MULTISCALE MODELING OF PERIODIC DISSIPATIVE COMPOSITES UNDER THERMOMECHANICAL LOADING CONDITIONS

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ABSTRACT The modern technological challenges on the engineering industry and the extensive advances in the materials science have caused a tremendous increase in the development of composites. Plenty of engineering and biomechanics applications demand smart materials and structures which combine high strength, multifunctionality and durability. At the same time, a crucial parameter in the choice of the most suitable composite material is the long lifetime during repeated loading cycles, thus fatigue is an essential parameter in design. To achieve the high demands in the modern applications, composite materials often operate under thermomechanical conditions that cause the appearance of dissipative phenomena like plasticity, viscoelasticity-viscoplasticity and damage. The present work deals with periodic composite media subjected to fully coupled thermomechanical loading. The material constituents of these composites are assumed to belong in the general class of generalized standard materials laws. The aim is to provide a proper homogenization framework that describes accurately the basic conservation laws in both microscopic and macroscopic levels. The study is based on the asymptotic expansion homogenization technique, which permits to deduce useful results about the energy potentials that characterize the material response in both scales. Moreover, the numerical implementation is based on an incremental, linearized formulation. This formulation allows to identify proper thermomechanical 3D tangent moduli for the macroscale problem and thus design an implicit computational scheme.

INTRODUCTION: This paper comprises results published in [Chatzigeorgiou *et al.* 2016]. The framework of generalized standard materials is adopted for writing the heterogeneous thermomechanical problem under homogenization:

$$\varepsilon^{\varepsilon}_{ij} = sym(grad u^{\varepsilon}_{i})_{j}\,, \quad \frac{\partial \sigma^{\varepsilon}_{ij}}{\partial x_j} = 0\,, \tag{1}$$

$$\frac{\partial q^{\varepsilon}_{i}}{\partial x_i} = \sigma^{\varepsilon}_{ij}\dot{\varepsilon}^{\varepsilon}_{ij} - \dot{U} = r^{\varepsilon}\,, \tag{2}$$

$$\theta^{\varepsilon}\dot{s}^{\varepsilon} + \sigma^{\varepsilon}_{ij}\dot{\varepsilon}^{\varepsilon}_{ij} - \dot{U}^{\varepsilon} - \frac{1}{\theta^{\varepsilon}} q^{\varepsilon}_{i}(grad\,\theta^{\varepsilon})_i \geq 0, \tag{3}$$

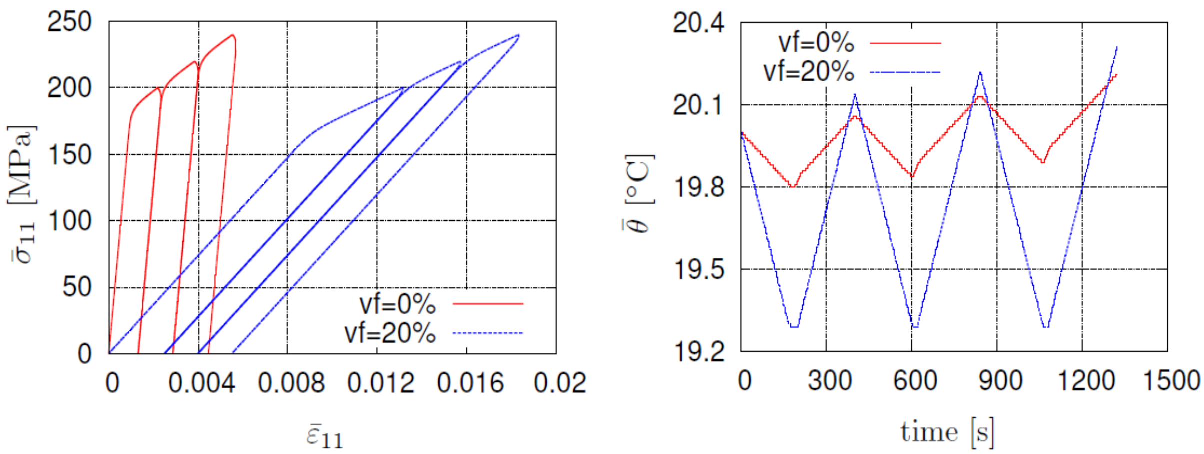
where ε denotes the parameter of heterogeneity, assumed to form periodically distributed microstructure. The Helmholtz free energy potential $\Psi^{\varepsilon} = U^{\varepsilon} - \theta^{\varepsilon} s^{\varepsilon}$ is assumed to depend on the temperature, the strain and internal variables $z^{\varepsilon} = (z^{\varepsilon}, z^{\varepsilon}_{i}, z^{\varepsilon}_{ij})$, as hardening parameters or plastic strain. The mechanical dissipation is given by $\gamma_{\varepsilon} = -\frac{\partial \Psi^{\varepsilon}}{\partial z_{\alpha}} \dot{z}_{\alpha}$, where z^{ε}_{α} stands for the set of scalars, vectors and second order tensors describing all possible internal variables, and satisfies $\gamma^{\varepsilon} - \frac{1}{\theta^{\varepsilon}} q^{\varepsilon}_{i} (grad\,\theta^{\varepsilon})_{i} \geq 0$.

Asymptotic expansion homogenization consists in expanding every function entering the above system as an asymptotic series, for instance $u_i(\mathbf{x},\mathbf{y})=u_i^0(\mathbf{x},\mathbf{y})+\varepsilon\, u_i^1(\mathbf{x},\mathbf{y})+\varepsilon^2 u_i^2(\mathbf{x},\mathbf{y})+\ldots$, where x and $\mathbf{y}=\mathbf{x}/\varepsilon$ denote respectively the macro-coordinate and the micro-coordinate and every function of the expansion is periodic with respect to **y**. Substituting in the system yields to the homogenized or macro-scale variables and to the homogenized or macro-scale equations, that comprise two cell problems corresponding to the equilibrium and to the energy equation. The main difficulty is the existence of the product in dissipation, which cannot be homogenized as a product of two functions, contrarily to the product of stress and strain (Hill-Mandell principle) and the product of heat flux and temperature gradient $\prec q^{\varepsilon}_{i} grad\,\theta^{\varepsilon}_{i} \succ \rightarrow \prec q^{0}_{i} \succ \prec grad\,\theta^{0}_{i} \succ, \prec \sigma^{\varepsilon}_{ij} \varepsilon^{\varepsilon}_{ij} \succ \rightarrow \prec \sigma^{0}_{ij} \succ \prec \varepsilon^{0}_{ij} \succ$ where $\prec . \succ$ denotes the mean value over the cell. We note that these values are the first terms of the corresponding expansions. Macroscopic dissipation is obtained only as the mean value of the micro-dissipation thus causing problems in expressing explicitly the macroscopic energy equation. In [Chatzigeorgiou *et al* 2016] this is done by linearization of the incremental thermomechanical problem. The return mapping algorithm is used in order to linearize with respect to the time the problem [Simo and Hughes 1998, Terada and Kikuchi 2001, Chatzigeorgiou *et al* 2015] and an iterative scheme is applied in order to solve simultaneously the whole problem and the unit cell problem: from the macroscale problem, the macroscopic strain, temperature and temperature gradient are calculated using macroscopic tangent moduli corresponding to incremental loading, which in turn feed the cell problem in order to obtain the microvariables and the rest of the unknown macrovariables, as the stress, needed for the new macroscopic tangent modulus. **PROCEDURES, RESULTS AND DISCUSSION:** The linearized, incremental cell equations are

$$\delta\varepsilon^{0}_{ij} = \delta E_{ij} + sym(grad u^{1}_{i})_{j}, \delta\, grad\,\theta^{0}_{i} = \delta\, grad\,\Theta_{i} + \partial\delta\theta^{1} / \partial x_{i}, \quad (4)$$

$$\frac{\partial}{\partial x_j}(\sigma^{0}_{ij} + \delta\sigma^{0}_{ij}) = 0, \frac{\partial}{\partial x_j}(q^{0}_{j} + \delta q^{0}_{j}) = 0, \quad (5)$$

These equations are accompanied by the incremental constitutive law and $r^0 = \sigma^0_{ij}\dot{\varepsilon}^0_{ij} - \dot{U}^0$. In return mapping algorithm the constitutive expressions take the forms $\delta\sigma^0_{ij} = D^{\varepsilon}_{ijkl}\delta\varepsilon^0_{kl} + D^{\theta}_{ij}\delta\Theta$, $\delta r^0 = R^{\varepsilon}_{ij}\delta\varepsilon^0_{ij} + R^{\theta}\delta\Theta$, $\delta q^0_i = -k_{ij}\delta\, grad\,\theta^0_j$, in terms of the coupled tangent moduli that are assumed constant during the iteration step.



(a) (b)

Fig. 1 Uniaxial loading under constant stress (1 MPa/s) of a multilayered composite under adiabatic conditions: (a) stress-strain and (b) temperature-time diagram at macroscale. The vf stands for the epoxy volume fraction.

The homogenization framework has been successfully implemented in the case of multilayered composites. In the discussed example, the unit cell of the examined composite consists of two layers, one made of an elastic epoxy and the other made of a viscoplastic metal. This bilayered material is subjected to macroscopic uniaxial loading and adiabatic conditions. The mechanical loading has 3 repeated cycles, where at each cycle the maximum stress increases by 20 Mpa. Fig. illustrates the obtained results for the pure metal and for the composite with 20% epoxy volume fraction. As it can be observed, the presence of epoxy causes a drastic decrease of the material's stiffness as well as a higher range of temperature changes during cyclic loading.

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