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# Control and evaluation of a 2-D Multimodal Controlled-Friction Display

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**Abstract**—The multimodal control of a 2D controlled-Friction Device is presented. We use the Vector control method because the phase and amplitude of two vibration modes at a same frequency can be precisely set. The closed loop response time of 10 ms is achieved. The device is then associated with a finger position sensor. The algorithm of the multimodal approach is then tested. In spite of the inevitable limitations of the system - saturation of amplifiers, low sampling frequency of the sensor - low friction could be imposed under a finger while a high friction was imposed on a pre-determined position. This confirms the validity of the modal approach to create multi touch interactions.

## I. INTRODUCTION

When controlling the friction between a flat plate and the fingertip it is possible to control the tangential force felt by a user, and thus simulate virtual surfaces [1], [2]. Several technological implementations of this principle have been proposed recently, using electrostatic forces [3], [4], or using ultrasonic vibrations of a glass plate [5]. These devices can modify the level of friction between a fingertip and their active surface. However, by nature, a given excitation will lead to almost the same modification of friction all over the active surface. To create the illusion of fine tactile details, like gratings for example, the level of friction has to vary with time and the position of the fingertip on the plate. Hence, the variable friction tactile interfaces compensate for their poor spatial resolution by a dynamic change of the stimulation. By this way, gratings smaller than fingertip's length can be produced with a good level of user acceptance [6].

By principle, if two fingers are touching the active area in the same time, they are stimulated with the same intensity, and two different tactile patterns cannot be simulated independently of each other. To cope with this issue, and propose multi-finger interactions, several authors have proposed solutions. For example the time reversal wave focusing [7] allows to create different tactile stimulation on two positions independently. However, the sensation felt is vibrotactile and simulation of fine gratings have not been confirmed yet. In [8] the authors use several vibration shapes at several frequencies to produces programmable tactile maps. The authors presented 4 different tactile maps, but they also point out that the method has the disadvantage to produce audible

noise by principle. In [9], [10], we presented a multimodal approach which allows a programmable tactile stimulation on two fingers located on two independent positions, on a 1D device. It consists in using two vibration modes which are energized at the same frequency, and which are superimposed in order to control the vibration amplitude at the positions of two fingers. The method doesn't have the disadvantage of audible noise, but it requires vibration modes at large wavelength, and should be more suited for large tactile displays.

In this paper, we present in a first section the extension of the principle to a 2D device. The multimodal control, which allows the settings of independent vibration amplitude of two vibration modes is presented in detail in a second section. Finally, an experimental study shows the performance of the device, including a psychophysical study.

## II. DESIGN OF THE PROTOTYPE

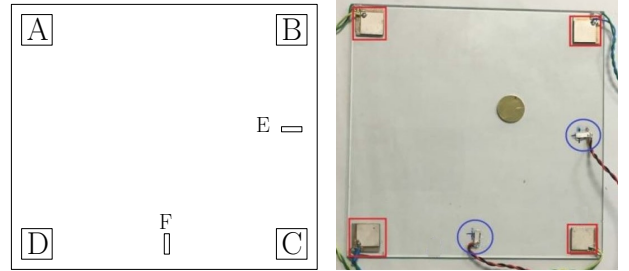


Fig. 1. Design of transparent thin plate of  $137.5 \times 137.5 \times 2$  mm<sup>3</sup> with four exciters of  $12 \times 12 \times 1$  mm<sup>3</sup> and two sensors of  $9 \times 2.5 \times 0.5$  mm<sup>3</sup> glued around its periphery

The multimodal approach in controlled friction display consists in using two vibration modes namely mode  $X$  and mode  $Y$  with different vibration patterns, named  $\phi_X$  and  $\phi_Y$ . We name  $\underline{W}_X$  and  $\underline{W}_Y$  the vibration amplitude of each mode; we use the complex form because the vibration are sinusoidal functions of time, and to take into account the information of amplitude and phase, without lack of generality. If the two modes are established at a same frequency, then the modal superimposition consists in the sum of the contribution of each mode, and the deformation amplitude  $\underline{W}$  is given by:

$$\underline{W} = \underline{W}_X \phi_X + \underline{W}_Y \phi_Y \quad (1)$$

If we consider two fingers at two positions  $P_1$  and  $P_2$ , the vibration which occurs under each of them is a contribution of the two vibration modes which is different for  $P_1$  and  $P_2$ . If we name  $\underline{W}_1$  and  $\underline{W}_2$  the two vibration amplitudes under each finger, then equation 1 leads to:

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$$\begin{pmatrix} W_1 \\ W_2 \end{pmatrix} = \begin{pmatrix} \phi_X(P_1) & \phi_Y(P_1) \\ \phi_X(P_2) & \phi_Y(P_2) \end{pmatrix} \begin{pmatrix} W_X \\ W_Y \end{pmatrix} = \phi_{X,Y} \begin{pmatrix} W_X \\ W_Y \end{pmatrix} \quad (2)$$

In [9] we have shown that the friction at  $P_1$  (resp.  $P_2$ ) is a function of  $W_1$  (resp.  $W_2$ ) at the condition that the wavelength of the two mode shapes is high enough to produce very different friction reduction felt by a user moving one finger at the nodes and antinodes of vibration. If we suppose that  $\phi_{X,Y}$  is a non-singular matrix, then it is possible to find the amplitude of each mode in order to impose the vibration amplitude under each fingertip, as proposed in equation 3:

$$\begin{pmatrix} W_X \\ W_Y \end{pmatrix} = \phi_{X,Y}^{-1} \begin{pmatrix} W_1 \\ W_2 \end{pmatrix} \quad (3)$$

It should be emphasized here that the matrix  $\phi_{X,Y}$  depends on the position  $P_1$  and  $P_2$ , and has to be dynamically calculated.

To validate this principle, a thin square plate with free boundary conditions was chosen. Theoretically we can obtain two similar modes, orthogonal in space, with a same wavelength and resonance frequency, which is an advantage for actuation. Moreover, to avoid discomfort due to noise, the size of the plate is chosen in order to obtain an ultrasonic resonant frequency. Consequently, the plate size is  $2 \text{ mm} \times 137.5 \text{ mm} \times 137.5 \text{ mm}$  as presented in figure 1. The 10<sup>th</sup> modes with wavelength of 27 mm and frequency of 29210 Hz were selected.

The actuation of the two modes is obtained with 4 piezoelectric patches (NOLIAAC, Denmark) which are placed at the four corners of the plate, according to the procedure already described in [10]. The voltage applied on each patch is named  $v_A$ ,  $v_B$ ,  $v_C$  and  $v_D$  respectively. Two additional piezoelectric patches are used as sensors in direct mode in order to measure the vibration amplitude  $W_X$  and  $W_Y$  for control purpose. A cartography of the two modes is realized with a grid step of 4.5 mm using an interferometer laser (OFV - 5000, Polytec, Germany) as shown in figure (2).

The measurements show that the inevitable discrepancies between real and theoretical plate's dimensions lead to unmatched resonance frequency for the two modes, while their vibration shapes are similar. In fact, we practically obtained a resonance frequency of 29195 Hz and 29215 Hz for the mode X and Y respectively. The consequence is twofold. First, it is necessary to actuate the plate in the middle of the two modes in order to not favour to one mode, which has the disadvantage to reduce the efficiency of the system. Moreover, a closed loop control is necessary, because open-loop operation would lead to uncontrolled phase shift between one mode and the other. This control is presented in the next part of the paper.

### III. MULTIMODAL CONTROL

#### A. Equivalent electromechanical model

The electromechanical equivalent circuit of the plate actuated by two pairs of piezoelectric actuators, can be derived

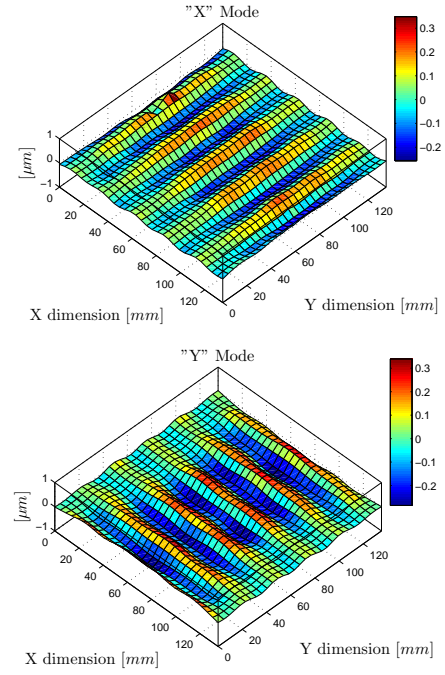


Fig. 2. Measured deformation mode shapes at resonance frequency of "X" mode 29195 Hz and "Y" mode 29215 Hz, with 20 V supply voltage for each mode

from the Mason's model applied to the two vibration modes. This is depicted in figure (3) where only the mechanical domain is presented.  $\gamma_x v_x$  is the piezoelectric force generated by the applied voltage  $v_x = V_A - V_C + V_B - V_D$ , and similarly  $\gamma_y v_y$  is the piezoelectric force generated by the applied voltage  $v_y = V_A - V_C - V_B + V_D$ . The two vibration modes

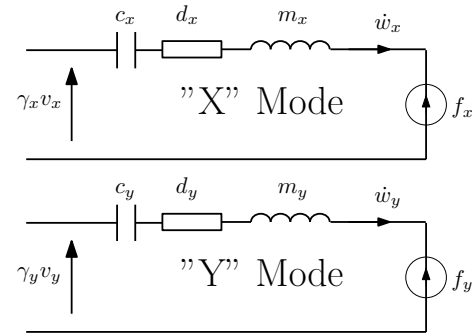


Fig. 3. Electromechanical equivalent circuit representation of a plate actuated by two pairs of piezoelectric actuators by accounting two vibration modes (X and Y) in vicinity of their resonance, [11].

are represented as a second order circuit with stiffness  $c_k$ , damping  $d_k$  and modal mass  $m_k$ , the vibration velocity is written  $\dot{w}_k$  with  $k = \{x, y\}$ . In this study, the effect of the finger on the plate is assumed to be limited, and to not lead to a change of the boundary conditions, nor to the deformation mode shapes. Hence it is modelled by a reaction force  $f_k$ .

Again, it is necessary to write finger force, voltage and vibration amplitude in complex form:  $x(t) = Re(\underline{x})$  with  $\underline{x} = \underline{X}e^{j\omega t}$ , where  $\omega$  is the angular frequency of the vibration,

and  $x_k = w_k, v_k$  or  $f_k$ . The equation of motion around two vibration modes can then be written as:

$$m_k \ddot{w}_k + d_k \dot{w}_k + c_k w_k = \gamma_k v_k - f_k \quad (4)$$

In order to elaborate the control laws of the plate vibration amplitude, (4) will be established in a rotating reference frame presented hereafter.

### B. Vector control method for plate vibration amplitude

The Vector Control Method is applied to the system in order to control the amplitude and phase of  $W_X$  and  $W_Y$  simultaneously. This transformation can be illustrated as in figure (4). In this figure the vectors  $W_X$  and  $W_Y$  are rotating vectors in a fixed frame  $(\alpha, \beta)$ . The projection of  $W_X$  on the axis  $\alpha$  gives  $w_X(t)$ . A rotating frame  $(d, q)$  is defined, with an angular position equal to  $\omega t$ . The projection of  $W_k$  on axis  $d$  and  $q$  are named  $W_{kd}$  and  $W_{kq}$  respectively, with  $k = \{x, y\}$ . These projections are constant in steady state, therefore simplifying the control design. The phases of each vibration amplitude can be accurately controlled by controlling the four projections.

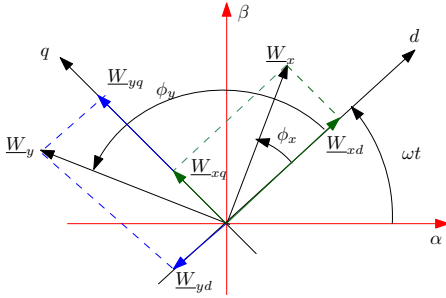


Fig. 4. Vector representation of the vibration amplitude of the two modes in a rotating reference frame.

The equation 4 can be projected onto each axis [12], leading to:

$$(c_k - m_k \omega^2) W_{kd} - \omega(2m_k \dot{W}_{kq} + d_k W_{kq}) = \gamma_k V_{kd} - F_{kd} \quad (5)$$

$$(c_k - m_k \omega^2) W_{kq} + \omega(2m_k \dot{W}_{kd} + d_k W_{kd}) = \gamma_k V_{kq} - F_{kq} \quad (6)$$

The dynamic of the plate's vibration, described by an ordinary second order differential equation (4), is then transformed into a system of two coupled first-order equations presented in (5) and (6), with:

- The voltage amplitude  $V_{kd}$  controls the vibration amplitude  $W_{kq}$
- The voltage amplitude  $V_{kq}$  controls the vibration amplitude  $W_{kd}$

It is then possible to design four controllers, one by axis and by mode, as presented figure 5. The controllers should be able to cancel the perturbations induced by the environment (like the finger) through  $F_{kd}$  and  $F_{kq}$ , or by the coupling terms  $(c_k - m_k \omega^2) W_{kd}$  and  $(c_k - m_k \omega^2) W_{kq}$ .

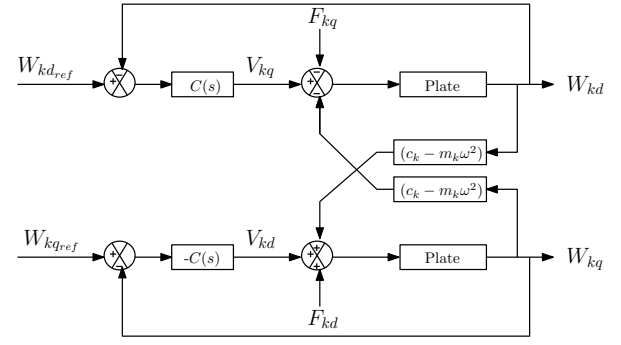


Fig. 5. Control strategy of the vibration amplitude of "X" mode and "Y" mode independently in rotating frame  $(d, q)$

### C. Control validation

The control is embedded into a micro-controller (STM32F4 from ST-MicroElectronics), which measures the vibration amplitude of each mode through the two sensor patches. The two DAC outputs of the micro-controller are connected to two high bandwidth linear amplifiers (HSA 4051 from NF - Japan) in order to supply power to the piezoelectric actuators. References are sent to the micro-controller from a laptop computer using a serial interface. The experimental test bench is presented figure 6.

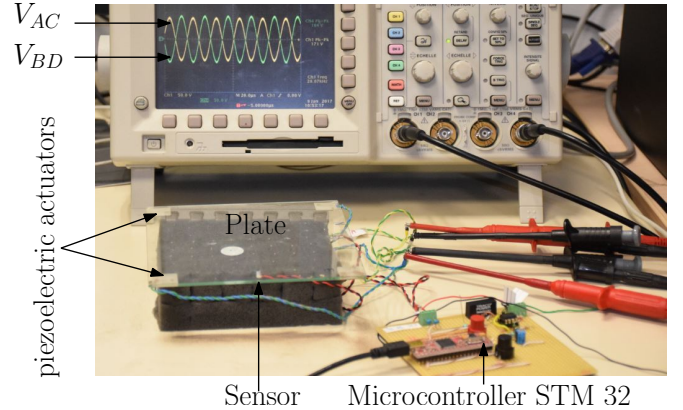


Fig. 6. Experimental setup

We used four proportional Integral controllers to control the vibration amplitude of each mode. The control is running at 2 kHz, while a DDS (Direct Digital Synthesis) at 1 MHz is used to produce the rotating reference frame. Figure (7) depicts the transient responses of  $W_{kd}$  for a reference step from 0 to 1000 nm, while  $W_{kq}$  is zero, and for no load operation, i.e. no finger is touching the plate.

The closed loop operation shows a response time of approximately 10 ms; lower response time could be achieved by compensating the coupling terms, or by using an other type of controller. This value however was found to be low enough for our study. Figure 8 shows an example of the superimposition of modes X and Y, with a same vibration amplitude.

Hence, it is possible to dynamically control the amplitudes of the two modes X and Y on the tactile plate. In the next



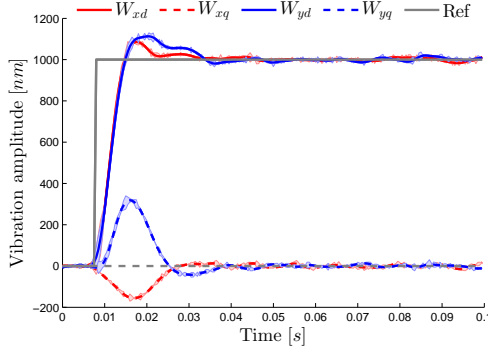


Fig. 7. Vibration amplitude control of  $W_{kdq}$  in a rotating frame.

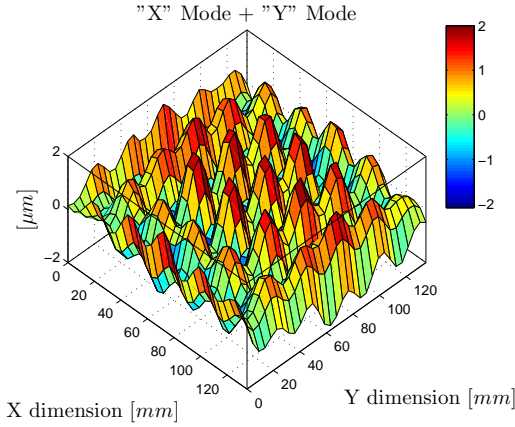


Fig. 8. Experimental cartography of the superimposition of the vibration modes X and Y with the same amplitude, at frequency of 29205 Hz

section of the paper, the plate is associated with a tactile sensor, and the references for  $W_{kd}$  and  $W_{kq}$  are deduced from the position of two fingers on the plate.

#### IV. MULTITOUCH OPERATION

##### A. Presentation of the experimental test bench

The position of fingers on the plate are measured thanks to a multi-touch track-pad (Apple) placed underneath. To fix the relative position of the plate on the track-pad, four stamps of adhesive tape are glued close to the corners. To avoid unwanted damping, these stamps are precisely located at positions which correspond to vibration nodes of mode X and Y simultaneously. Figure 9 shows the new experimental test bench.

A laptop computer running Matlab software (The Mathworks) retrieves the position of the fingers and calculates the reference values of  $W_{kd}$  by using 3. For that purpose, a specific virtual environment is created. For this study, it consists in:

- producing a constant vibration amplitude of  $W_1 = 1\mu\text{m}$  peak-peak underneath the finger 1, thus creating the condition of low friction,
- cancelling the vibration underneath the finger 2 with  $W_2 = 0$ , to create the condition of high friction

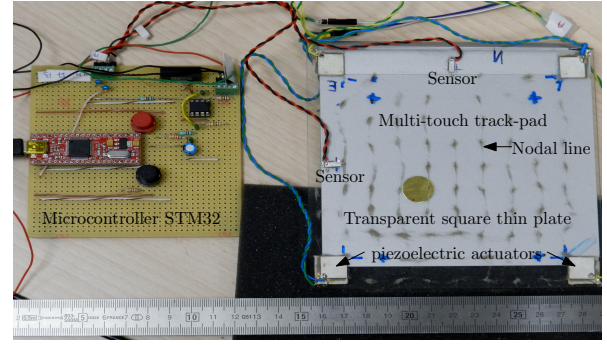


Fig. 9. Experimental setup for multitouch operation.

In spite of its simplicity, this virtual environment is not possible to obtain on single mode Variable Friction Display, for which the friction reduction is uniform over the touched surface. Hence, it is a good example of what can be achieved with the modal superimposition.

As a preliminary evaluation, and to illustrate the modal decomposition, we virtually displaced two fingers on the plate, by imposing the positions  $P_1$  and  $P_2$ . The resulting references and measurements for  $W_X$  and  $W_Y$  are depicted figure 10.

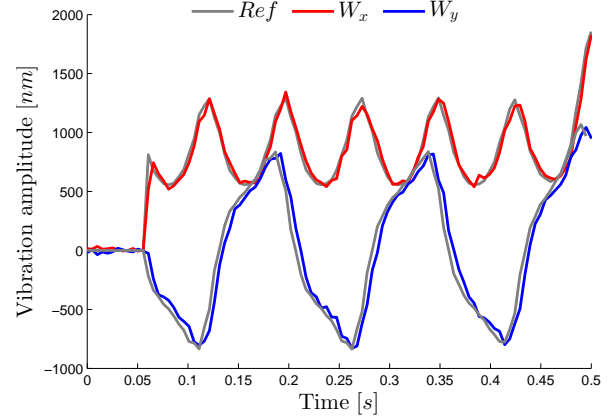


Fig. 10. Comparison between modal amplitude measurements and references in a dynamic operation.

These measurements show the dynamic behaviour of the modal amplitudes. First, we have  $W_{kq} = 0$ , which derives from the condition that  $W_1$  and  $W_2$  are real figures. Hence, the two modal deformation  $W_X$  and  $W_Y$  has to be in phase. Moreover, in spite of constant references for  $W_1$  and  $W_2$ ,  $W_{kd}$  are not constant. This is due to the deformation mode shapes which result in non constant matrix  $\phi_{XY}$ .

Measuring the actual vibration amplitudes at the positions  $P_1$  and  $P_2$  to confirm their consistency with the references would need to move the plate under the beam of the laser interferometer, which was complicated in our experimental environment. This is why, we propose in the following section of the paper a psychophysical study which aims at conclude whether or not finger 1 feels the surface more slippery than the finger 2. Before that study, we present the

limitation of the experimental test bench.

### B. Limitations of the experimental test bench

The first limitation comes from the position sensor which can only send its data every 10 ms. The consequence is that the references are not updated as often as needed, leading to a difference between the actual and the estimated position. In addition to that, the serial port communication is slow, and 5 ms are needed to send the reference to the micro-controller. This is why, it is necessary to move the finger slowly with this device; for example, moving the finger at approximately 10 cm/s will lead to a position error of 1.5 mm.

The second limitation comes from the voltage amplifiers, which have a maximum output voltage of  $\pm 150V$ . The consequence is that the vibration amplitudes are limited, and this limit depends on the global damping of the system, which increases when fingers are touching the plate [13]. Unfortunately, when two users are touching the device, the damping is too high, and the limit is too low to obtain a good stimulation. This is why, in use, the reference values of  $W_X$  are  $W_Y$  are limited to  $W_{LIM} = 4 \mu m$ . When a reference is limited, the other is recalculated in order to maintain the condition  $W_2 = 0$ . Hence, the condition  $W_1 = 1 \mu m$  is not guaranteed due to this limitation of the system. To illustrate the consequences of the limitation, we present figure 11 the actual values of the two modal amplitudes, and the calculated vibration amplitudes under the two fingers.

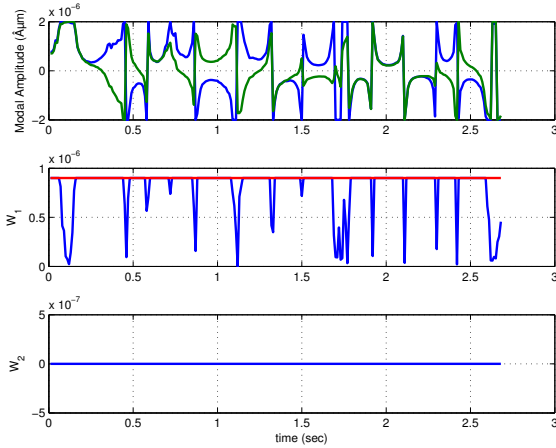


Fig. 11. Effect of the limitations on the vibration amplitude at  $P_1$  and  $P_2$ ; blue is mode X, green is mode Y.

The result shows that the condition  $W_2 = 0$  is obtained. However, the limitation induces additional spikes in the response of  $W_1$ . The spikes can occur for example when finger 2 is located close to an anti-node of vibration of mode X and mode Y, while the finger 1 is close to a node of vibration of the two modes. The result is a singular matrix  $\phi_{XY}$ . The spikes are felt by finger 1. Reducing them would be an improvement of the device. When exploring the surface as with the finger 2, spikes are also felt, while they do not show up on the curve of figure 11. This is due to the fact

that the fingertip is not punctual. Under some circumstances, the combination of the two modes is steep under the finger, in particular when the vibration amplitudes is high. Then in average over the contact area, the friction decreases, and this is felt by the user.

However, in average, the friction is theoretically lower for the finger 1 than for the finger 2. We expect that user can feel the difference. This is evaluated in the following section of the paper.

### C. Evaluation study: result and discussion

During the experiment, users are asked to move one finger over the tactile plate, and to evaluate the friction produced under their fingertip. While exploring the surface, a virtual finger is moving back and forth on the plate, on a pre-calculated and linear trajectory. Hence, on the control point of view, user's finger can be associated with the finger 1 (low friction) while the virtual finger is associated with the finger 2 (low friction). This combination is numbered "1". The other combination is also possible (user's finger is 2, virtual finger is 1) and it is named "2".

7 individuals have participated in this experiment, 1 female, 6 males, all right handed and aged between 25 and 54 years. All of them were familiar with tactile feedback, and considered as expert in the field of variable friction devices. The experiment starts with a training session, when a high friction and a low friction surface is presented; during this session, participants are also trained to adapt their speed, and to not move their finger too fast. When the participant estimates that he can distinguish the two friction levels, the training session stops.

We then presented either the combination "1" or the combination "2" to the participant which has to chose between high or low friction. The experiment is repeated 20 times, with randomization of the trials. A positive response is counted if the participant recognizes the associated stimulus. Results of psycho-physical testing is presented in figure (12).

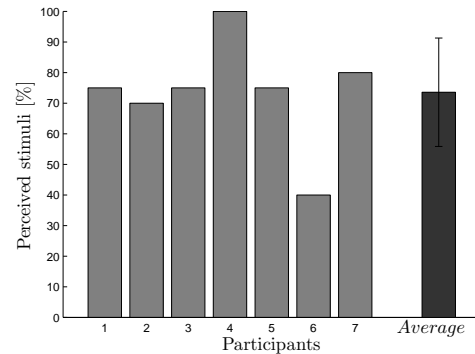


Fig. 12. Perceived stimuli for each participant (from 1 to 7) ; light and average with standard deviation; dark

The participants were able to discriminate the two friction conditions, with an average success rate of 73.57 % with a standard variation of 17.72 % . The hypothesis test against the random answer was refused with a  $p \leq 0.1$ .

This result shows that users could discriminate a high and a low level of friction, on a plate playing a different environment to two fingers exploring the surface independently. One of the fingers is a virtual one and moves along a pre-defined straight line, while the other is free to move at a small speed. One participant failed to discriminate the two sensations. He explained later that he has been disturbed by the spikes, and he had difficulties to clearly estimate the level of friction. This remark has also been reported by other participants, who felt disturbed by the spikes, which occur randomly.

## V. CONCLUSION

In this paper, we present a 2D device which is able to vibrate according to two vibration modes at a same frequency. A control of the vibration amplitude is presented. It uses the vector control method in order to precisely control amplitude and phase of each mode. The closed loop response time is about 10 ms.

Then, a multitouch sensor is associated with the device; it allows to measure the positions of fingers moving on the tactile plate. The calculation of the references of the two modes is presented. The limitations of the set-up, in terms of sampling frequency or saturation of the voltage amplifiers, is studied. We find that it is difficult to maintain the condition of a perfect virtual environment. This results in spikes which occurs randomly on the vibration amplitude at two positions.

We then lead a study to confirm that one finger can feel more friction than the other. The results have shown that it is possible to control two independent levels of friction at two positions on the plate. However, spikes are clearly reducing the level of accuracy of the stimulation, and we did not actually measure the vibration amplitude at these positions.

This study gives us directions for future work. First of all, we will try to reduce the sampling period of the device. It will be necessary to reduce the response time of the control loop, and need an other type of fingers sensor. Also, we need to confirm that the fingers does not change the mode shapes. Finally, the spikes in the vibration response have to be cancelled. This may be achieved by using a third mode, in order to remove positions where  $\phi_{XY}$  is singular. This will require an optimization process in order to choose the three modal amplitudes with only two objectives.

## ACKNOWLEDGMENT

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