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Right Harmonic Spectrum for the Back-Electromotive Force of a *n*-phase Synchronous Motor

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***Abstract*—This paper deals with a vector control of *n*-phase permanent magnet synchronous machine. To use control algorithms already developed for sine-wave 3-phase machines, the spectrum of back electromotive force (EMF) must contain only odd *2k+1* harmonics which verify the following inequality, $1 \le 2k+1 < n$. In an experimental vector control of a 5-phase drive, two usual algorithms of sine-wave 3-phase machine are thus used to supply a machine with trapezoidal waveform back EMF. In this case, the first and third harmonics are used to produce torque: the other harmonics, and particularly the 7th one, induce effects as torque ripples and parasitic currents.**

Keywords- multi-phase; synchronous motor; multi-machine.

I. INTRODUCTION

Multi-phase machines have several advantages over conventional 3-phase ones [1]-[2] such as higher reliability and reduction of the torque pulsation magnitude. Nevertheless, their vector controls are not known as well as those of 3-phase BLDC machines [3]. For these last ones, two kinds of controls are usually distinguished on the basis of back EMF waveforms. If the back EMFs have a sinusoidal waveform then current controls in synchronous frame achieve high efficient brushless DC machines. If the back EMFs have trapezoidal waveforms, the control in stationary frame is simpler but also less efficient [3]. For *n*-phase machines, particular controls have already been proposed [4]-[6] but there is no synthetic approach. This simple classification is no more obvious for *n*-phase machines. In fact, a more accurate harmonic analysis of back EMF is necessary to distinguish between two kinds of machines which make possible two kinds of performances. We consider only the case of machines whose number of phases is a odd number. If the harmonic spectrum of a *n*-phase machine contains only odd harmonics strictly inferior to *n*, then it is possible to achieve equivalent controls as those of sine-wave 3-phase machines. In this paper, the particular case of a 5-phase experimental (Fig1. Fig.2 Fig.3) drive is used to highlight the influence of harmonics on the vector control.

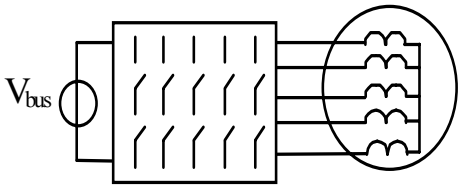

Figure 1. Symbolic representation of the 5-leg PWM-VSI and wye-coupled 5-phase machine.

Figure 2. Experimental 5-phase motor.

II. MULTI-MACHINE CHARACTERIZATION OF A *n*-PHASE MACHINE

Under assumptions of no saturation, no reluctance effect and regular spatial distribution of windings, the vectorial approach developed in [7]-[9] leads to define the equivalence of a *n*-phase wye-connected machine to a set of *(n-1)/2* fictitious independent 2-phase machines.

A. Assumptions, notations, nomenclature

1) Nomenclature

All quantities relating to the phase n° *k* are written x_k.

- j_{sk}: current in the phase n° *k*;
- u_{sk}: voltage across the phase n° *k*;
- ϕ_{sk}: flux through the phase n° *k* exclusively produced by the stator currents;
- ϕ_{srk}: flux through the phase n° *k* exclusively due to the rotor magnets;
- e_k: electromotive force (EMF) induced in the phase n° *k* only due to the rotor magnets;
- θ: mechanical angle.

2) Assumptions

Usual assumptions are used to model the machine:

- all phases are identical and regularly shifted by an angle (Fig.4):

$$\alpha = \frac{2\pi}{n} \tag{1}$$

- effects of saturation and damper windings are neglected.

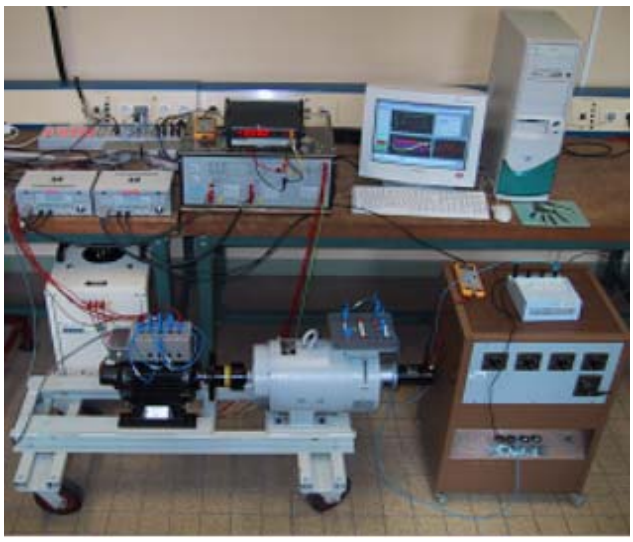

Figure 3. Experimental drive.

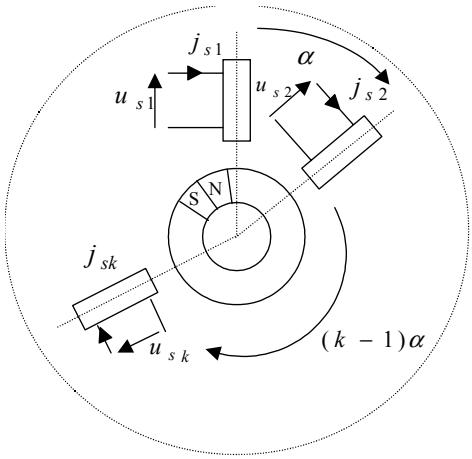

Figure 4. Presentation of *n*-phase PM machine.

B. *Usual modeling in a natural base*

In the usual matricial approach of *n*-phase machines, a vector *n*-space is implicitly considered since vectors with *n* components are defined. This space is provided with an orthonormal base $B_\sigma = \left\{\overrightarrow{x_1^\sigma}, \overrightarrow{x_2^\sigma}, ..., \overrightarrow{x_n^\sigma}\right\}$ that can be called "natural" since the coordinates of a vector in this base are the measurable values relative to each phase.

In this paper, we consider that this space is an Euclidean vectorial space with the usual canonical dot product.

In this natural base, different vectors are defined:

$$\overrightarrow{u_s} = u_{s1}\overrightarrow{x_1^\sigma} + u_{s2}\overrightarrow{x_2^\sigma} + ... + u_{sn}\overrightarrow{x_n^\sigma} \quad (2)$$

$$\overrightarrow{j_s} = j_{s1}\overrightarrow{x_1^\sigma} + j_{s2}\overrightarrow{x_2^\sigma} + ... + j_{sn}\overrightarrow{x_n^\sigma} \quad (3)$$

$$\overrightarrow{\phi_s} = \phi_{s1}\overrightarrow{x_1^\sigma} + \phi_{s2}\overrightarrow{x_2^\sigma} + ... + \phi_{sn}\overrightarrow{x_n^\sigma} \quad (4)$$

$$\overrightarrow{e} = e_1\overrightarrow{x_1^\sigma} + e_2\overrightarrow{x_2^\sigma} + ... + e_n\overrightarrow{x_n^\sigma} \quad (5)$$

Taking into account R_s, the stator resistance per phase, the vectorial voltage equation is:

$$\overrightarrow{u_s} = R_s\ \overrightarrow{j_s} + \left[\frac{d\vec{\phi}_s}{dt}\right]_{/B_\sigma} + \vec{e} \quad (6)$$

The B_σ index reminds that differentiation is operated according to the B_σ natural base.

This equation can be projected onto each vector of the natural base to find again the more usual equation:

$$u_{sk} = \overrightarrow{u_s}.\overrightarrow{x_k^\sigma} = R_s\ j_{sk} + \frac{d\phi_{sk}}{dt} + e_k \quad (7)$$

The assumptions previously defined allow to write a linear relation between the stator current vector and the stator flux vector:

$$\overrightarrow{\phi_s} = \Lambda_s\left(\overrightarrow{j_s}\right) \quad (8)$$

usually written with the circular matrix notation:

$$\left[L_s^\sigma\right] = Mat\left[\Lambda_s, B_\sigma\right] = \begin{pmatrix} M_{s_1s_1} & M_{s_1s_2} & ... & M_{s_1s_n} \\ M_{s_1s_n} & M_{ss1} & ... & M_{s_1s_{n-1}} \\ ... & ... & ... & ... \\ M_{s_1s_2} & M_{s_1s_3} & ... & M_{s_1s_1} \end{pmatrix} \quad (9)$$

The modeling of the machine in the natural base is not simple because of the complexity of the matrix $\left[L_s^\sigma\right]$. It would be interesting to find a simpler form for matrix of Λ_s.

C. *Vectorial decomposition in eigenspaces of Λ_S*

The equation (8) is true whatever is the chosen base associated with the stator coils. The symmetry and circularity of the inductance matrix $\left[L_s^\sigma\right]$allows to state that:

- Λ_s belongs *F* eigenvalues L_g with $F=(n+1)/2$ and

$$L_g = \sum_{e=1}^{e=n} M_{s_1s_e}\ cos\left(\frac{2\pi}{n}g(e-1)\right) \quad (10)$$

- the *F* associated eigenspaces G_g are orthogonal each other;
- the G_0 eigenspace, associated with the eigenvalue L_0, is a vectorial line;
- the others $(n-1)/2$ eigenspaces G_{2g} $(1 \le g \le (n-1)/2)$, associated with the others $(n-1)/2$ eigenvalues L_g are vectorial planes. A base of G_{2g} plane is:

$$\begin{cases} \overrightarrow{x_g^d} = \sqrt{\frac{2}{n}}\sum_{h=1}^{h=n}\cos\left(\frac{2\pi}{n}g(h-1)\right)\overrightarrow{x_h^\sigma} \\ \overrightarrow{x_{g+1}^d} = \sqrt{\frac{2}{n}}\sum_{h=1}^{h=n}\sin\left(\frac{2\pi}{n}g(h-1)\right)\overrightarrow{x_h^\sigma} \end{cases} \quad (11)$$

The orthogonality of the G_{2g} planes allows to decompose every vector $\overrightarrow{y}$ into a sum of vectors $\overrightarrow{y_g}$ obtained by orthogonal projection of $\overrightarrow{y}$ onto G_{2g}:

$$\overrightarrow{y} = \sum_{g=0}^{g=F-1} \overrightarrow{y_g} \tag{12}$$

This kind of decomposition has already been achieved in particular cases [10] for induction machines.

Finally, as L_g are eigenvalues of Λ_s, a simpler expression between flux and currents can be expressed:

$$\overrightarrow{\phi_s} = \sum_{g=0}^{g=F-1} \overrightarrow{\phi_g} = \sum_{g=0}^{g=F-1} L_g \overrightarrow{j_g} \tag{13}$$

D. Multi-machine concept

It is now possible to show that the torque of a multi-phase machine can be decomposed into a sum of torques of machines which are magnetically decoupled.

First, the electric power in the stator is expressed:

$$p_s = \sum_{k=1}^{k=n} u_{sk}\ j_{sk} = \overrightarrow{u_s} \cdot \overrightarrow{j_s} \tag{14}$$

From (6) we obtain:

$$p_s = \overrightarrow{u_s} \cdot \overrightarrow{j_s} = R_s\ \overrightarrow{j_s} \cdot \overrightarrow{j_s} + \left[\frac{d\overrightarrow{\phi_s}}{dt}\right]_{B_\sigma} \cdot \overrightarrow{j_s} + \overrightarrow{e_s} \cdot \overrightarrow{j_s} \tag{15}$$

With the previous assumptions, the eigenvalues and eigenvectors are constant, the electromotive force does not depend on currents. We have then:

$$\left[\frac{d\overrightarrow{\phi_s}}{dt}\right]_{B_\sigma} = \sum_{g=1}^{g=F} L_g \frac{d\overrightarrow{j_g}}{dt} \tag{16}$$

and

$$\overrightarrow{e_s} = \left(\frac{d\theta}{dt}\right) \frac{d\overrightarrow{\phi_{sr}}}{d\theta} \tag{17}$$

Therefore, as the eigenspaces are orthogonal each other, (14) becomes:

$$p_s = \overrightarrow{u_s} \cdot \overrightarrow{j_s} = \sum_{g=0}^{g=F-1} \left(R_s \left(\overrightarrow{j_g}\right)^2 + L_g \frac{d\overrightarrow{j_g}}{dt} \cdot \overrightarrow{j_g} + \left(\frac{d\theta}{dt}\right) \frac{d\overrightarrow{\phi_{srg}}}{d\theta} \cdot \overrightarrow{j_g} \right) =$$

$$\sum_{g=0}^{g=F-1} \left(R_s \left(\overrightarrow{j_g}\right)^2 + \frac{d\frac{1}{2} L_g \left(\overrightarrow{j_g}\right)^2}{dt} + \left(\frac{d\theta}{dt}\right) \frac{d\overrightarrow{\phi_{srg}}}{d\theta} \cdot \overrightarrow{j_g} \right) \tag{18}$$

The first term can be considered as stator Joule losses, the second term as derivative of stator magnetic energy and the third as the product of angular speed $d\theta/dt$ by torque:

$$T_{2g} = \frac{d\overrightarrow{\phi_{srg}}}{d\theta} \cdot \overrightarrow{j_g} \tag{19}$$

The wye-coupling implies that $j_0 = \sum_{k=1}^{k=n} j_{sk} = 0$.

The total torque T_{em} is then:

$$T_{em} = \sum_{g=1}^{g=F-1} T_{2g} \tag{20}$$

So we can consider *(F-1)* fictitious machines, each one of them being associated with one eigenspace. These machines are magnetically independent and have two phases since the dimension of their associated eigenspaces is two. These 2-phase machines are electrically and mechanically coupled.

A very well-known case is the wye-coupled 3-phase machine that is considered to be equivalent to only one 2-phase machine in d-q theory. The control of the torque of such 3-phase machine is then equivalent to this one of the 2-phase fictitious machine. In this case, it is obvious that the proposed approach is not interesting.

In the case of a wye-coupled 5-phase machine, there are two fictitious 2-phase machines named MM (Main Machine, *g=1*) and SM (Secondary Machine, *g=2*) which produces respectively the torque T_2 and T_4. Consequently, for the control of the 5-phase machine, a choice must be done to define the two references $T_{2\text{-}ref}$ and $T_{4\text{-}ref}$ from a unique torque reference $T_{em\text{-}ref}$. Of course, the choice depends on the characteristics of the two fictitious machines but also of the supply of the 5-phase machine. The designer must simultaneously take into account these two aspects [11].

This kind of problem is the same as found into multi-machine control. So, we propose to use a formalism developed for these systems [12]-[13]. The considered approach (Fig.5) highlights the electrical and mechanical couplings (the intersecting rectangles and triangles) between the two fictitious machines (the circles).

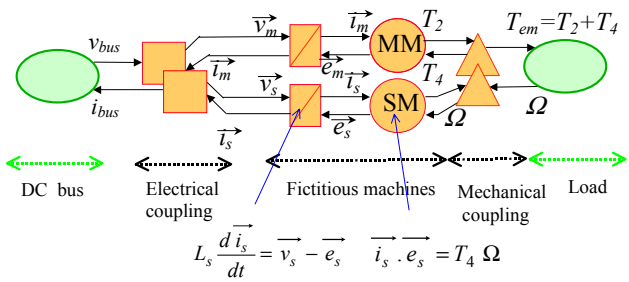

Figure 5. Multi-machine Representation of a 5-phase machine.

To control a multi-machine system, it is necessary to characterize each machine. In the following section, an

harmonic characterization of the fictitious machines is presented.

III. HARMONIC CHARACTERIZATION OF FICTITIOUS MACHINES

We prove that for a *n*-phase machine, a machine n° *g* is associated with the family of $nh \pm g$ odd harmonics ($1 \le g \le (n-1)/2$). In the case of the 5-phase machine (*n=5*), the first machine (MM, *g=1*) is associated with the odd harmonics $5h \pm 1$, and the Secondary Machine (SM, *g=2*) with the odd harmonics $5h \pm 2$.

1) *Projection onto eigenspace G_{2g}*

Expression (19) shows that the torque T_{2g} is the dot product of projections of two vectors. The first one is only depending on the design of the machine, the second one is imposed by the power supply. To characterize the fictitious machine, $d\overrightarrow{\phi_{sr}} / d\theta = \overrightarrow{q(\theta)}$ is analyzed.

As $\overrightarrow{q(\theta)}$ is a $2\pi/p$ periodic function, each one of its *n* components, q_k, can be decomposed into a Fourier series. Properties of symmetry, due to the regular manufacturing assumption of the rotating machine, involve, as usual, the cancellation of cosine terms and of even sine terms.

$$q_k(\alpha) = \sum_{e=1, e\neq 2h}^{+\infty} w_e \sin\left[e\,p\left(\alpha - (k-1)\frac{2\pi}{np}\right)\right] \tag{21}$$

with

$$w_e = \frac{p}{\pi}\int_{-\frac{\pi}{p}}^{\frac{\pi}{p}} q_1(\alpha)\sin(e\,p\alpha)\,d\alpha \tag{22}$$

To get components of $\overrightarrow{q(\alpha)}$ in the different eigenspaces, the equation must be inserted in the following vectorial expression of $\overrightarrow{q(\alpha)}$ in the natural basis:

$$\overrightarrow{q(\alpha)} = \sum_{k=1}^{k=n} q_k(\alpha)\,\overrightarrow{x_k^{\sigma}} \tag{23}$$

By combination of (21) and (23), $\overrightarrow{q(\alpha)}$ becomes:

$$\overrightarrow{q(\alpha)} = \sum_{e=1, e\neq 2h}^{+\infty} w_e\,\overrightarrow{g_e(\alpha)} \tag{24}$$

with

$$\overrightarrow{g_e(\alpha)} = \sum_{k=1}^{k=n} \sin\left(ep\left(\alpha - \frac{2\pi}{np}(k-1)\right)\right)\overrightarrow{x_k^{\sigma}} \tag{25}$$

$\overrightarrow{q(\alpha)}$ has been expressed as a sum of n vectors $\overrightarrow{g_e(\alpha)}$. It is sufficient to study the projections of $\overrightarrow{g_e(\alpha)}$.

2) *Family of harmonics*

Let us calculate the projection of the vector $\overrightarrow{g_e(\alpha)}$ onto the subspace G_{2g}. The dot product of $\overrightarrow{g_e(\alpha)}$ by $\overrightarrow{x_g^d}$ and $\overrightarrow{x_{g+1}^d}$ (two vectors of a base of G_{2g}) is evaluated:

$$\overrightarrow{g_e(\alpha)}\,.\,\overrightarrow{x_g^d} = \sqrt{\frac{2}{n}}\sum_{k=1}^{k=n} \sin\left(ep\alpha - \frac{2\pi}{n}e(k-1)\right)\cos\left(\frac{2\pi}{n}g(k-1)\right) =$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \sin\left(ep\alpha - \frac{2\pi}{n}e(k-1) + \frac{2\pi}{n}g(k-1)\right) +$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \sin\left(ep\alpha - \frac{2\pi}{n}e(k-1) - \frac{2\pi}{n}g(k-1)\right) =$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \sin\left(ep\alpha - \frac{2\pi}{n}(k-1)(e-g)\right) +$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \sin\left(ep\alpha - \frac{2\pi}{n}(k-1)(e+g)\right) =$$

$$\sqrt{\frac{1}{2n}}\,Im\left(\sum_{k=1}^{k=n} e^{jep\alpha}\left[e^{\left[-j\frac{2\pi}{n}(e-g)\right](k-1)} + e^{\left[-j\frac{2\pi}{n}(e+g)\right](k-1)}\right]\right.$$

With the same approach:

$$\overrightarrow{g_e(\alpha)}\,.\,\overrightarrow{x_{g+1}^d} = \sqrt{\frac{2}{n}}\sum_{k=1}^{k=n} \sin\left(ep\alpha - \frac{2\pi}{n}e(k-1)\right)\sin\left(\frac{2\pi}{n}g(k-1)\right) =$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \cos\left(ep\alpha - \frac{2\pi}{n}e(k-1) + \frac{2\pi}{n}g(k-1)\right) -$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \cos\left(ep\alpha - \frac{2\pi}{n}e(k-1) - \frac{2\pi}{n}g(k-1)\right) =$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \cos\left(ep\alpha - \frac{2\pi}{n}(k-1)(e-g)\right) -$$

$$\sqrt{\frac{1}{2n}}\sum_{k=1}^{k=n} \cos\left(ep\alpha - \frac{2\pi}{n}(k-1)(e+g)\right) =$$

$$\sqrt{\frac{1}{2n}}\,Re\left(\sum_{k=1}^{k=n} e^{jep\alpha}\left[e^{\left[-j\frac{2\pi}{n}(e-g)\right](k-1)} - e^{\left[-j\frac{2\pi}{n}(e+g)\right](k-1)}\right]\right.$$

As $1 + x + x^2 + \ldots + x^{n-1} = \dfrac{1-x^n}{1-x}$ unless *x=1,* we get the following results:

For *h* positive integer	$\overrightarrow{g_e(\alpha)}\,.\,\overrightarrow{x_g^d}$	$\overrightarrow{g_e(\alpha)}\,.\,\overrightarrow{x_{g+1}^d}$
$e = nh+g$ and odd number	$\sqrt{\frac{n}{2}}\sin(ep\alpha)$	$\sqrt{\frac{n}{2}}\cos(ep\alpha)$
$e=nh-g$ and odd number	$\sqrt{\frac{n}{2}}\sin(ep\alpha)$	$-\sqrt{\frac{n}{2}}\cos(ep\alpha)$
$e+g \neq nh$ and $e-g \neq nh$	0	0

It appears that the family of odd harmonics verifying $e = nh \pm g$ is associated with the eigenspace G_{2g}. According to the considered eigenspace associated with the fictitious studied machine, the null terms of the Fourier series are not the same. There is a distribution of the different space harmonics between the eigenspaces.

3) *References and examples*

This kind of harmonic characterization has already been observed in [14] by E. Klingshirn, studying multi-phase induction motors supplied by PAM VSI: in steady states, an harmonic analysis of the periodic voltages shows that there are families of harmonics which are each one associated with a particular impedance of the motor.

For transient states, [10] has also in the particular case of a 6-phase induction machine exhibited, using a matrix approach, harmonic families associated with particular eigenspaces.

Other harmonic studies [15]-[18] have been also realized for *n*-phase machines but have not exhibited neither the fictitious machines nor the independence of the different families of harmonics.

In the following tables, examples of Multi-machine characterization are given.

TABLE I. HARMONIC CHARACTERIZATION FOR WYE-COUPLED 3-PHASE MACHINE

Fictitious machine	Eigenspace	Families of odd harmonics
First 2-phase machine	G_2	1, 5, 7,…, $3h \pm 1$

TABLE II. HARMONIC CHARACTERIZATION FOR WYE-COUPLED 5-PHASE MACHINE

Fictitious machine	Eigenspace	Families of odd harmonics
First 2-phase machine	G_2	1, 9, 11,…, $5h \pm 1$
Second 2-phase machine	G_4	3, 7, 13…, $5h \pm 2$

TABLE III. HARMONIC CHARACTERIZATION FOR WYE-COUPLED 7-PHASE MACHINE

Fictitious machine	Eigenspace	Families of odd harmonics
First 2-phase machine	G_2	1, 13, 15,…, $7h \pm 1$
Second 2-phase machine	G_4	5, 9, 19…, $7h \pm 2$
Third 2-phase machine	G_6	3, 11, 17…, $7h \pm 3$

TABLE IV. HARMONIC CHARACTERIZATION FOR WYE-COUPLED 9-PHASE MACHINE

Fictitious machine	Families of odd harmonics
First 2-phase machine	1, 17, 19,…, $9h \pm 1$
Second 2-phase machine	7, 11, 25…, $9h \pm 2$
Third 2-phase machine	3, 15, 21…, $9h \pm 3$
Fourth 2-phase machine	5, 13, 23…, $9h \pm 4$

IV. RIGHT HARMONIC SPECTRUM OF BACK-EMF FOR EFFICIENT CONTROL

The Multi-machine modeling of *n*-phase machine is used to find right characteristics of the machine for efficient vector control.

If the back EMFs of a 3-phase machine have a sinusoidal waveform then current controls in synchronous frame permit to achieve high efficient brushless DC machines particularly at low speeds [3].

It has been shown that the control of a *n*-phase machine is equivalent to the control of *(n-1)/2* 2-phase fictitious machines. To be able to use algorithms developed for sine wave 3-phase machines, it is enough to have only one harmonic per fictitious machine. Each fictitious machine is then a sine wave one. For each fictitious machine it is then possible to work in a frame where the back EMF has a constant value. To get constant torque the reference current i_{ref} is then constant: a simple Proportional Integral controller is convenient for each fictitious machine.

Consequently, the equivalent case of a sine-wave 3-phase machine for a *n*-phase one is that the spectrum of back electromotive force must contain only odd *2k+1* harmonics which verify the following inequality, $1 \le 2k+1 < n$.

For a 5-phase machine for example the back-EMF has only to contain the first and third harmonics. The corresponding synopsis of control is presented in Fig.7.

After a choice of repartition of the torque reference $T_{em\text{-}ref}$ between the two fictitious machines, two independent controls using the same algorithm as for vector control of 3-phase machines are used. The unique difference is that the angle of the matrix rotation relative to the secondary machine is 3θ instead of θ. Such a control has been implemented for an experimental 5-phase machine whose characteristics of back EMF (Fig.6) are given in TABLE V.

One can remark that the main machine is almost a machine with a sinusoidal back-EMF since the 9th represents only 1,7% of the first harmonic. For the secondary machine, the 7th harmonic of back-EMF, which is only 5,1% of the fundamental, represents 18% of the 3rd harmonic. Consequently, the back-EMF of SM can not be considered as sinusoidal because it is composed of 3rd and 7th harmonics.

TABLE V. HARMONIC DECOMPOSITION OF BACK-ELECTROMOTIVE OF ONE PHASE

Order of harmonic	1	3	5	7	9
Relative RMS amplitude	100%	28,5%	12,4%	5,1%	1,7%

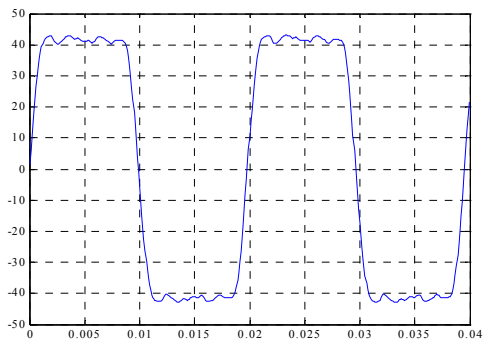

Figure 6. Back electromotive force of 5-phase machine.

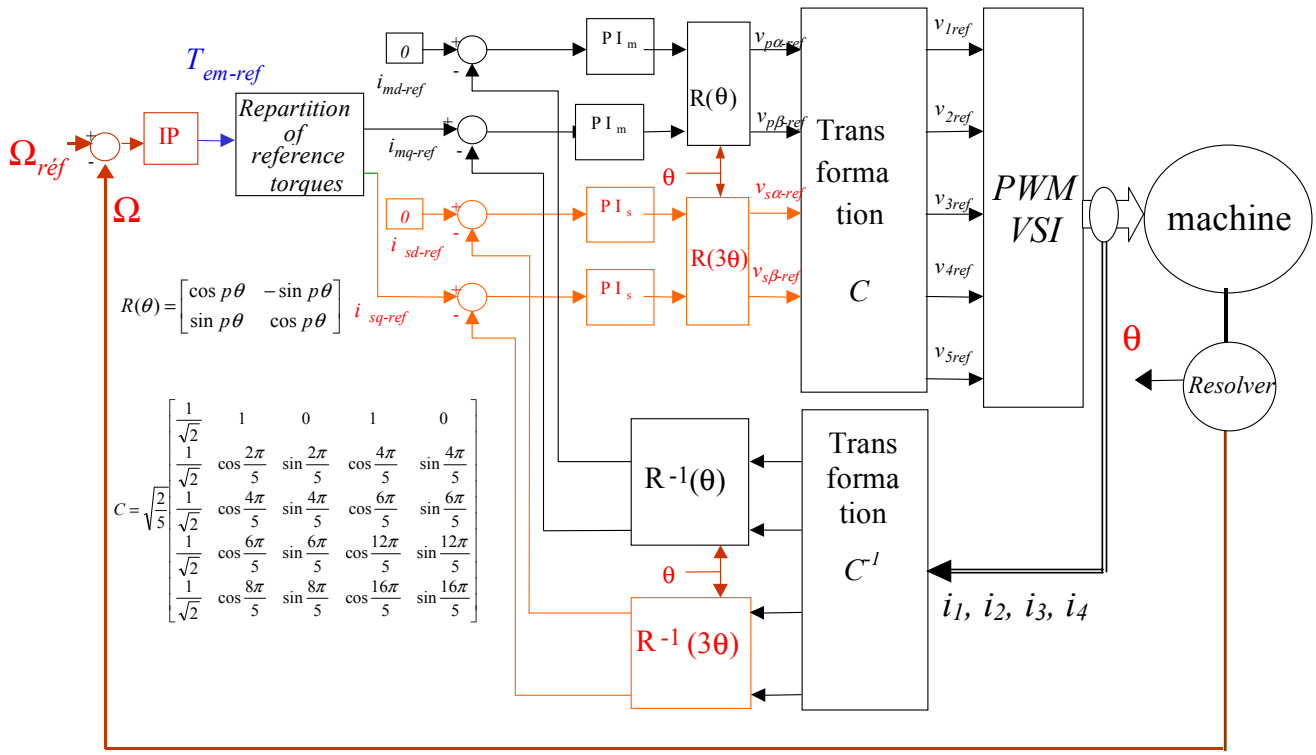

Figure 7. Synopsis of control.

To exhibit the influence of the 7th harmonic on the performance of the drive, it has been chosen to use only the Main Machine to produce the torque. In the ideal case of a machine with only the 1st and 3rd harmonics in the back emf it should appear only the first harmonic in the spectrum of the current. In Fig.8 and Fig.10 effects of 7th harmonic of EMF are visible on the current in the phase n°1. Fig.12 shows currents in each frame of the fictitious machines: it is effectively in the Secondary Machine that the 7th harmonic appears. It is consequently necessary to compensate the negative effect of the 7th harmonic in the control of Secondary Machine. In Fig.9 and Fig.11 effects of compensation of 7th harmonic of EMF are shown.

In a second strategy, the two fictitious machines have been used to produce torque with a compensation of the 7th harmonic. Fig.14., Fig. 15, Fig. 16 and Fig.17 gives results of the vector control. More explanations about this control are given in [19].

Conclusion

In this paper, a Multi-machine modeling of *n*-phase machines has been presented. This approach exhibits particularities of *n*-phase machine beyond 3-phase ones: it is possible to get the total torque using different fictitious machines. The choice of the repartition of the total torque between the different machines depends on the characteristic of the fictitious machines whose harmonic characterization has been given. To use algorithms already developed for vector control of 3-phase machine, it is sufficient that each fictitious machine has sinusoidal waveform EMF.

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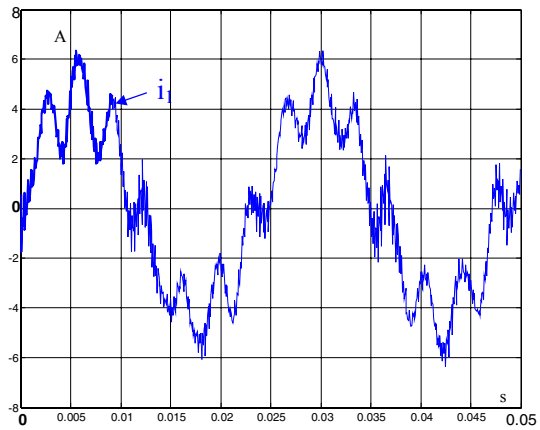

Figure 8. Experimental i_1 current in phase n°1 at 1000 rpm without compensation of 7th harmonic of back-EMF.

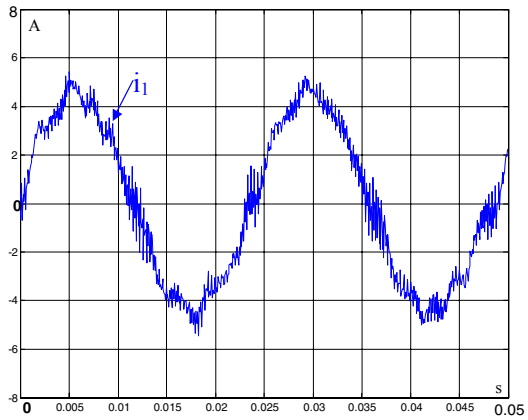

Figure 9. Experimental i_1 current in phase n°1 at 1000 rpm with compensation of 7th harmonic of back-EMF.

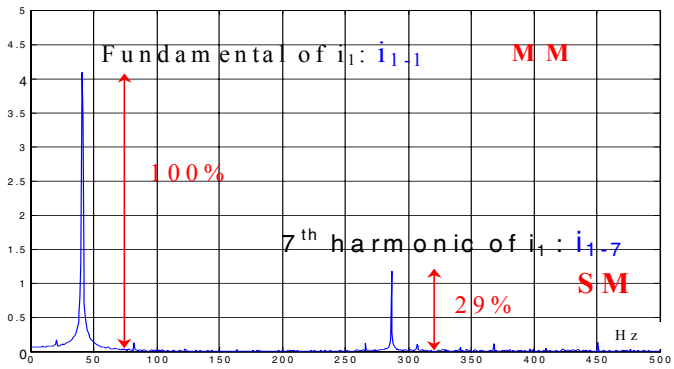

Figure 10. Spectrum of experimental current i_1 at 1000 rpm without compensation of 7th harmonic of back-EMF.

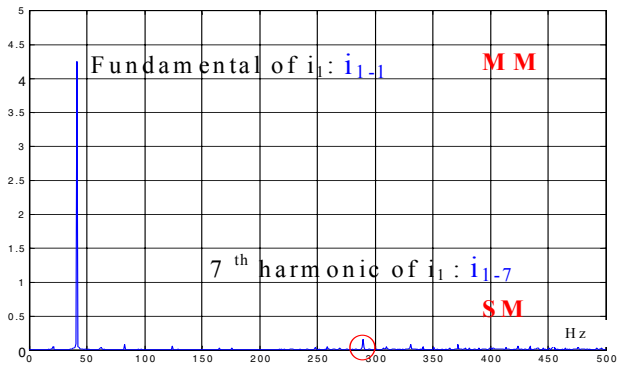

Figure 11. Spectrum of experimental current i_1 at 1000 rpm with compensation of 7th harmonic of back-EMF.

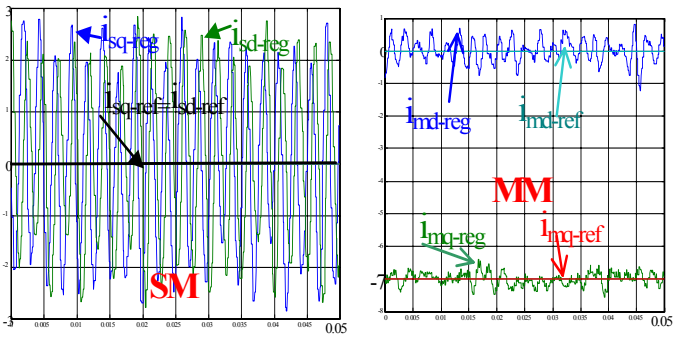

Figure 12. Currents in the two fictitious 2-phase machines 'MM' and 'SM' without compensation of 7th harmonic of back-EMF.

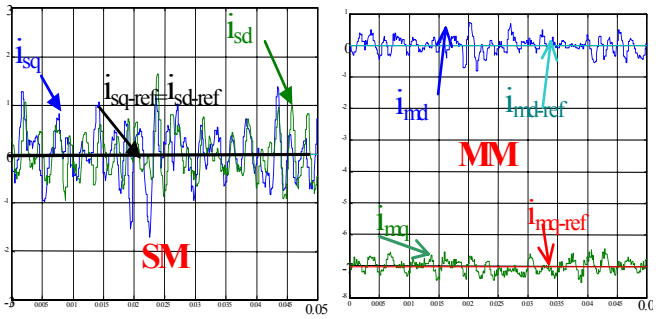

Figure 13. Currents in the two fictitious 2-phase machines 'MM' and 'SM' with compensation of 7th harmonic of back-EMF.

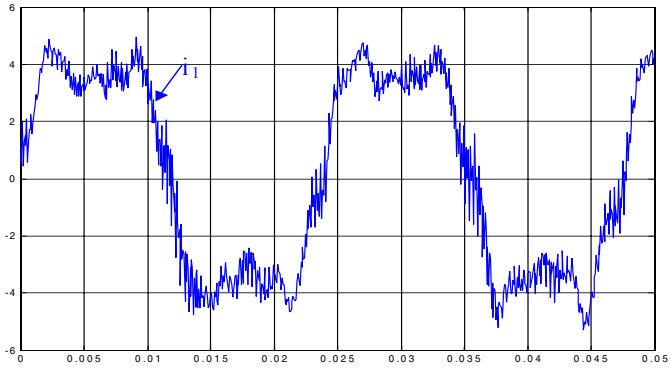

Figure 14. Experimental i_1 current in phase n°1 at 1000 rpm with compensation of 7th harmonic of back-EMF and the repartition between MM and SM $i_{sq\text{-}ref}$ = (29%) $i_{mq\text{-}ref}$.

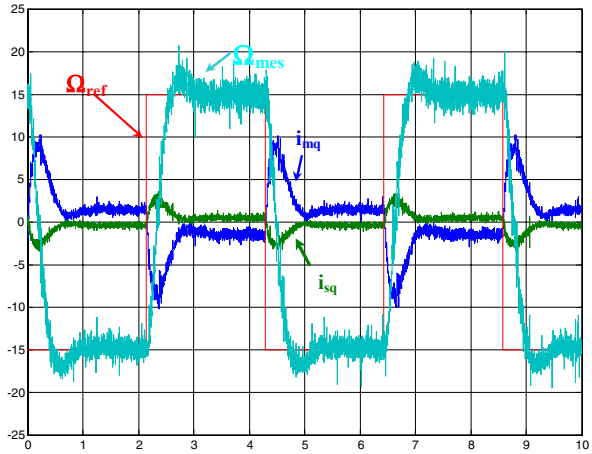

Figure 15. Experimental active currents in MM and SM with compensation of 7th harmonic of back-EMF and $T_{es\text{-}ref}$ = (29 %) $T_{ms\text{-}ref}$.

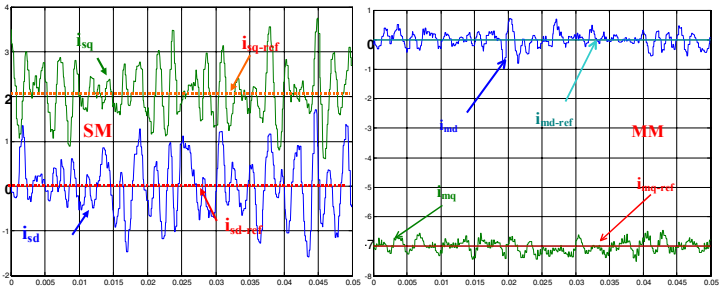

Figure 16. Currents in the two fictitious 2-phase machines 'MM' and 'SM' with compensation of 7th harmonic of back-EMF.

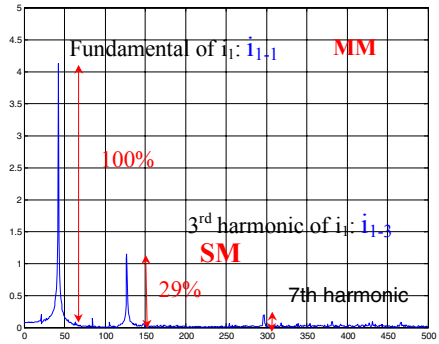

Figure 17. Spectrum of Figure 14 experimental current i_1.