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Eulerian/Lagrangian Sharp Interface Schemes for Multimaterials

Y. Gorsse, A. Iollo and T. Milcent

Abstract We present multi-material simulations using both Eulerian and Lagrangian schemes. The methods employed are based on classical Godunov-like methods that are adapted to treat the case of interfaces separating different materials. In the models considered the gas, liquids or elastic materials are described by specific constitutive laws, but the governing equations are the same. Examples of gas-gas and gas-elastic material interactions in one and two spatial dimensions are presented.

1 Introduction

Physical and engineering problems that involve several materials are ubiquitous in nature and in applications. The main contributions in the direction of simulating these phenomena go back to [1] for the model and [2] for numerical simulations. However, the numerical scheme presented in that paper is relatively complicated and has the disadvantage that the interface is diffused over a certain number of grid points. We propose a simple second-order accurate method to recover a sharp interface description keeping the solution stable and non-oscillating. This scheme can be adapted to both Eulerian and Lagrangian frameworks.

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2 Eulerian model

The conservative form of elastic media equations in the Eulerian framework are

$$\begin{cases} \rho_t + \mathrm{div}_x(\rho u) = 0 \\ (\rho u)_t + \mathrm{div}_x(\rho u \otimes u - \sigma) = 0 \\ (\rho e)_t + \mathrm{div}_x(\rho e u - \sigma^T u) = 0 \\ (\nabla_x Y)_t + \nabla_x (u \cdot \nabla_x Y) = 0 \end{cases} \tag{1}$$

The unknowns are the density $\rho(x,t)$, the velocity $u(x,t)$, the total energy per unit mass $e(x,t)$ and the backward characteristics of the problem $Y(x,t)$. Here $\sigma(x,t)$ is the Cauchy stress tensor in the physical domain.

3 Lagrangian model

The counterpart of these equations in the lagrangian framework are

$$\begin{cases} (\rho_0 X_t)_t - \mathrm{div}_\xi(\mathscr{T}) = 0 \\ (\rho_0 e)_t - \mathrm{div}_\xi(\mathscr{T}^T X_t) = 0 \end{cases} \tag{2}$$

The unknowns are the velocity $X_t(\xi,t)$ (the direct characteristics of the problem are $X(\xi,t)$) and the total energy per unit mass $e(\xi,t)$. Here $\mathscr{T}(\xi,t)$ is the first Piola-Kirchoff stress tensor in the reference domain and ρ_0 is the initial density.

4 Constitutive law

To close the system, a constitutive law is chosen:

$$\varepsilon = e - \frac{1}{2}|u|^2 = \frac{\exp\left(\frac{s}{c_v}\right)\, \rho^{\gamma-1}}{\gamma - 1} + \frac{p_\infty}{\rho} + \frac{\chi}{\rho_0}(\mathrm{Tr}(\overline{B}) - 2) \tag{3}$$

where $s(x,t)$ is the entropy and $\overline{B}$ is the modified left Cauchy-Green tensor which depends on $\nabla_x Y$. The constants $c_v, \gamma, p_\infty, \chi$ characterize a given material. The two first terms of (3) represent a stiffened gas and the third one represents a Neo-Hookean elastic solid. The stress tensors σ and $\mathscr{T}$ are then derived from this constitutive law as a function of the problem unknowns.

5 Approximate Riemann Solver

We have already developed a numerical scheme to solve the Eulerian equations of conservation (1). This scheme is based on a directional splitting on a fixed cartesian mesh where the fluxes are computed by an HLLC approximate Riemann solver. The interface is kept sharp by using a non-conservative numerical flux for the computational cells that are crossed by the contact discontinuity. We refer for the 1D numerical results to [3] and [4].

For the Lagrangian scheme the ideas are the same but the contact discontinuity turns out to be steady by definition in this framework.

6 Results

6.1 Eulerian

For the Eulerian framework, we provide numerical results of fluid-structure interaction involving 2D impacts. The initial configuration is a ”T” shaped domain (filled with copper or water) and surrounded by air. We impose an initial horizontal velocity on the protruding square of the ”T”. In Fig.1 we present the numerical results obtained with our multimaterial scheme of air-copper (left) and air-water (right) media after the impact.

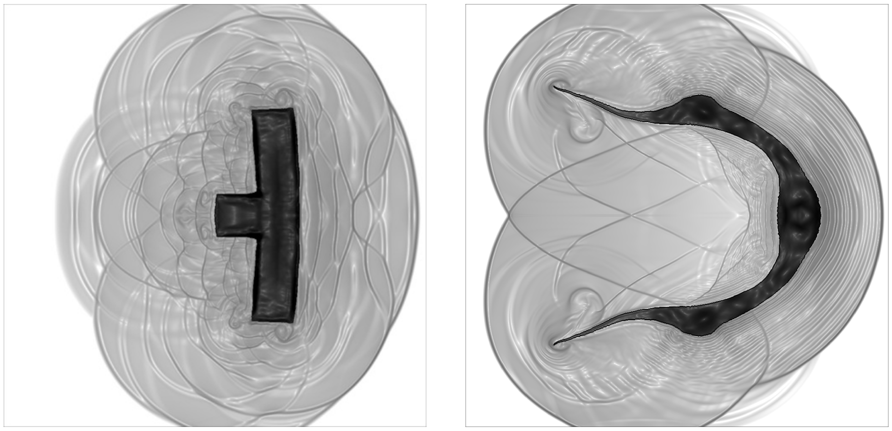

Fig. 1 Schlieren image of the density for the air-copper (left) and air-water (right) after the impact.

6.2 Lagrangian

As for the Lagrangian equations of conservation (2), we start by solving classical 1D shock-tube test cases for a perfect gas ($\gamma = 1.4$). First of all we consider a Sod shock tube. The initial condition consists two piece-wise constant states. For the left state $\rho_0 = 1$, $-\mathscr{T}^{11} = 1$, $X_t = 0$; for the right $\rho_0 = 0.125$, $-\mathscr{T}^{11} = 0.1$, $X_t = 0$. The results at the same times in the reference and in the physical plane are shown in Fig. 2 with second order accuracy in space and 1000 grid points. A similar test case has been performed with an elastic material whose characteristics model those of copper ($\gamma = 4.22$, $p_\infty = 3.42\ 10^{10}$, $\chi = 3.42\ 10^{10}$). We have also for the left state $\rho_0 = 8900$, $-\mathscr{T} = 10^9$, $X_t^1 = 0$, $X_t^2 = 0$; for the right $\rho_0 = 8900$, $-\mathscr{T} = 10^5$, $X_t^1 = 0$, $X_t^2 = 100$. We have discontinuous normal tensor components and discontinuous vertical velocity components. In Fig. 3 the results are shown for the physical domain. They compare well with those presented in [2] and [3].

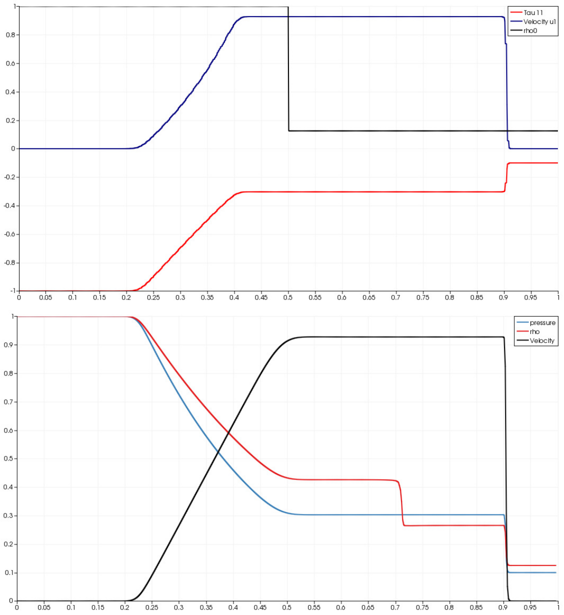

Fig. 2 Sod test case. Reference domain (up) and physical domain (down).

7 Conclusions

We have synthetically presented a model to deal with multimaterials in Eulerian and Lagrangian framework. For the Eulerian case we show 2D results of stiff gas-

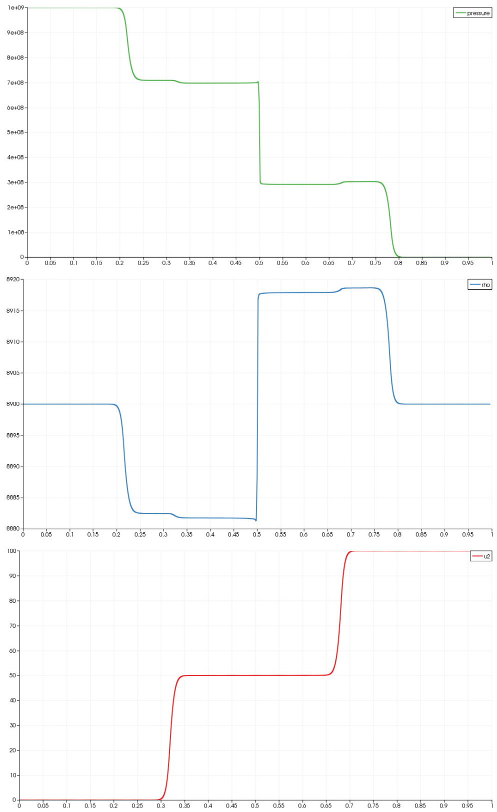

Fig. 3 Copper test case. Five waves are present: the contact discontinuity is sharp; pressure is discontinuous at the contact discontinuity (but the stress tensor is continuous); the vertical speed is continuous at the contact discontinuity and at the normal perturbations, discontinuous at the shear waves.

structure, liquid-structure interaction. For the Lagrangian case, we have limited our scrutiny to 1D cases. Ongoing research adresses 2D models.

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