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# A general surface reconstruction method for post-processing of topology optimisation results

Giulia Bertolino, Giulio Costa, Marco Montemurro, Nicolas Perry and Franck Pourroy

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# Outline

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**Context and scientific objectives**



**Surface Reconstruction strategy for genus 0 open surfaces**



**Poly-patches strategy for genus  $N$  surfaces (open and closed)**



**Conclusions and perspectives**

## Context and scientific objectives

Surface Reconstruction strategy for genus 0 open surfaces

Poly-patches strategy for genus  $N$  surfaces (open and closed)

Conclusions and perspectives

Appendix

## Context and scientific objectives

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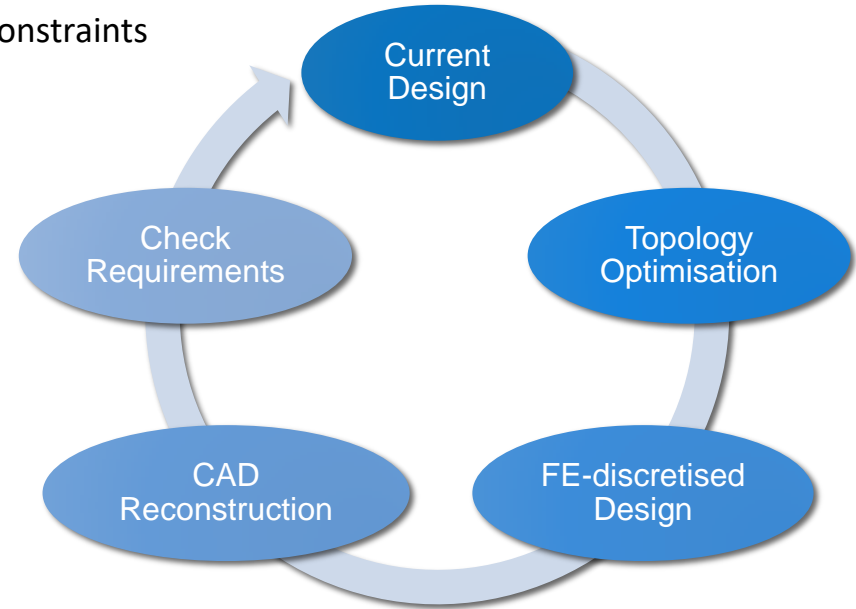
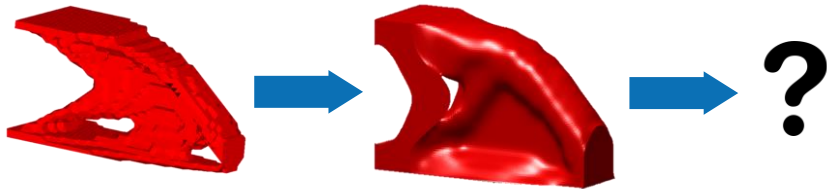
# Context

## Topology optimisation:

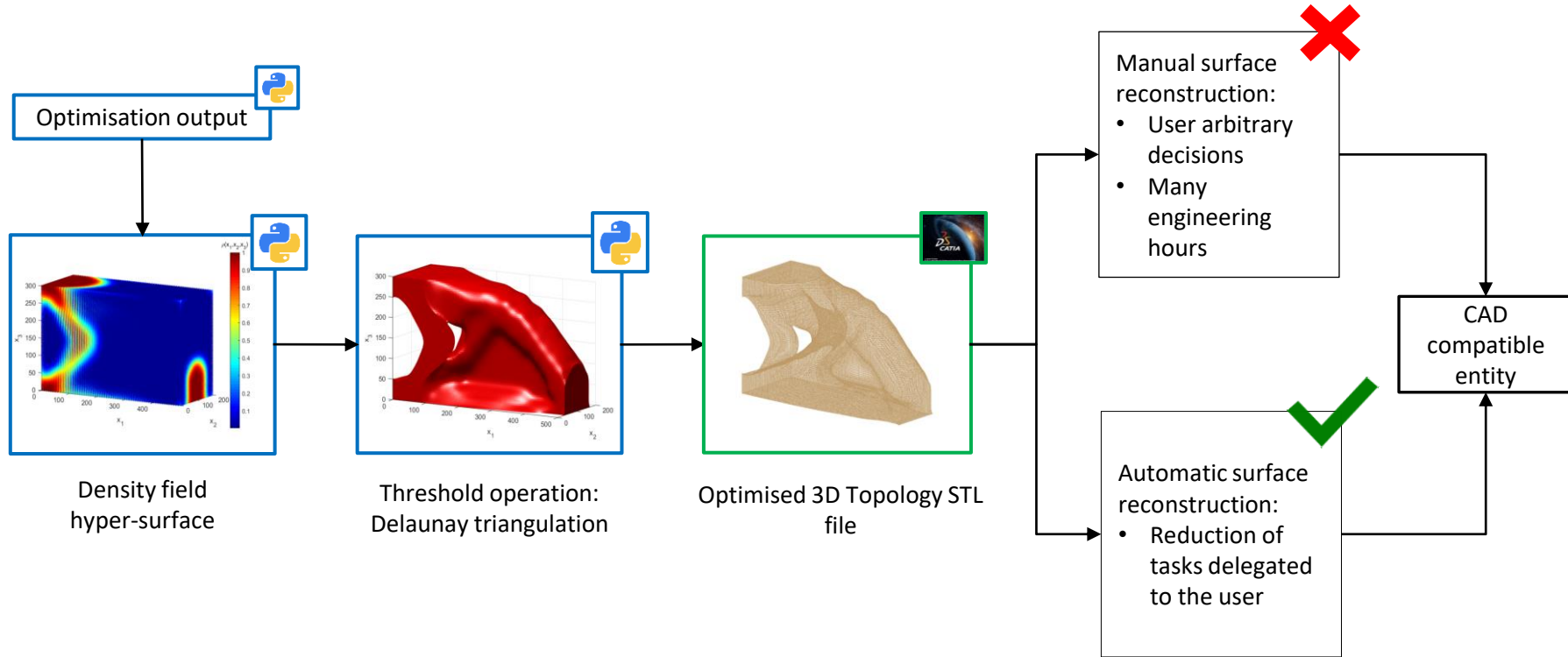
- Optimal distribution of material in a prescribed domain
- Minimise an objective/cost function + meet optimisation constraints

## Results of topology optimisation strategy:

- Density field described by element-wise format
- Need to obtain smooth surfaces
- How is it possible to obtain CAD compatible entity?



# Objectives



Context and scientific objectives

**Surface Reconstruction strategy for genus 0 open surfaces**

Poly-patches strategy for genus  $N$  surfaces (open and closed)

Conclusions and perspectives

Appendix

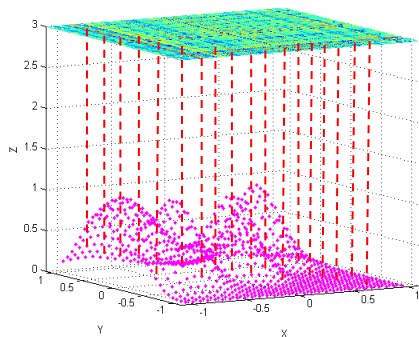
# Surface Reconstruction strategy for genus 0 open surfaces

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# Proposed strategy: main ingredients

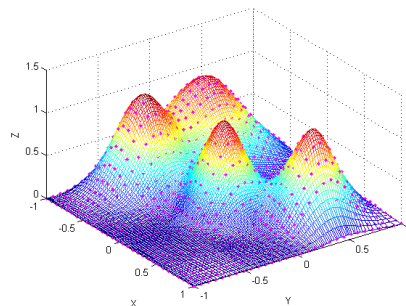
## Mapping the STL points

Find the planar triangulation  $P$  isomorphic to the given triangulated graph  $G$



## 1 - Parameterisation

## 2 - Fitting

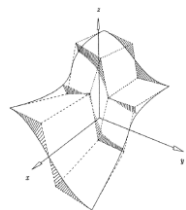


## Least squares minimisation

Obtain the optimal set of NURBS parameters (Degrees, Knot vector, Weights)

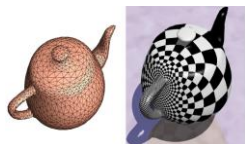
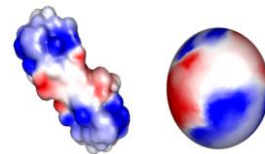


# Parameterisation



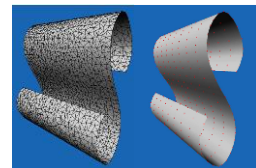
Projection method [Piegl,1995] - open, genus 0, not folded

Mercator's projection method [Rahi,2007] - closed, genus 0



Global conformal method [Gu,2003] - closed, genus N

Shape preserving method [Floater,1997] - open, genus 0, folded



# Shape preserving method: capabilities and main features

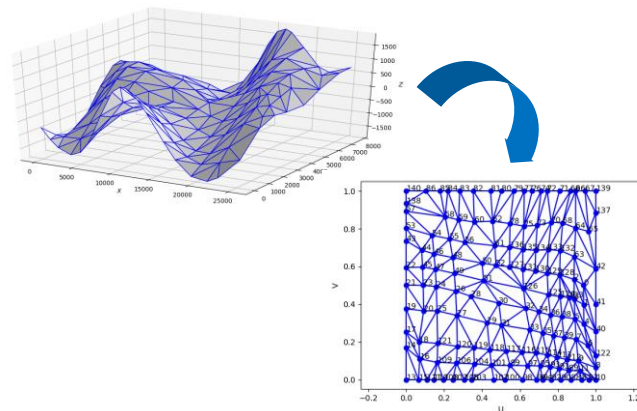
Find  $(u, v)$  parameters associated to the Cartesian coordinates of the 3D Euclidean space

**Relabel nodes** of STL file: **internal nodes** and **boundary nodes** (ordered in anti-clockwise sequence)

**Parameterisation of the boundary nodes** by chord length method into the boundary of a convex polygon  $D \in \mathbb{R}^2$   $[0,1] \times [0,1]$

**Expression of each internal node as linear convex combination of neighbours.**

- Evaluation of the weights  $\lambda_{i,j_k}$  for each neighbour
- Preserving distances and angles between 3D and 2D



$$\begin{cases} [\Delta]\{u\} = b_1 \\ [\Delta]\{v\} = b_2 \end{cases}$$

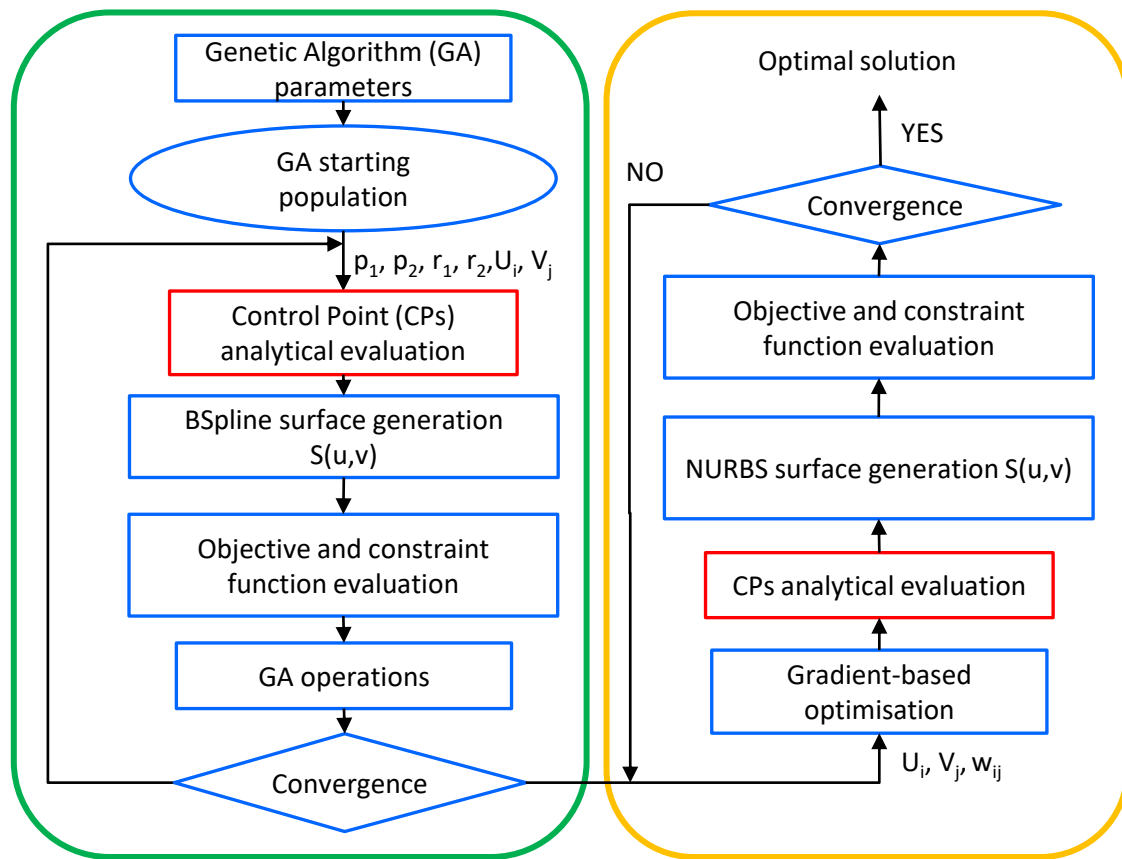
$u, v$  internal nodes

$b_1 = \lambda_{i,j} u_j \rightarrow u_j$  boundary nodes

$b_2 = \lambda_{i,j} v_j \rightarrow v_j$  boundary nodes

$$[\Delta] \rightarrow \lambda_{i,j_k} = \frac{1}{d_i} \sum_{k=1}^{d_i} \mu_{k,l} \text{ weights}$$

# Surface fitting: Optimisation strategy



## Part A

**Originality:** NURBS surface parameters (degree, CPs number, Knot Vector (KV) components) will be found automatically by the GA (in the literature there are no rules to set these parameters)

## Part B

Local refinement of the minimum found by the GA  $\Rightarrow$  improvement of the solution in terms of KV COMPONENTS and WEIGHTS

# Problem formulation and numerical aspects: genetic optimisation

## Part A

### Objective function

$$\min f(\mathbf{x}) = \left( \sum_{k=0}^{n_{tp}} \|S(u_k, v_k) - Q_k\|^2 + \lambda J \right)^{\frac{1}{r_1 + r_2}} \quad \text{such that:}$$

- Distance between BSpline entity and target points
- Thin-plate spline energy functional<sup>[Floater,2000]</sup>: smoothing term

$$J = \int_{a_1}^{b_1} \int_{a_2}^{b_2} S_{uu}^2 + 2S_{uv}^2 + S_{vv}^2 du dv$$

### Constraint function:

Non singularity of Basis Functions (BF) matrix

$$g_1(\mathbf{x}) = \dim(BF) - \rho(BF) \rightarrow BF = [N_u N_v]^T [N_u N_v] + \lambda E$$

- Dimension of the basic functions matrix
- Rank of the matrix of the basic functions matrix
- Smoothing matrix

### Design variables

#### Discrete variables:

- $p_1, p_2 \rightarrow$  Degrees of the BSpline entity
- $r_1, r_2 \rightarrow$  Number of non-trivial components of KV

#### Continuous variables:

- $U_{p1+2}, \dots, U_{p1+r1+2}, V_{p2+2}, \dots, V_{p2+r2+2} \rightarrow$  Knot vector components

Design space dimension =  $4 + r_1 + r_2$



Discrete variables values affect the dimension of the Continuous variables module.

# Problem formulation and numerical aspects: genetic optimisation

## Part A

### GA ERASMUS<sup>[Montemurro,2018]</sup> capabilities:

- Reproduction among individuals: crossover and mutation operations
- Reproduction among species: on individuals with different number of chromosomes
- Penalisation: Automatic Dynamic Penalisation (ADP)
  - Automatically and adaptively updating the coefficients of penalisation
  - Preventing infeasible solutions
  - Efficient exploration of the boundary of the feasible domain

### Genotype

Gene 1		Gene 2	Individual Standard section
Ch 1	$p_1$	$p_2$	
Ch 2	$r_1$	$r_2$	
Gene 1			Individual Modular Section n. 1
Ch 1	$U_{p1+2}$		
...	...		
Ch $r_1$	$U_{p1+r1+2}$		Individual Modular Section n. 2
Gene 1			
Ch 1	$V_{p2+2}$		
...	...		
Ch $r_1$	$V_{p2+r2+2}$		

Ch = chromosome

# Problem formulation and numerical aspects: deterministic optimisation

## Part B

### Design variables (only continuous)

$$\mathbf{x} \begin{cases} \mathbf{U} = \{0, \dots, 0, u_{p+1}, \dots, u_n, 1, \dots, 1\} \\ \mathbf{V} = \{0, \dots, 0, v_{q+1}, \dots, v_m, 1, \dots, 1\} \\ \mathbf{W} = \begin{pmatrix} w_{11} & \cdots & w_{1n_2} \\ \vdots & \ddots & \vdots \\ w_{n_1 1} & \cdots & w_{n_1 n_2} \end{pmatrix} \end{cases}$$

### Objective function

$$\min f(\mathbf{x}) = \sum_{k=0}^{n_{tp}} \|S(u_k, v_k) - Q_k\|^2 + \lambda J \text{ such that:}$$

### Constraint function

$$g_1(\mathbf{x}) = \dim(BF) - \rho(BF) \rightarrow BF = [N_u N_v]^T [N_u N_v] + \lambda E$$

Non trivial KV  
components

- Numerical evaluation of  $\nabla f(\mathbf{x})$  respect to KV

Weights

- Analytical evaluation of  $\nabla f(\mathbf{x})$  respect to weights

# Focus on the analytical CPs evaluation

## Part A

GA optimisation

CPs analytical  
evaluation

BSpline surface generation  
 $S(u,v)$

## Part B

Gradient-based  
optimisation

CPs analytical  
evaluation

NURBS surface generation  $S(u,v)$

## CPs analytical evaluation

Evaluation of  $\frac{\partial f(\mathbf{x})}{\partial P_{ij}}$

Find the critical point  
 $\nabla f(\mathbf{x}) = 0$

$$([N_u N_v]^T [N_u N_v] + \lambda E)[P] = [N_u N_v]^T [Q]$$

Check on the singularity due to the absence of  
parameters in the knot span

$[N_u N_v]^T [N_u N_v] + \lambda E$  matrix inversion to find  $[P]$

- Basis function matrix evaluated at  $(u_k, v_k)$
- Smoothing constant
- Smoothing matrix
- Control points coordinates
- Target points coordinates

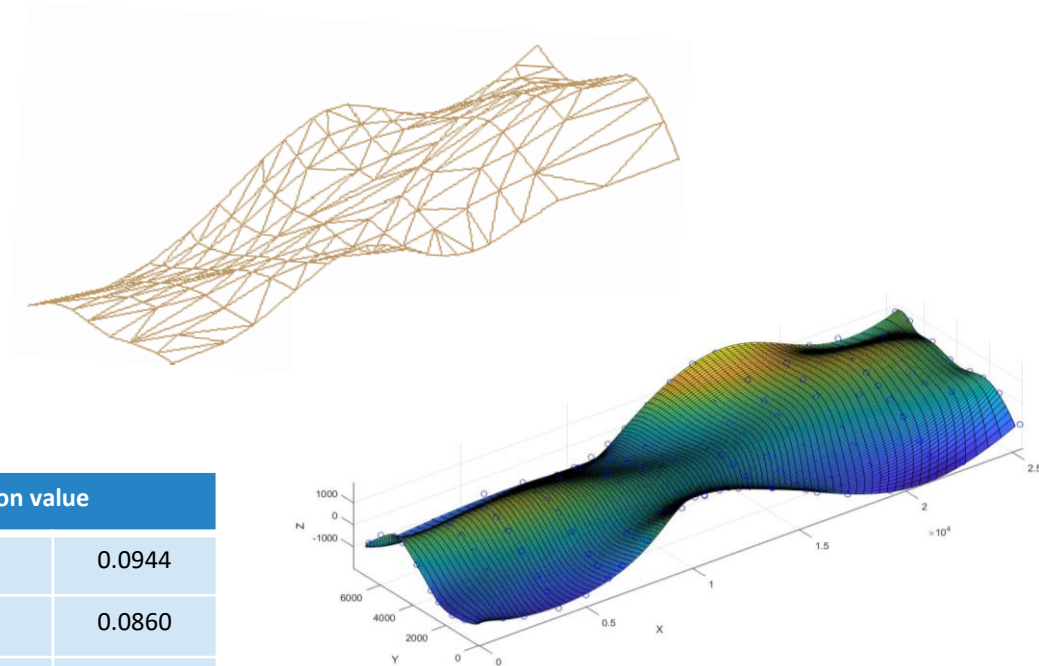
# Numerical results: 1<sup>st</sup> benchmark

	Design variables	Lower - Upper bounds
GA optim	Degree (p)	2 – 6
	N° of KV's components (r)	1 – 17
	KV's components values (U, V)	0.001 – 0.999
Grad optim	Alpha, Beta (KV's components values)	0.001 – 0.999
	Weights (w)	0 – 10

Design space

Optimised values		Objective function value	
Degree ( $p_1 - p_2$ )	5 – 5	GA phase	0.0944
N° of KV's components ( $r_1 - r_2$ )	1 – 1	Grad KV	0.0860
KV's components values (U, V)	0.374 – 0.599	Grad KV + Weights	0.0844

Optimised design variables at the end of the Surface Reconstruction algorithm and objective function value evolution along the different phases



Results of the Surface Reconstruction strategy



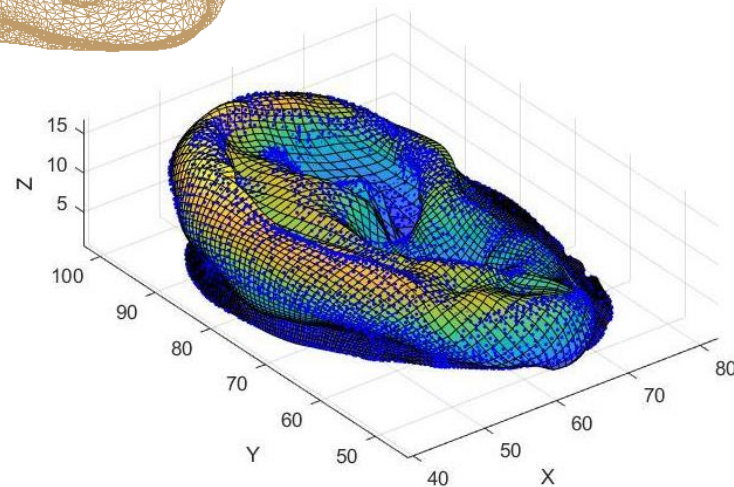
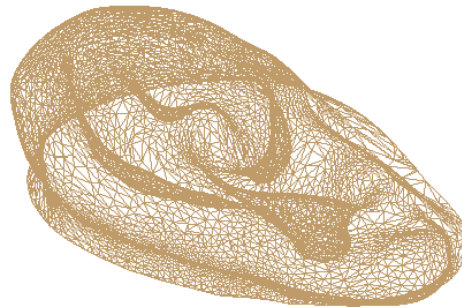
# Numerical results: 2<sup>nd</sup> benchmark

	Design variables	Lower - Upper bounds
GA optim	Degree (p)	2 – 6
	N° of KV's components (r)	16 – 35
	KV's components values (U, V)	0.001 – 0.999
Grad optim	Alpha, Beta (KV's components values)	0.001 – 0.999
	Weights (w)	0 – 10

Design space

Optimised values		Objective function value	
Degree ( $p_1 - p_2$ )	2 – 2	GA phase	0.963576
N° of KV's components ( $r_1 - r_2$ )	19 – 19	Grad KV	0.954653
		Grad KV + Weights	0.892313

Optimised design variables at the end of the fitting algorithm and objective function value evolution along the different phases



Results of the Surface Reconstruction strategy

Context and scientific  
objectives

Surface Reconstruction strategy  
for genus 0 open surfaces

**Poly-patches strategy for  
genus N surfaces (open and  
closed)**

Conclusions and perspectives

Appendix

## Poly-patches strategy for genus N surfaces (open and closed)

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# Poly-patches strategy for genus N surfaces (open and closed)

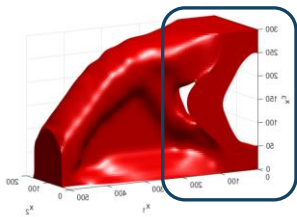
**Aim:** Application of the Surface Reconstruction strategy to **surfaces** (open and closed) with **holes** (genus  $> 0$ )

## Strategy:



Domain divided into  
opened patches of genus 0

Manual segmentation



Adjacent patches have  
same parameters along  
boundary

Proper roto-translation of  
patches according to the  
global reference system

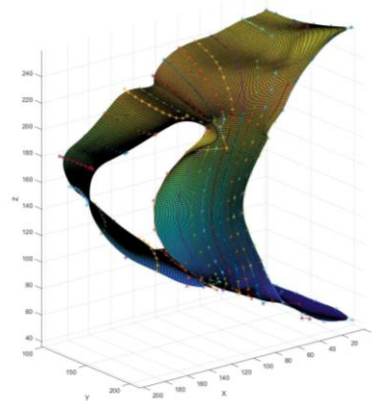
Parameterisation



Automatic calculation of  
NURBS parameters

Automatic imposition of C0  
and C1 continuity condition  
between patches

Patch fitting



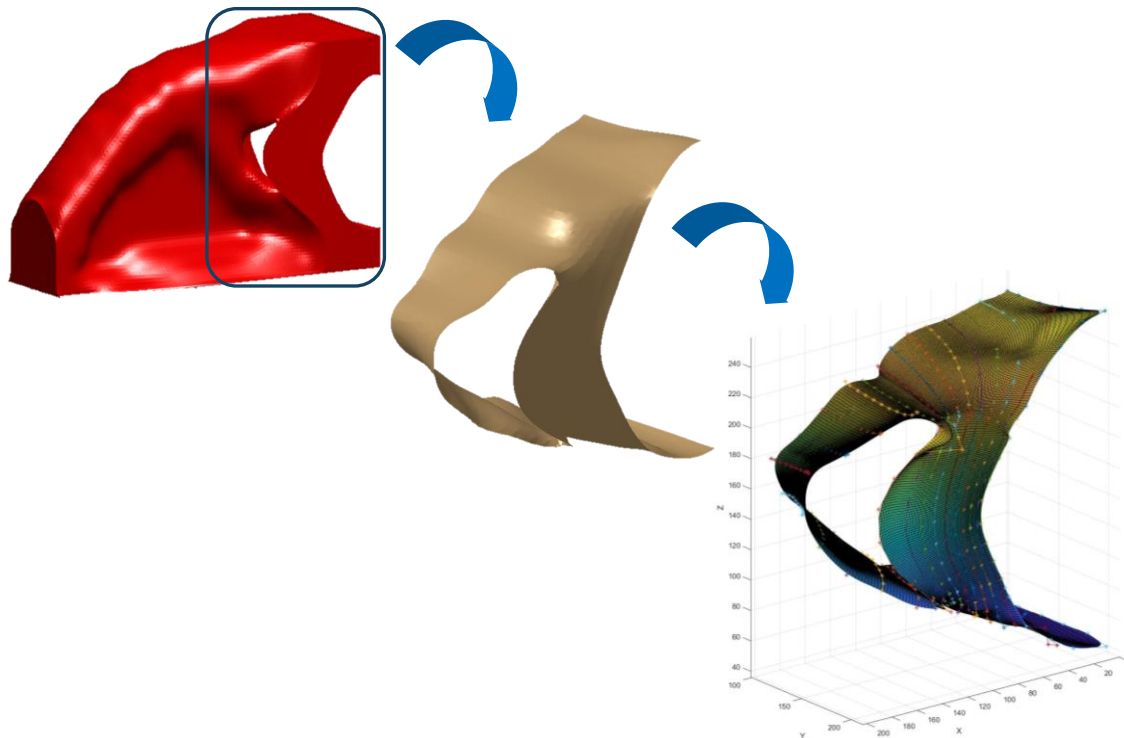
# Numerical result

	Design variables	Lower - Upper bounds
GA optim	Degree (p)	2 – 6
	N° of KV's components (r)	4 – 20
	KV's components values (U, V)	0.001 – 0.999
Grad optim	Alpha, Beta (KV's components values)	0.001 – 0.999
	Weights (w)	0 – 10

Design space

Optimised values	
Degree ( $p_1 - p_2$ )	2 – 2
N° of KV's components ( $r_1 - r_2$ )	8 – 8

Optimised design variables at the end of the fitting algorithm.



Results of the Surface Reconstruction strategy

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N surfaces (open and closed)

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Appendix

## Conclusions and perspectives

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# Conclusions and perspectives

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## **Conclusions**

- Automatic optimisation of approximation surface parameters
- Reduction of tasks delegated to the user

## **Perspectives**

- Mapping methods for genus  $> 0$  surfaces
- Automatic segmentation of the triangulation (STL file)
- Integration of Tspline entities in the surface fitting

Thank you for your attention

Context and scientific  
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N surfaces (open and closed)

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## **Appendix**

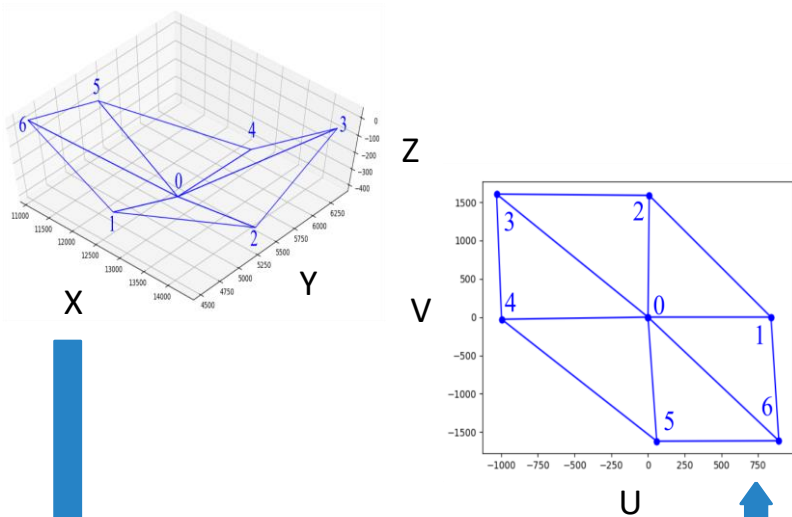
# Appendix

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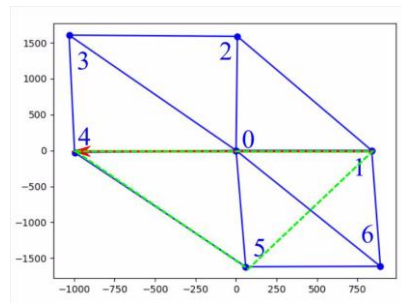
# Shape preserving method<sup>[Floater,1997]</sup>

## FIRST STEP



$$\begin{cases} \|p_k - p\| = \|x_{j_k} - x_i\| \\ \text{ang}(p_k, p, p_{k+1}) = \frac{2\pi \text{ang}(x_{j_k}, x_j, x_{j_{k+1}})}{\theta_i} \end{cases}$$

## SECOND STEP



Barycentric coordinates

$$\begin{cases} \mu_k = \frac{\text{area}(pp_r p_{r+1})}{\text{area}(p_k p_r p_{r+1})} \\ \mu_r = \frac{\text{area}(p_k p_r p_{r+1})}{\text{area}(p_k p_r p)} \\ \mu_{r+1} = \frac{\text{area}(p_k p_r p)}{\text{area}(p_k p_r p_{r+1})} \end{cases}$$

$\mu_{1,1}$	0	$\mu_{3,1}$	$\mu_{4,1}$	0	0
0	$\mu_{2,2}$	0	$\mu_{4,2}$	$\mu_{5,2}$	$\mu_{6,2}$
0	0	$\mu_{3,3}$	0	$\mu_{5,3}$	$\mu_{6,3}$
$\mu_{1,4}$	$\mu_{2,4}$	0	$\mu_{4,4}$	0	0
$\mu_{1,5}$	$\mu_{2,5}$	0	0	$\mu_{5,5}$	0
0	0	$\mu_{3,6}$	0	0	$\mu_{6,6}$

$$\lambda_{i,1} = \frac{1}{6} \sum_{l=1}^6 \mu_{l,1}$$

...

$$\lambda_{i,6} = \frac{1}{6} \sum_{l=1}^6 \mu_{l,6}$$

# Thin-plate spline energy<sup>[Floater,2000]</sup>

Adding a smoothing term in the surface approximation of unstructured data aims to find a unique solution.

$$f(\mathbf{x}) = \sum_{k=0}^{n_{tp}} \|S(u_k, v_k) - Q_k\|^2 + \lambda J \quad \begin{cases} J = \int_{a_1}^{b_1} \int_{a_2}^{b_2} S_{uu}^2 + 2S_{uv}^2 + S_{vv}^2 du dv \rightarrow \text{simple thin plate energy functional} \\ \lambda \rightarrow \text{constant measuring the trade off between approximation and smoothing} \end{cases}$$

Find the minimum  $\rightarrow$  normal equations

$$\frac{\partial f(\mathbf{x})}{\partial P_{ij}} = ([N_u N_v]^T [N_u N_v] + \lambda E)[P] - [N_u N_v]^T [Q] = 0$$

Where  $E$  is a  $(n_1 n_2) \times (n_1 n_2)$  matrix whose elements are:

$$E_{ijrs} = A_{ijrs} + 2B_{ijrs} + C_{ijrs} \quad \begin{cases} A_{ijrs} = \int_{a_1}^{b_1} N_i''(u) N_j''(u) du \int_{a_2}^{b_2} N_j(v) N_s(v) dv \\ B_{ijrs} = \int_{a_1}^{b_1} N_i'(u) N_j'(u) du \int_{a_2}^{b_2} N_j'(v) N_s'(v) dv \\ C_{ijrs} = \int_{a_1}^{b_1} N_i(u) N_j(u) du \int_{a_2}^{b_2} N_j''(v) N_s''(v) dv \end{cases}$$

And  $\lambda$  is:

$$\lambda = \frac{\|([N_u N_v]^T [N_u N_v])^2\|}{\|E^2\|}$$