Investigation of Some Localization Criteria and Their Relevance to Prediction of Forming Limit Diagrams

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Because prevention of forming defects has become one of the major industrial challenges, various experimental and theoretical approaches have been developed to predict sheet metal formability. The main theoretical models can be summarized as: the maximum force principle, according to which necking is associated with the maximum load in a uniaxial tensile test; Marciniak–Kuczyński (M–K) two-zone analysis, based on an initial thickness defect in the sheet; bifurcation theory, predicting diffuse necking with general bifurcation criterion or localized modes corresponding to loss of ellipticity; and more recently, linear stability analysis by means of linearization of perturbed equilibrium equations. Considering this variety of models, a careful comparison of numerical Forming Limit Diagrams (FLDs) along with in-depth understanding of their theoretical foundations is required to help select relevant localization criteria. In this paper, the theoretical bases of M–K and Rice's criteria are first reviewed, which are then applied to steels modeled by elasto-plastic constitutive equations coupled with damage. It is shown that the FLDs obtained with the M–K model tend to those yielded by Rice's criterion in the limit of vanishing initial imperfections.

Keywords: Sheet metal forming, Necking, Strain localization, Forming limit diagrams, Elasto–plasticity, Damage, Rice's bifurcation criterion, Marciniak–Kuczyński criterion

Introduction

In the literature dealing with material instabilities, many instability criteria have been developed and some of them have been extensively applied to sheet metals in order to investigate their formability limits. An exhaustive list of those criteria is difficult to be given, considering the multitude of variants derived from some approaches. A short review reveals, however, that those criteria could be classified into at least four distinct categories, depending on their fundamental basis as well as their theoretical or physical background. Early instability criteria were based on the Maximum Force Principle, originated by Considère [1] and its two-dimensional extension by Swift [2] for application to sheet metals. These criteria, in their original form, were known to predict diffuse necking. Later these Maximum-Force-based criteria were extended by Hora et al. [3] and subsequently by Mattiasson et al. [4] in order to predict localized necking, and some enhanced versions were developed to take into account some effects such as thickness effect… Concurrently, Hill's zero-extension criterion [5] was developed to predict localized necking on the left-hand side of the FLD. Another approach, postulating a pre-existing defect in the material sheet, was proposed by Marciniak and Kuczyński [6]. In its original version, M–K model can be regarded, in a sense, as a complementary approach to Hill's zero-extension criterion, which is only applicable to left-hand side of FLDs as no zero extension direction exists for positive biaxial stretching. However, since localized necking in biaxial stretching is observed in practice, a pre-existing defect has to be introduced in the M–K model to capture this effect, which may provide some justification to this imperfection theory. Drucker's [7] and Hill's [8] general bifurcation theory represents another class of approach for diffuse necking. Belonging to the same class, limit point bifurcation appeared later (see, e.g., Valanis [9]), and it has been shown that for associative plastic materials, limit point bifurcation coincides with general bifurcation. For localized necking, Rudnicki and Rice's bifurcation criterion [10], based on loss of ellipticity of the acoustic tensor, was established. In the same way, some authors (see Bigoni and Hueckel [11]) suggested the use of loss of strong ellipticity, which was shown to coincide with Rice's criterion for associative elasto–plastic models. More recently, stability analysis approaches based on linearized perturbation techniques have been developed (Dudzinski and Molinari [12]) and applied in the framework of soil mechanics as well as sheet metal formability.

It is worth noting that, while M–K analysis has been widely used in the literature, few applications of Rice's ellipticity loss theory, mostly restricted to plane-stress assumptions and simple behavior models, have been attempted in sheet metal forming for quantifying metals in terms of formability. It should be noted, however, that some recent contributions (Franz et al. [13], Haddag et al. [14]) have coupled Rice's bifurcation analysis with advanced physically based behavior models. In this paper, the theoretical formulations of these criteria are reviewed in order to demonstrate some similarities in their respective bases. The application of these localization criteria to steels modeled by elasto–plastic laws coupled with damage allows us to determine the associated FLDs; the role of the initial imperfection size in M–K analysis is especially emphasized.

Constitutive Modeling

Elasto–Plastic Model. The adopted model is based on a phenomenological approach aiming to reproduce the behavior of a large class of metallic materials, including steels. This model is able to account for the initial anisotropy of the sheet encountered in deep-drawing operations, but it is restricted here to cold deformation. A hypo-elastic law describes the evolution of the Cauchy stress $\boldsymbol{\sigma}$:

$$\dot{\boldsymbol{\sigma}} = \mathbf{C} : \dot{\boldsymbol{\varepsilon}}^e \tag{1}$$

where $\mathbf{C}$ is the elasticity modulus and $\dot{\boldsymbol{\varepsilon}}^e$ the elastic strain rate, defined by an additive decomposition of the total strain rate $\dot{\boldsymbol{\varepsilon}}$ into its elastic and plastic parts. The plastic strain rate evolution is given by an associative flow rule:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\lambda}\frac{\partial F}{\partial \boldsymbol{\sigma}} = \dot{\lambda}\mathbf{V} \tag{2}$$

where $\dot{\lambda}$ is the plastic multiplier and $\mathbf{V}$ the flow direction, normal to the yield surface defined by F. The yield criterion can then be written under the Kuhn–Tucker form:

$$F = \bar{\sigma} - Y \le 0 \qquad \dot{\lambda} \ge 0 \qquad \dot{\lambda}F = 0 \tag{3}$$

where $\bar{\sigma}$ is here the Hill'48 anisotropic yield function and Y denotes the current size of the yield surface. The evolution of Y is related to that of isotropic hardening R:

$$Y = Y_0 + R \tag{4}$$

where Y_0 is the initial yield stress, while the evolution of the isotropic hardening is given by a saturating Voce, respectively, a non-saturating Swift law such as:

$$\dot{R} = H_R \dot{\lambda} \tag{5}$$

and:

$$H_R = \begin{cases} C_R\left(R_{sat} - R\right) & \text{for Voce law} \\ nk\left(\dfrac{R + Y_0}{k}\right)^{\frac{n-1}{n}} & \text{for Swift law} \end{cases} \tag{6}$$

where C_R and R_{sat} are the Voce material parameters, while n and k are the Swift parameters. H_R is called isotropic hardening modulus. Combining these equations with the consistency condition, one can obtain the relation between the stress and strain rates during elastic–plastic loading:

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{L} : \dot{\boldsymbol{\varepsilon}} = \left(\mathbf{C} - \alpha\frac{(\mathbf{C}:\mathbf{V})\otimes(\mathbf{V}:\mathbf{C})}{\mathbf{V}:\mathbf{C}:\mathbf{V} + H_R}\right) : \dot{\boldsymbol{\varepsilon}} \tag{7}$$

where $\alpha = 1$ for plastic loading and zero otherwise. For Rice localization criterion and associative elasto-plastic laws, material instability is predicted during softening regime, an effect that can be introduced by coupling the constitutive equations with damage.

Elasto–Plastic Model Coupled with Damage. During large deformations that occur in deep-drawing operations, a deterioration of material properties may take place. This deterioration is related to the evolution of micro-defects. Adopting the continuum damage mechanics, the deterioration of the material properties is modeled by a damage variable d related to the surface density of micro-defects. The evolution of d is given by a Lemaitre-type law:

$$\dot{d} = H_d \dot{\lambda} = \begin{cases} \dfrac{1}{(1-d)^{\beta}}\left(\dfrac{Y_e - Y_{ei}}{S_d}\right)^{s_d} \dot{\lambda} & \text{if } Y_e \ge Y_{ei} \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where Y_e is the strain energy density release rate and β, S_d, s_d, Y_{ei} are material parameters. By adopting the strain equivalence principle, Equation (1) becomes:

$$\dot{\boldsymbol{\sigma}} = (1-d)\mathbf{C} : \left(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p\right) - \frac{\dot{d}}{1-d}\boldsymbol{\sigma} \tag{9}$$

and, due to coupling with damage, the relation between the stress and strain rates can be rewritten as $\dot{\boldsymbol{\sigma}} = \boldsymbol{L}_d : \dot{\boldsymbol{\varepsilon}}$, with:

$$\boldsymbol{L}_d = (1-d)\mathbf{C} - \alpha\frac{(1-d)(\mathbf{C}:\mathbf{V})\otimes(\mathbf{V}:\mathbf{C}) + H_d\boldsymbol{\sigma}\otimes(\mathbf{V}:\mathbf{C})}{\mathbf{V}:\mathbf{C}:\mathbf{V} + (1-d)H_R} \tag{10}$$

is the tangent modulus affected by damage.

Localization Criteria

Marciniak–Kuczyński Criterion. M–K criterion is based on the semi empirical observation according to which strain localization occurs at an imperfection of the structure. In this model, a heterogeneity with degraded properties is initially introduced into the metal sheet; although different geometrical or material heterogeneities could be used, the defect is usually introduced in the form of a band of reduced thickness, as shown in **Figure 1**.

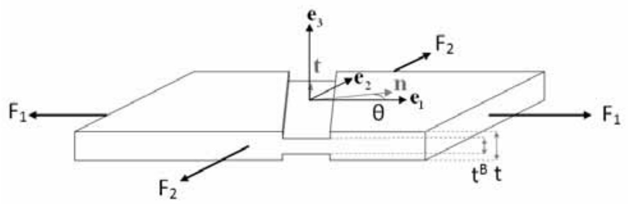

Figure 1. Metal sheet with band of reduced thickness used in M–K model.

The ratio between the initial thickness of the defect area and that of the safe one is denoted by f_0. During loading, components of strain or stress tensors are applied on the unaffected area. The mechanical state of this area can then be computed using Eqs. (1)-(10). Compatibility of deformations, force equilibrium conditions and evolution of the current defect size f are then used to determine the mechanical state inside the band:

$$\begin{cases} \sigma_{nn}^B t^B = \sigma_{nn} t \\ \sigma_{nt}^B t^B = \sigma_{nt} t \\ \dot{\varepsilon}_{tt}^B = \dot{\varepsilon}_{tt} \\ f = f_0 \exp\left(\varepsilon_{33}^B - \varepsilon_{33}\right) \end{cases} \tag{11}$$

where superscript ^{B} refers to a variable in the band. Strain localization is predicted when the strain or the strain rate becomes concentrated in the band, i.e. is much larger in the affected area. In practice, once the strain or strain rate tensors are known in both zones, localization is predicted if the ratio between two strain-related variables, for example the equivalent strain rates $\dot{\bar{\varepsilon}}^B$ and $\dot{\bar{\varepsilon}}$, exceeds a user defined threshold S_{M-K}:

$$\frac{\dot{\bar{\varepsilon}}^B}{\dot{\bar{\varepsilon}}} > S_{M-K} \tag{12}$$

Hutchinson and Neale [15] observed that the original M–K model tends to overestimate the formability in the left side of the FLD. Hence, they proposed to take into account the influence of the orientation of the defect band, given by the normal $\mathbf{n}$, to improve the formability predictions.

Rice Criterion. In this approach a solid subjected to a given loading path is considered; localization is viewed as a rapid evolution from a homogeneous state of the velocity gradient to a heterogeneous state exhibiting discontinuity planes for the velocity gradient. Two discontinuity planes of normal $\mathbf{n}$ define a localization band. Localization here also means a bifurcation of the governing equations associated with a discontinuous compatible strain rate.

Let us consider the velocity gradient fields inside and outside a possible localization band, denoted $\mathbf{L}^B$ and $\mathbf{L}$ respectively. The difference between $\mathbf{L}$ and $\mathbf{L}^B$ is then:

$$[\mathbf{L}] = \mathbf{L} - \mathbf{L}^B \tag{13}$$

A discontinuity of the velocity gradient across the localization band leads to the existence of a non-zero vector $\dot{\mathbf{c}}$, representing the relative velocities between the areas situated at each side of the discontinuity planes, such that Hadamard's compatibility condition is satisfied:

$$\mathbf{L} = \mathbf{L}^B + \dot{\mathbf{c}} \otimes \mathbf{n} \tag{14}$$

The jump in the velocity gradient is then:

$$[\mathbf{L}] = \dot{\mathbf{c}} \otimes \mathbf{n} \tag{15}$$

which can also be written:

$$[\mathbf{L}] \cdot \mathbf{n} = \dot{\mathbf{c}} \tag{16}$$

A second condition, the continuity of the force across the planes defining the localization band, has to be verified. Written in its rate form and with the first Piola–Kirchhoff stress tensor, this condition leads to:

$$\left[\dot{\mathbf{\Pi}}\right] \cdot \mathbf{n} = \mathbf{0} \tag{17}$$

By introducing the nominal stress rate tensor $\dot{\mathbf{N}}$ (with $\dot{\mathbf{N}} = \boldsymbol{L}_d : \mathbf{L}$) as the transpose of first Piola–Kirchhoff stress rate tensor and using Equation (17), one can obtain:

$$\mathbf{n} \cdot [\boldsymbol{L}_d : \mathbf{L}] = \mathbf{0} \tag{18}$$

It is usually admitted that, at the onset of localization, the tangent moduli are the same inside and outside the localization band:

$$\boldsymbol{L}_d^B = \boldsymbol{L}_d \tag{19}$$

Taking into account the equality of the tangent moduli at the onset of localization (Eq. (19)) and the jump of the velocity gradient (Eq. (15)), the continuity of the force across the planes defining the localization band Eqs. (17)-(19) leads to:

$$\mathbf{n} \cdot \left(\boldsymbol{L}_d : [\mathbf{L}]\right) = \left(\mathbf{n} \cdot \boldsymbol{L}_d \cdot \mathbf{n}\right) \cdot \dot{\mathbf{c}} = \mathbf{0} \tag{20}$$

As previously mentioned, localization occurs for non-zero values of $\dot{\mathbf{c}}$. Non-trivial solution of system (20) is then obtained if:

$$\det\left(\mathbf{n} \cdot \boldsymbol{L}_d \cdot \mathbf{n}\right) = 0 \tag{21}$$

In practice, the numerical prediction of localization is carried out by searching for the first value of the tangent modulus leading to a singularity of the acoustic tensor $\mathbf{n} \cdot \boldsymbol{L}_d \cdot \mathbf{n}$ during loading of the metal sheet. For each loading increment, the determinant of the acoustic tensor is computed for different orientations of the normal to the band. As long as the determinant of the acoustic tensor remains positive for all orientations of the normal to the plane of the band, no localization is predicted.

Application of Marciniak–Kuczyński and Rice Criteria to Forming Limit Diagrams

The M–K and Rice localization criteria were applied to predict the FLDs for steel sheets. The material behavior is described with the models introduced in the first section of the paper. The material parameters are given in **Table 1**. The hardening parameters corresponding to mild steel are combined with two sets of fictitious damage parameters – also corresponding, roughly, to the behavior of mild steels.

Table 1. Two material parameter sets for Voce saturating isotropic hardening law coupled with Lemaitre isotropic damage.

set	C_R	R_{sat} (MPa)	Y_0 (MPa)	β	S_d	s_d	Y_{ei} (MPa)
1	10	550	350	12	20	0.01	0
2	10	550	350	10	10	1	0.5

Figures 2 and **3** show the FLDs corresponding to the two sets of parameters, as predicted by the Rice and Marciniak–Kuczyński criteria – for different values of initial heterogeneity for the latter criterion.

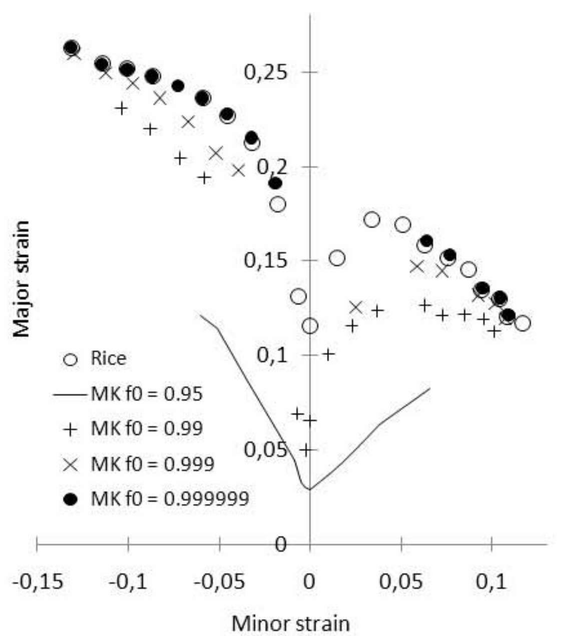

Figure 2. FLDs of steel 1 obtained with Rice criterion and with M–K model for various values of initial defect size.

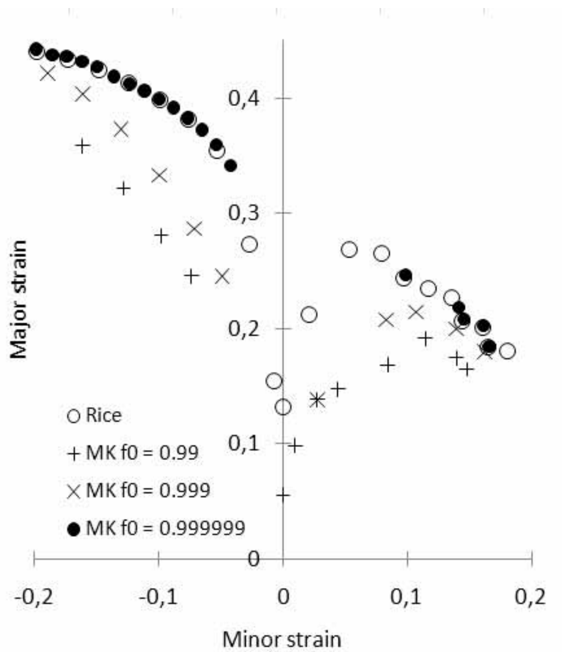

Figure 3. FLDs of steel 2 obtained with Rice criterion and with M–K model for various values of initial defect size.

According to these results, the M–K predictions converge towards the FLDs predicted by Rice's criterion when the size of the initial defect tends to vanish. Moreover, when the material response exhibits no softening (here, no damage), the determinant of the acoustic tensor is always positive, thus no localization is detected by Rice's criterion. In these conditions, no realistic FLD predictions are obtained by the M–K criterion either, when the initial defect value is kept reasonably small. These numerical findings suggest that a theoretical link may exist between these two criteria (one being a limit case of the other); this needs however to be mathematically demonstrated. Also, the robustness of the numerical simulation needs further attention, as the M–K simulations exhibit convergence difficulties for loading paths close to plane strain, as soon as the value of the initial defect becomes very small.

Conclusions

Many models and criteria have been developed to predict the necking and localization phenomena; however, the theoretical or numerical relations between them have been seldom investigated. Starting from the observation of similarities in their mathematical formulations, the FLD predictions using the criteria of Rice and Marciniak–Kuczyński were compared, with material parameters corresponding to two (fictitious) mild steels. The numerical results show that the M–K predictions tend to the Rice predictions when the initial defect becomes very small.

Based on these promising numerical results, future work will concern rewriting these two criteria in a common mathematical framework, with the aim of deriving possible theoretical relations between the two models.

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