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# Tolerancing

## Definition

This word has a double meaning:

- Geometric dimensioning and tolerancing (GD&T) is a symbolic language used to specify the size, shape, orientation, and location of part features. GD&T is a concept widely used for specifying dimensions and tolerances of parts and subassemblies of a product according to their functional requirements.
- Tolerancing is the set of activities which manage the tolerances during the product development.

## Theory and Application

### Why Tolerancing?

Where Do Tolerances Come from?

Due to the variations associated with manufacturing process, it is not possible to attain the theoretical dimensions in a repetitive manner; it is accepted that there will be a certain amount of variation in terms of manufacturing geometry and dimensions. It causes a degradation of characteristics of the product. In order to ensure the interchangeability (interchangeability of manufactured parts is a critical element of present-day production – mass production, mass customization, modular product, maintenance, etc.), the desired behavior, and the functional requirements of the product in spite of variations, the part features are assigned a tolerance zone within which the value of the feature, i.e., situation and intrinsic, lies (Weill et al. 1988; BJORKE 1989).

Necessity of a Tolerance Model (Common Language)?

Geometrical dimensioning and tolerancing and geometrical product specification were developed to address the many problems that had been encountered over the years as companies tried to describe their nonideal geometry. They realized that everybody that read their drawings had a different interpretation of their dimensioning and tolerancing specifications and the limits they created (Mathieu and Ballu 2003). This led to mistakes, rejected parts, nonconforming parts, wasted time and money, and costly delays in production. Therefore, it is necessary to develop a common, univocal, and complete language to express the limits of the nonideal geometry.

A History of Tolerancing

Conventional tolerancing methods have been in use since the middle of the 1800s. Tolerancing began with the Industrial Revolution. Occasionally, the designer would write in the drawing a dimension (plus/minus). The craftsman realizes each product. The manufacturing process was different then. There are no assembly lines.

In 1935, American Standards Association (ASA) published the first standard for engineering drawings. Only 20 pages in length with five pages devoted to dimensioning. In the 1940s, drafting standards started in England. Stanley Parker (English worker in a torpedo factory in Scotland) devised a method of specifying cylindrical tolerance zones surrounding an absolute location from what was previously specified as rectangular plus/minus tolerances.

During World War II, the manufactured and shipped spare parts overseas for the war effort. Many of these parts, even though they were made to specifications, would not assemble. In 1945, the US Army wrote a publication of ordinance manual specified dimensioning and tolerance for the US Army. In 1949, Military Standard 8 was published to specify dimensioning and tolerancing standards for the US Army (Drake 1999).

In 1966, the first unified standard was published by American National Standards Institute (ANSI): Standard ANSI 14.1–14.5.

In the early 1990s, ISO created the ISO TC 213 “Dimensional and geometrical product specifications and verification” technical committee to improve the standards. The work in TC 213 was based on the idea that the field of geometrical product specifications can be described as a matrix: the rows are the various requirements, and the columns are the

various pieces that have to be in place to create a univocal specification. In this new approach, specifications are defined by an ordered set of operations, each of which is applied to a feature [ISO TS 17450-1] according to Ballu & Mathieu (Mathieu and Ballu 2003), based on these ordered set of operations (or operators) the uncertainties links to tolerancing activities are developed in [ISO 17450 Part 2 17450-2].

Over the last 30 years, the industrial needs, the rise of the CAx software, and the development of coordinate metrology justified lots of research works and an evolution of the standards.

### Central Role of Tolerancing

Dimensions and tolerances influence almost all aspects of product development. Hong illustrates the ubiquitous role of tolerances in a product life cycle (Fig. 1) (Hong and Chang 2002).

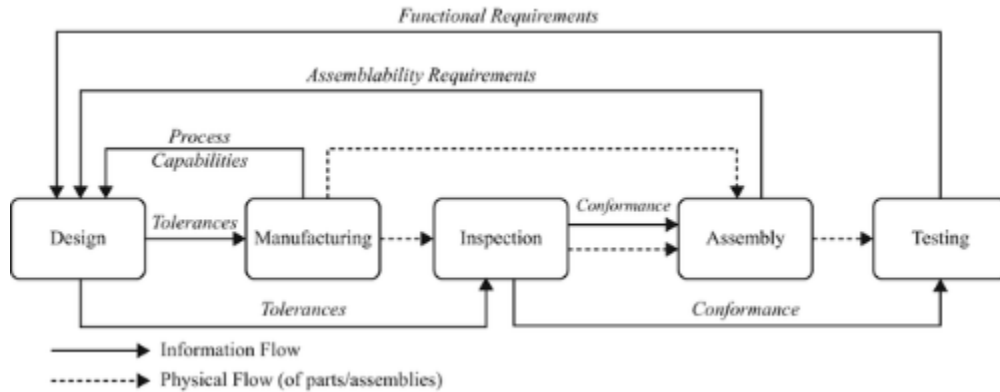


Fig. 1 Ubiquitous role of tolerances in a product life cycle

Srinivasan mentions that “We may consider the ubiquitous tolerances in various stages of a product life cycle. Since the role of tolerances in a life cycle varies from stage to stage, depending on their own respective objectives, it is not a trivial task to take all these different factors into account when a designer determines a tolerance.” Moreover, Chase mentions that “Tolerance requirements have a far-reaching influence that touches nearly every aspect of the manufacturing enterprise as shown in Fig. 2” (Srinivasan et al. 1996).

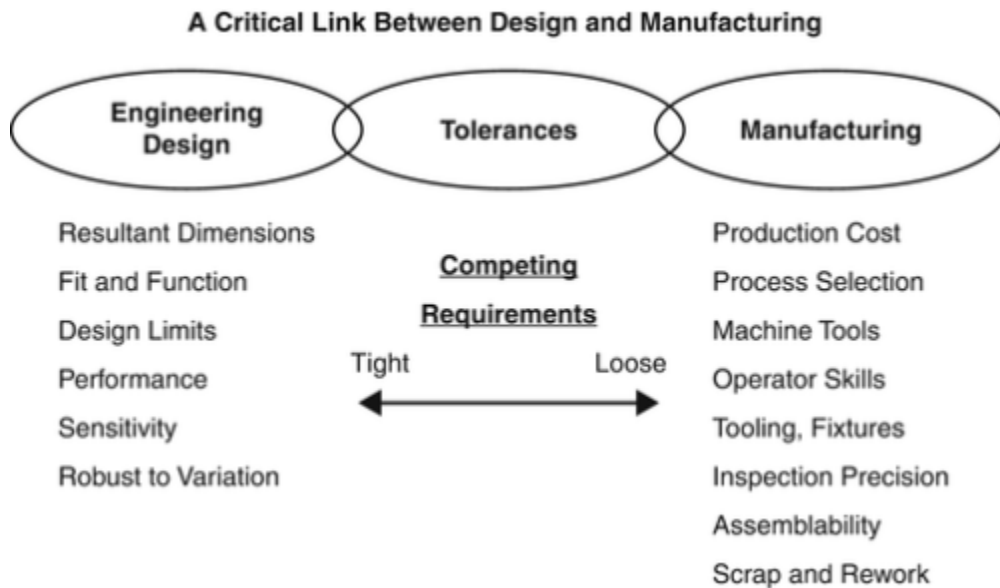


Fig. 2 The effects of assigned tolerances are far-reaching

Research into tolerancing in the field of engineering has been significant, particularly in the last three decades. The main objective of this section is to provide an overview of tolerancing.

In 1998, Salomons and van Houten proposed a review on “Current Status of CAT Systems” (Salomons et al. 1998). He has distinguished four aspects to analyze the CAT systems:

- Tolerance representation: it refers to how tolerances are represented computer-internally; this aspect points out directly on the models used for the description of the mechanism without and with geometric variations.
- Tolerance specification: it is an important activity for tolerancing. It tries to answer the question: Which tolerance types and values are needed on features to control functional requirements?
- Tolerance analysis: it concerns the verification of the value of functional requirements after tolerance has been specified on each isolated part. This verification is totally dependent on the models chosen before. A lot of tools are also generally provided to the designer to understand the results.
- Tolerance synthesis: it is regarded as a tolerance allocation and a tolerance optimization method taking into account manufacturing and inspection aspects.

We can add another aspect:

- Tolerance verification: it defines inspection planning and metrological procedures for functional requirements, functional specifications, and manufacturing specifications. It is very important to consider tolerance verification early in the design activities to be able to assess uncertainties. Tolerance verification permits to close the process loop, to check the product conformity, and to verify assumptions made by the designer.

## Various Aspects of Tolerancing

### Tolerance Representation

To be coherent, the tolerancing process has to use the same language based on a unified mathematical model to express tolerancing for each person involved during the process. To ensure this coherence between tolerance design and tolerance verification, there is a necessity of developing a complete tolerance model which should:

- Represent standard tolerance practices
- Be integrated in the computer-aided systems of design, manufacturing, and metrology
- Be verified by a CMM (coordinate measuring machine)
- Support automated tolerance analysis

Tolerancing is preferably carried out in conformance with the tolerancing standards. There are several standards available worldwide that describe the symbols and define the rules used in GD&T (Humieny 2009):

- ISO/TR 14638 gives an overview of the international standardization of geometrical product specification (GPS). Explains the concept of GPS and provides a framework of GPS including the existing and future standards.
- ISO 14660-1 defines the general terms and definitions for geometrical features.
- etc

GeoSpelling and ISO TS 17450 Part 1 (2011) allow to express the specification from the function to the verification with a common language. This model is based on geometrical operations which are applied not only to ideal features but also to the nonideal features which represent a real part. These operations are themselves defined by constraints on the form and relative characteristics of the features.

For tolerance analysis, the nonideal geometry of parts is apprehended by a variation of nominal dimension, or it is apprehended by a variation of the nominal geometry. The small displacement torsors or the linearized homogenous matrixes are generally used for the representation of the small geometrical displacements. Based on these displacement models, several vector space models map all possible manufacturing variations (geometrical displacements between manufacturing surfaces or between a manufacturing surface and a nominal surface) into a region of hypothetical parametric space (Kimura et al. 1986; Roy and Li 1999). The geometrical tolerances or the dimensioning tolerances are represented by deviation domain, T-Map® or specification hull.

Geometric tolerances are also used in design to limit the form deviations. Classical tolerance models are not yet satisfactory in simulating the effect of the form deviations on part geometry. The modal tolerancing method allows to decompose any real defects of any discretized feature in the natural mode shapes with an unambiguous mathematical language. The finite element method is used to compute those shapes, and a vectorial projection of the measured feature is made to obtain the modal coefficients of the defect (Samper and Formosa 2007). This tolerance representation is used

for the assessment of the manufacturing process effects on the form deviations and for the tolerance analysis of flexible assemblies.

### Tolerance Specification

Tolerance specification methods provide a complete framework to define the geometric dimensioning and tolerancing. In practice, the tolerances are specified by the designer, mainly based on experience and/or empirical information. However, the standards do not prescribe a method of how tolerances must be specified.

Tolerance specification usually involves the identification of the features to be toleranced and the required datum features, the determination of the types of tolerances needed and material conditions, and, finally, the assignment of the tolerance values as per functional requirement. To do so, several approaches based on graphs and/or rules are continuously being developed, tested, and implemented in industry:

- Tolerance specification method based on graph representation and TTRS analysis (Clement et al. 1996; Salomons et al. 1995; Sellakh et al. 2001)
- Tolerance specification method based on the concept called functional group and graph representation (Linares et al. 2002)
- Tolerance specification method based on the graph decomposition of the product and a kinematic analysis (Ballu and Mathieu 1999; Roy et al. 2001; Wang and Roy 2005)
- Tolerance specification method based on the influence of contacts and rules (Anselmetti et al. 2010)
- etc

### Tolerance Analysis

Usually, tolerance analysis can be either worst-case or statistical (Chase and Parkinson 1991; Nigam and Turner 1995):

- Worst-case analysis (also called deterministic or high-low tolerance analysis) involves establishing the dimensions and tolerances such that any possible combination produces a functional assembly, i.e., the probability of non-assembly is identically equal to zero. It considers the worst possible combinations of individual tolerances and examines the functional characteristic. Consequently, worst-case tolerancing can lead to excessively tight part tolerances and hence high production costs.
- Statistical tolerancing is a more practical and economical way of looking at tolerances and works on setting the tolerances so as to assure a desired yield. By permitting a small fraction of assemblies to not assemble or function as required, an increase in tolerances for individual dimensions may be obtained, and in turn, manufacturing costs may be reduced significantly. Statistical tolerance analysis computes the probability that the product can be assembled and will function under given individual tolerance.

Based on the mathematical point of view, the tolerance analysis methods could be divided into two distinct categories: displacement accumulation or tolerance accumulation.

- The aim of displacement accumulation is to simulate the influences of deviations on the geometrical behavior of the mechanism. Usually, tolerance analysis uses a relationship of the form  $Y = f(x_1, x_2, \dots, x_n; g_1, g_2, \dots, g_m)$  where  $f$  is the response (characteristic such as gap or functional characteristics) of the assembly;  $x_1, x_2, \dots, x_n$  are the values of some characteristics (such as situation deviations or/and intrinsic deviations) of the individual parts or subassemblies making up the assembly; and  $g_1, g_2, \dots, g_m$  are the values of gaps. The part deviations could be represented by kinematic formulation (Desrochers 2007; Laperrière and Lafond 1999; Loose et al. 2007), small displacement torsor (SDT) (Bourdet and Ballot 1995), matrix representation, vectorial tolerancing, etc. The function  $f$  is the assembly response function which represents the deviation accumulation (Soderberg and Johannesson 1999). It could be an explicit analytic expression, an implicit analytic expression, or numerical simulation for which it is possible to compute a value for  $Y$  given values of  $x_1, x_2, \dots, x_n$  and  $g_1, g_2, \dots, g_m$ . In a particular relative configuration of parts of an assembly consisting of gaps without interference between parts, the composition relations of displacements in some topological loops of the assembly permits to determine the

function  $f$  (Bourdet and Ballot 1995). For complex system, determination of explicit function  $f$  is very complex (Serré et al. 2010). Probabilistic approaches coupled to optimization are being developed and tested on industrial cases for this type of problems (Beaucaire et al 2012; Qureshi et al 2012; Vignat et al. 2010).

- The aim of tolerance accumulation is to simulate the composition of tolerances, i.e., linear tolerance accumulation and 3D tolerance accumulation. Based on the displacement models, several vector space models map all possible manufacturing variations (geometrical displacements between manufacturing surfaces or between manufacturing surface and nominal surface) into a region of hypothetical parametric space. The geometrical tolerances or the dimensioning tolerances are represented by deviation domain (Giordano et al. 2003), T-Map® (Ameta et al. 2007), or specification hull (Morse and You 2009; Roy and Li 1999). These three concepts are a hypothetical Euclidean volume which represents all possible deviations in size, orientation, and position of features. For tolerance analysis, the above mathematical representations of tolerances allow calculation of accumulation of the tolerances by Minkowski sum of deviation and clearance domains, to calculate the intersection of domains for parallel kinematic chain, and to verify the inclusion of a domain inside another one (Teissandier and Delos 2011). The methods based on this mathematical representation of tolerances are very efficient for the tolerance analysis of linear models (Shen et al. 2005).

#### Tolerance Synthesis/Allocation

Among all solutions available to perform tolerance allocations, three can be distinguished:

- The first consists in equally dividing the tolerance between the workpieces achieving functional requirements. This solution is the easiest; however it does not take into account the capability of the production resources. As a consequence, the cost of this distribution is unrefined and the risk of having out of range dimensions or not machinable workpieces is high.
- The second solution which tries to avoid the previous defects is based on the resource capability. This capability is an indicator commonly used to assess the relevance of using a resource to produce a tolerance objective. This indicator is mathematically defined:

$$Capability = \frac{Tolerance}{6.\sigma_i}$$

- $\sigma$  models the standard deviation of the statistical distribution of the resource's deviations. The aim of this method is to have the same capability (usually higher than 1.33) for all resources. However this solution is not efficient enough: although the risk of having out of limit dimensions is reduced, the production plan loses its flexibility. Moreover, since this solution selects only quality-controlled process plan, the cost of products becomes expensive.
- The third allocation is performed regarding the tolerance cost. Current methods designed to assess the tolerance cost are mainly based on parametric approaches. According to these methods, the cost depends on mathematical combination of the tolerance required and several constants. Among all parametric functions available, the reciprocal squared model, the reciprocal model, and the exponential model are currently the most used mathematical definitions (Lee and Johnson 1993; Spotts 1995; Nassef and ElMaraghy 1997; Lööf et al. 2005; Tsai and Cheng 2005). Another approach consists in assessing the cost impact by the designer's tolerances allocation (the cost-weighted risk concept assesses cost by analyzing the impact of the tolerance allocations on the manufacturing process) (Schmitt and Behrens 2007; Etienne et al. 2009).

## Tolerance Verification

Tolerance verification is all activities by which organizations try to ensure that their product conforms to specifications and meets customer expectations, connected with measurement and testing functions to be provided in the industrial development process of the product. Tolerance verification defines inspection strategy and metrological procedures for functional requirements, functional specifications, and manufacturing specifications. It is very important to consider tolerance verification early in the design activities to be able to assess uncertainties.

The uncertainties through the product life cycle begin from the design intent to the inspection activity. In the ISO TS 17450 Part 2 (Humienny 2009), the notion of the uncertainty is generalized to the specification and the verification. The transition between the functional requirements and their inspection is ensured by some models and tools. The total uncertainty of this transition is divided into correlation uncertainty, specification uncertainty, and measurement uncertainty. Correlation uncertainty characterizes the fact that the intended functionality and the controlled characteristics may not be perfectly correlated. The specification uncertainty characterizes the ambiguity in the specification expression. And the measurement uncertainty is however the best known type of uncertainties. It is considered by the metrologists and well described in GUM.

Tolerancing and uncertainties are ubiquitous in any engineering system at any stage of product development and throughout a product life cycle.

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