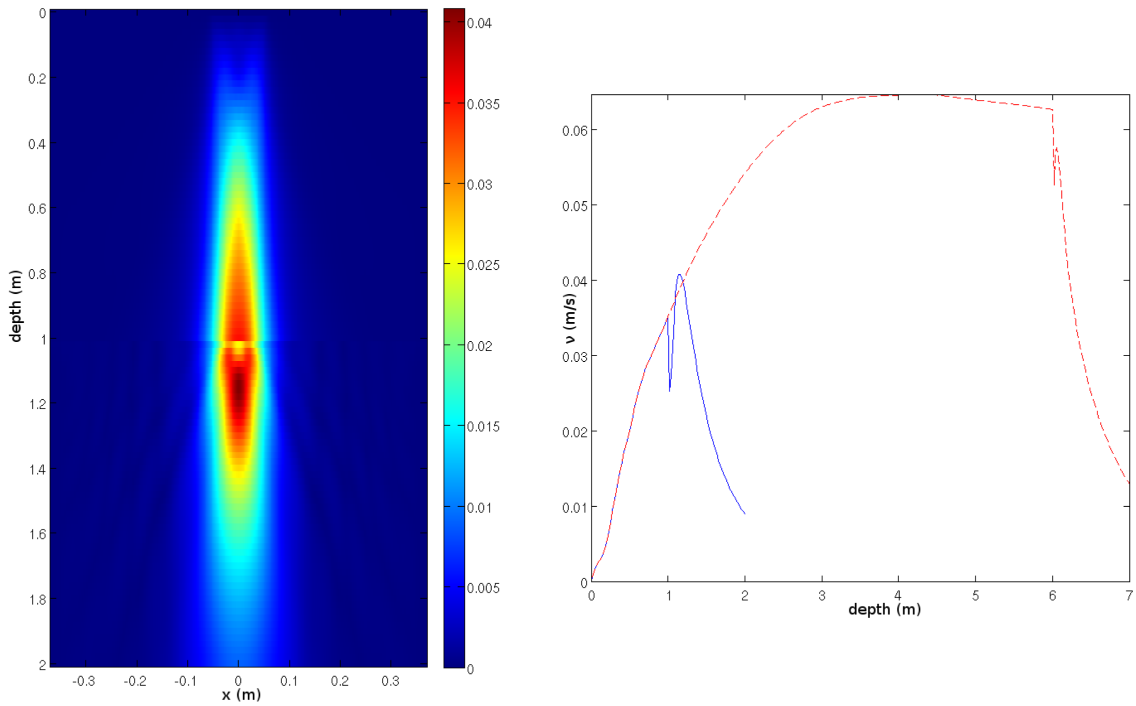
4. RESULTS

Simulations were performed with the following acoustic properties : $c_0 = 1520\,\text{m.s}^{-1}$ and $\rho_0 = 1025\,\text{kg.m}^{-3}$ for the water, $c_s = 1660\,\text{m.s}^{-1}$ and $\rho_s = 1600\,\text{kg.m}^{-3}$ for the sediment. The amplitude of the acoustic velocity at the source is constant for each primary wave. Tests were conducted for different primary frequencies f_h and parametric frequencies f_l. We also considered several water depths between the source and the sediment from 1 to 6 meters.



(a) *(b)*

Fig.1: Velocity field (m.s^{-1}) :
(a) for a depth of 6 m and a parametric frequency of 15 kHz
(b) for a depth of 3 m and a parametric frequency of 50 kHz.



(a) *(b)*

Fig.2: (a) Velocity field (m.s^{-1}) for a depth of 1 m and a parametric frequency of 15 kHz. (b) Axial propagation curves for depths of 1 m (—) and 6 m (---) for a parametric frequency of 15 kHz.

Figure (1a) shows the normal velocity field obtained for a parametric frequency of 15 kHz with a parametric ratio of 6.5 and a 6 m water height. For the frequency involved, attenuation is quite low and the parametric wave is generated through the entire water column. And despite high nonlinearity, velocity quickly decreases in the sediment. For a parametric frequency of 50 kHz (Fig.1b), as expected, maximal values of the parametric waves increase with frequency. Nevertheless, the growth of the parametric wave takes place only on the first meter due to attenuation.

Figure (2a) presents the velocity field obtained for a parametric frequency of 15 kHz for a depth of 1 m. In this case, the value is higher in the first 20 cm of the sediments. As shown in figure (2b) with the axial propagation curves, when the source is 1 m above the sediments, their nonlinearity implies an increase in the parametric amplitude on the first 20 cm. But this one decreases quickly with the attenuation.

5. CONCLUSION

A parametric acoustic emission has been simulated in seawater and marine sediments with a numerical model based on a fractional-step method. The normal velocity is calculated plane by plane from the source considering the effects of nonlinearity, attenuation and diffraction. Simulations show that this numerical model gives results in concordance with the theory and that primary frequencies have to be optimal : if they are too low, the nonlinear effects do not create a sufficient level of parametric wave, and if they are too high, the attenuation will prevent any significant level on sediment surface.

Simulations were also performed with different focalizations but did not give significant differences in sediments.

6. ACKNOWLEDGEMENTS

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