Analysis of the role of structural disorder on the inertial correction to Darcy's law

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Abstract:

This work focuses on the stationary one-phase Newtonian flow in a class of homogeneous porous media at large enough flow rates leading to a non-linear relationship between the filtration velocity and the pressure gradient. A numerical analysis of the non linear -inertial- correction to Darcy's law is carried out for model periodic structures made of arrays of square-section cylinders. The global aim is to determine and analyze the effective properties appearing in the macroscopic model resulting from the volume averaging of the mass and momentum (Navier-Stokes) equations at the pore scale

$$\left\langle \mathbf{v}_\beta \right\rangle = -\frac{\mathbf{K}}{\mu_\beta}.\left(\nabla\left\langle p_\beta \right\rangle^\beta - \rho_\beta\right) - \mathbf{F}.\left\langle \mathbf{v}_\beta \right\rangle = -\frac{\mathbf{K}}{\mu_\beta}.\left(\nabla\left\langle p_\beta \right\rangle^\beta - \rho_\beta\right) + \mathbf{f_c}.\left|\left\langle \mathbf{v}_\beta \right\rangle\right|$$

The effective permeability tensor, **K**, and the inertial tensor, **F,** are obtained by solving the microscopic flow and also the closure problems resulting from upscaling. From extensive analysis, general features of the tensor **F** and normalized correction $\mathbf{f}_c$ for ordered structures (regular array of cylinders) have been studied (Interpore Conf. 2009). It was shown, in particular, that the quadratic correction, classically referred to as the “Forchheimer correction”, is an approximation which does not hold at all for certain particular orientations of the pressure gradient for this type of regular structures. The aim of the present work is to investigate the robustness of the quadratic “Forchheimer correction” when disorder in introduced in the medium.

The correction to Darcy’s law is studied for weakly and strongly disordered media for a given porosity with a pressure gradient aligned with the principal axes of the structure. Weak structural disorder is obtained as a result of random placement of identical square-section cylinders in a periodic unit cell, while strong disorder corresponds to both random placement and random size of the cylinders. The correction obtained for 10 to 20 realizations is studied for each case as a function of the Reynolds number.

It is shown that for disordered structures, the linear velocity -dependence of the correction, $\mathbf{f}_c$, is a robust approximation in a very large interval of Reynolds numbers, featuring a quadratic Forchheimer correction to the filtration velocity in the strong inertia regime. Moreover, the crossover Reynolds number, corresponding to the transition from the weak inertia cubic regime to the strong inertia quadratic regime, as well as the magnitude of the correction in the weak inertia regime stay small in comparison to an ordered structure. This explains the fact that this regime is generally overlooked during experiments on naturally disordered porous media.