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Frequency and amplitude modulations of a moving structure in unsteady non-homogeneous density fluid flow

T. Emerry RAJAOMAZAVA III ^{1,*}, Mustapha BENAOUICHA ^{2,3}, Jacques-André ASTOLFI ⁴ and Abdel-Ouahab BOUDRAA ⁴ 

¹ Computed Wing Sail, 151 Boulevard de l'Hopital, 75013 Paris, France ; emerry.rajaomazava@computedwingsail.com

² SEGULA TEHCHNOLOGIES, Research and Innovation Unit in Naval and Energy engineering, 19 Rue d'Arras, 92000 Nanterre, France; mustapha.benaouicha@segula.fr

³ Normandie University, Unicaen, Cherbourg University Laboratory of Applied Sciences (Lusac)

⁴ Ecole Navale/Arts et Metiers Institute of Technology, IRENav, BCRM Brest, CC 600, 29240 BREST Cedex 9, France; Jacques-Andre.astolfi@ecole-navale.fr; abdel.boudraa@ecole-navale.fr

* Correspondence: emerry.rajaomazava@computedwingsail.com; Tel.: +33-6829-894-85 (E.R.)

Abstract: A fluid-structure interactions effects on the dynamics of a hydrofoil immersed in a fluid flow of non-homogeneous density is presented and analyzed. A linearized model is applied to solve the fluid-structure coupled problem. A fluid density variations along the hydrofoil upper surface, based on the sinusoidal cavity oscillations, is used. It is shown that for the steady cavity case, the value of cavity length L_p does not affect the amplitude of the hydrofoil displacements. However, the natural frequency of the structure increases according to L_p . In the unsteady cavity case, the variations of the added mass and added damping (induced by the fluid density rate of change) generate frequency and amplitude modulations in the hydrofoil dynamics. In order to analyse this phenomena, the empirical mode decomposition, a well established data-driven method to handle such modulations, is used.

Keywords: fluid-structure interaction; added mass; added damping; frequency modulation; amplitude modulation ; non-homogeneous fluid density; cavity oscillations; empirical mode decomposition; intrinsic mode functions

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1. Introduction

Fluid structure interaction (FSI) problems occur when the fluid loading greatly affects the structure's dynamics and the structure displacement locally affects the fluid flow. Initially studied with simplified models, **the simulation of complex coupled problems has developed considerably in recent years**. The state-of-the-art in this field is now **very** mature and several papers with different fields of application domains can be found in literature [1–3].

The new challenge of FSI problem analysis consists in taking into account complex phenomena, observed **both in fluid and solid mechanics, especially in the field of fluids where the dynamics are subjected to many physical** quantities such as velocity, pressure, density or temperature. **This work focuses on FSI** effects in a non-homogeneous density flow. **Recent work has pointed out that two-phase flow has an impact on the** fluid structure interaction for various devices, such as propeller blades or hydrofoils [4–9]. However, **very few published works address** the problem of estimating this impact on the structure dynamics. This work is strongly motivated by recent advances in experimental and modeling studies **carried out** by the authors. It is shown that modal response of the structure could be modified in the presence of cavitation [10]. This modification can be attributed to the presence in the flow of a non stationary liquid-vapor mixture with **a strong variation in density** at the fluid structure interface. Previous works proposed the decomposition of the fluid variables into two components: the first component is related

34 to the fluid flow around the non-vibrating structure while the second one describes
 35 the fluid flow induced by the structure vibrations [11]. This approach can be used to
 36 compute the added mass and the added damping operators for complex geometries and
 37 complex fluid flow behaviour. Here, the fluid flow is characterized by oscillating cavity
 38 on the fluid-structure interface. Unlike to homogeneous fluid case, it is shown that the
 39 added mass operator is not symmetrical and depends on the flow through fluid density
 40 variations at the fluid-structure interface. Also, it is evidenced that variation rate of the
 41 fluid density induces an added damping operator. This suggest to a possible variation
 42 of the natural frequency of the structure related to the variation of added mass. It is
 43 reported in [11] that the fluid density variations on the fluid-structure interface have
 44 an effect on the added mass operator and the variation rate of this density induces an
 45 added damping operator.

46 The aim of this paper is to study the effect of these variations on the structure
 47 dynamics. First, the modeling of the structure dynamics is carried out. A rigid section of
 48 a hydrofoil immersed in a 2D fluid flow and supported by a linear spring, is considered.
 49 Equations of the hydrofoil motion are thus provided. **Second, a model of the fluid**
 50 **flow generated by the displacements of the structure is considered to determine the**
 51 **hydrodynamic loads.** This is given by the solution of a Laplace equation, with the space
 52 variations of the fluid density taken into account. Applied to the structure dynamics, the
 53 hydrodynamic loads act as an added mass and an added damping. A simplified model
 54 of an unsteady cavity, based on a sheet cavitation oscillation, is used to take into account
 55 the time and space variations of the fluid density on the fluid-structure interface. The
 56 empirical mode decomposition (EMD) [12] is used to analyze the structure displacement.
 57 The displacement signal is decomposed by EMD into intrinsic mode functions (IMFs),
 58 followed by the instantaneous frequencies estimation of these sifted IMFs that evidence
 59 the frequencies modulations.

60 2. Fluid loads acting on the immersed structure

61 A 2D rigid section of hydrofoil type NACA0012 (Ω_S), immersed in a 2D fluid flow
 62 (Ω_F) and animated by a heave motion, is considered (Figure 1). The fluid domain bound-
 63 aries are **respectively** the flow inlet Γ_I , the flow outlet Γ_O , the fixed boundary (wall)
 64 Γ_W and the fluid-structure interface (moving boundary) $\Gamma_{FS} = \Omega_F \cap \Omega_S$. \mathbf{n} denotes the
 65 outward normal unit vector at $\partial\Omega_F = \Gamma_I \cup \Gamma_O \cup \Gamma_W \cup \Gamma_{FS}$. \vec{U}_∞ is the uniform velocity
 66 field of the fluid upstream of the hydrofoil. Parameter θ corresponds to the angle of
 67 incidence of the hydrofoil.
 68

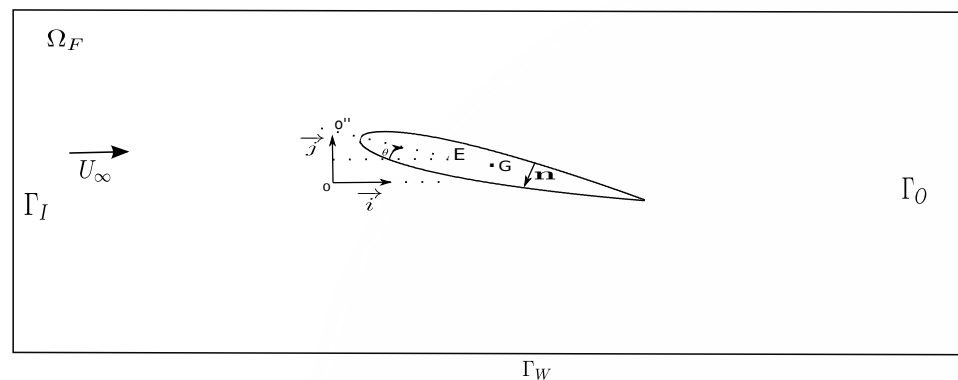


Figure 1. Fluid and Structure domains

In this study, a non-homogeneous inviscid fluid flow is considered. The corresponding conservation equations are given by:

$$\left\{ \begin{array}{ll} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 & \text{on } \Omega_F \quad (a) \\ \frac{\partial(\rho u)}{\partial t} + \nabla \cdot (\rho u \otimes u) = -\nabla p & \text{on } \Gamma_{FS} \quad (b) \end{array} \right. \quad (1)$$

where u , p and ρ are respectively the time and space-dependent fluid velocity, fluid pressure and two-phase fluid density. Density ρ can change from liquid, ρ_L , to vapor ρ_V (or vice versa). The boundary conditions are given by:

$$\left\{ \begin{array}{ll} u = u_\infty & \text{on } \Gamma_I \quad (c) \\ -p \mathbf{n} = 0 & \text{on } \Gamma_O \quad (d) \\ u \cdot \mathbf{n} = 0 & \text{on } \Gamma_W \quad (e) \\ u \cdot \mathbf{n} = \dot{\xi}_y n_y & \text{on } \Gamma_{FS} \quad (f) \end{array} \right. \quad (2)$$

69 where n_y and $\dot{\xi}_y$ are respectively the j components of the normal unit vector \mathbf{n} and the
70 velocity $\dot{\xi}$ of a point $A(x,y)$ on the interface Γ_{FS} given by Equation (12).

71

Let us assume the following decomposition:

$$u = \bar{u} + u', \text{ and } p = \bar{p} + p' \quad (3)$$

Assuming that u' is small and uncorrelated to \bar{u} , the problem described by the system of Equations (1) can be subdivided into two separate problems [11]: the first problem is related to the fluid flow equation around a non-vibrating structure and the second one is about the fluid flow equation induced by the structure vibrations. The later is described by the following system:

$$\left\{ \begin{array}{ll} \Delta p' = 0 & \text{on } \Omega_f \quad (a) \\ \nabla p' \cdot \mathbf{n} = -\rho \ddot{\xi}_y n_y - \frac{\partial \rho}{\partial t} \dot{\xi}_y n_y & \text{on } \Gamma_{FS} \quad (b) \\ p' = 0 & \text{on } \partial\Omega_f \setminus \Gamma_{FS} \quad (c) \end{array} \right. \quad (4)$$

where $\ddot{\xi}_y$ is the j component of the acceleration at the point $A(x,y)$ on the interface Γ_{FS} . It is given by Equation (12). This formulation is used for cambered hydrofoil and for other geometries [13]. Equations system (4) is coupled to the structure dynamic's equation through the boundary condition (4b), defined on the fluid-structure interface Γ_{FS} . It follows that the structure loading due to the pressure field p' is given by:

$$F(t) = \int_{\Gamma_{FS}} p' \mathbf{n} ds \quad (5)$$

72 In this paper, we are particularly interested in the effect of $F(t)$ on the dynamic of the
73 hydrofoil. Therefore, the main goal is to perform the coupling of the Equation (4) and
74 the structure dynamics equation (Equation (12)).

75 2.1. Added mass and added damping

Due to the linearity of Equation (4), superposition principle holds and the solution can be expressed as $p' = p_1 + p_2$, where p_1 and p_2 are respectively the solutions of the following systems:

$$\begin{cases} \Delta p_1 = 0 & \text{on } \Omega_F & (a) \\ \nabla p_1 \cdot \mathbf{n} = -\rho \ddot{\zeta}_y n_y & \text{on } \Gamma_{FS} & (b) \\ p_1 = 0 & \text{on } \partial\Omega_F \setminus \Gamma_{FS} & (c) \end{cases} \quad (6)$$

and

$$\begin{cases} \Delta p_2 = 0 & \text{on } \Omega_F & (a) \\ \nabla p_2 \cdot \mathbf{n} = -\frac{\partial \rho}{\partial t} \dot{\zeta}_y n_y & \text{on } \Gamma_{FS} & (b) \\ p_2 = 0 & \text{on } \partial\Omega_F \setminus \Gamma_{FS} & (c) \end{cases} \quad (7)$$

76 Solution p_1 of Equation (6) represents the inertial effect of the fluid on the structure as
 77 it is proportional to the acceleration $\ddot{\zeta}_y$ of the structure. Solution of Equation (7) shows
 78 that the fluid density rate of change induces a fluid load acting as an added damping
 79 on the structure, as it is proportional to the velocity $\dot{\zeta}_y$ of the latter (cf. Equation (17)).
 80 Equation (6-b) and Equation (7-b) show that both solutions p_1 and p_2 depend on space
 81 and time variations of the fluid density throughout the fluid-structure interface. It is
 82 easy to see that solution p_2 is zero for the homogeneous case.

In the other hand, it can be shown that the fluid load $F_a(t) = \begin{pmatrix} F_{a1} \\ F_{a2} \end{pmatrix}$, defined by the integral

$$F_a(t) = \int_{\Gamma_{FS}} p_1 \mathbf{n} ds, \quad (8)$$

is proportional to the structure acceleration. It can be expressed as

$$\begin{aligned} F_{a1}(t) &= -m_a^{11} \ddot{X}_1 - m_a^{12} \ddot{X}_2 \\ F_{a2}(t) &= -m_a^{21} \ddot{X}_1 - m_a^{22} \ddot{X}_2 \end{aligned} \quad (9)$$

where, for $A \in \Gamma_{FS}$, $\ddot{X}_1(t) = \ddot{\zeta}_x(A, t)$ and $\ddot{X}_2(t) = \ddot{\zeta}_y(A, t)$ are the accelerations according to the 2d-coordinates axis and $(m_a^{ij})_{i,j=1,2}$ are the added mass coefficients. The matrix \mathbf{M}_a such that

$$F_a(t) = -\mathbf{M}_a \ddot{\mathbf{X}} = (m_a^{ij} \ddot{X}_j)_{i=1,2} \quad (10)$$

83 is the added mass matrix.

84 By following the same analysis as before, we can define the added damping operator
 85 (induced by the fluid density rate of change) from Equation (7). The fluid load $F_d(t) =$
 86 $\begin{pmatrix} F_{d1} \\ F_{d2} \end{pmatrix}$ is proportional to the velocity of Γ_{FS} .

87 The same approach used for Equations ((8) and (9)) leads to the added damping matrix
 88 \mathbf{D}_a , given by the following relation:

$$F_d(t) = -\mathbf{D}_a \dot{\mathbf{X}} = (d_a^{ij} \dot{X}_j)_{i=1,2} \quad (11)$$

89 where $\dot{X}_1(t) = \dot{\zeta}_x(A, t)$ and $\dot{X}_2(t) = \dot{\zeta}_y(A, t)$ are the velocity and $(d_a^{ij})_{i,j=1,2}$ are the
 90 added damping coefficients (induced by the fluid density rate of change).

91 3. Structure dynamics modeling

Hydrofoil motion can be defined by its interface displacement $\zeta = \zeta(t)$ where $\zeta = \zeta_x \mathbf{i} + \zeta_y \mathbf{j}$. A linear spring with mass m and stiffness k_y is applied in order to model the heave motion ζ_y of the hydrofoil in \mathbf{j} direction (Figure 2). The angle of attack θ is assumed to be fixed at 8 degrees. The dynamic of the hydrofoil in heave motion is

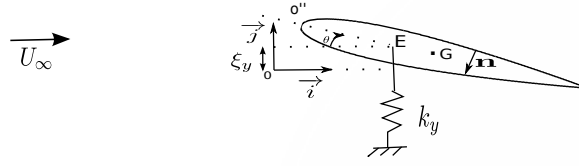


Figure 2. Modeling of the hydrofoil in heave motion with a spring mass system.

governed by the following equation:

$$m \ddot{\zeta}_y + k_y \zeta_y = F_y \quad (12)$$

with the following initial conditions:

$$\begin{cases} \ddot{\zeta}(0) = \ddot{\zeta}_0 & (a) \\ \zeta(0) = \zeta_0 & (b) \end{cases} \quad (13)$$

where F_y is the **second** component of the force vector F (Equation (5)), induced by the fluid flow around the vibrating hydrofoil. Moreover, due to linearity, the solution of Equations (6) and (7) can be expressed as $p_1 = -\ddot{\zeta}_y p'_1$ and $p_2 = -\dot{\zeta}_y p'_2$, where p'_1 and p'_2 are respectively the solutions of the following systems:

$$\begin{cases} \Delta p'_1 = 0 & \text{on } \Omega_F & (a) \\ \nabla p'_1 \cdot \mathbf{n} = \rho n_y & \text{on } \Gamma_{FS} & (b) \\ p'_1 = 0 & \text{on } \partial\Omega_F \setminus \Gamma_{FS} & (c) \end{cases} \quad (14)$$

and

$$\begin{cases} \Delta p'_2 = 0 & \text{on } \Omega_F & (a) \\ \nabla p'_2 \cdot \mathbf{n} = \frac{\partial \rho}{\partial t} n_y & \text{on } \Gamma_{FS} & (b) \\ p'_2 = 0 & \text{on } \partial\Omega_F \setminus \Gamma_{FS} & (c) \end{cases} \quad (15)$$

It follows that the Lift force F_y is given by

$$F_y = -\ddot{\zeta}_y \int_{\Gamma_{FS}} p'_1 n_y ds - \dot{\zeta}_y \int_{\Gamma_{FS}} p'_2 n_y ds \quad (16)$$

and Equation (12) can be rewritten as:

$$(m + m_a) \ddot{\zeta}_y + d_a \dot{\zeta}_y + k_y \zeta_y = 0 \quad (17)$$

where m_a and d_a are respectively the added mass and added damping,

$$m_a = \int_{\Gamma_{FS}} p'_1 n_y ds \quad \text{and} \quad d_a = \int_{\Gamma_{FS}} p'_2 n_y ds$$

The resolution of the coupled problem can be summarized by the resolution of Equations (14), (15) and (17). On the one hand, Equations (14) and (15) give the added mass and added damping (fluid load on the structure). On the other hand, Equation (17) provides the structure dynamics (structure displacement, velocity and acceleration). Note that, for fixed angle of attack (8° for our case), n_y has a fixed value and the variation of the solutions p'_1 and p'_2 depend only on the density ρ and its variation rate ($\frac{\partial \rho}{\partial t}$).

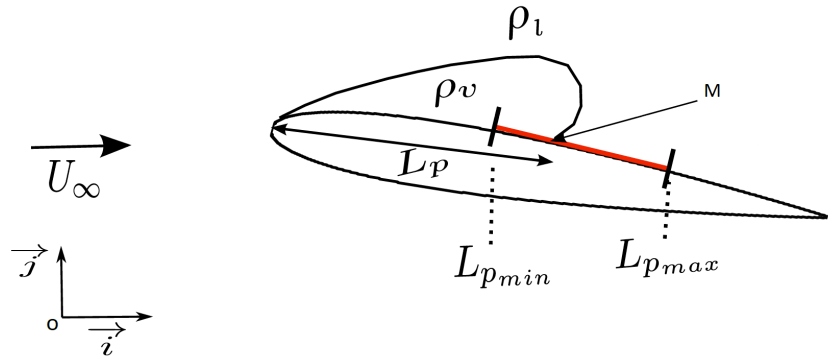


Figure 3. Modeling of sheet cavitation. M oscillation belongs to $[L_{p_{min}}, L_{p_{max}}]$.

99 4. Non-homogeneous density model

Modeling of the density variation is made in order to approximate the sheet cavitation behavior on the hydrofoil, as described in the literature [10,14,15] and shown in Figure 17. Sheet cavitation is characterized by unsteady behavior of cavity length L_p at the hydrofoil upper surface (Figure 3). Attached at the leading edge, the cavity extends on the upper surface and oscillates between the minimum length cavity ($L_{p_{min}}$) to the maximum length ($L_{p_{max}}$). Inside the cavity, the vapor density ρ_v is equal to $1\text{kg}/\text{m}^3$. Outside the cavity, the hydrofoil is surrounded by liquid (water) with the density ρ_l equal to $1000\text{Kg}/\text{m}^3$. At the interface Γ_{FS} , the density ρ is given by

$$\rho(x, y, t) = \begin{cases} \rho_v = 1 \text{ kg}/\text{m}^3 & \text{if } 0 \leq x < L_p(t) & (a) \\ \rho_l = 1000 \text{ kg}/\text{m}^3 & \text{if } L_p(t) \leq x < L_{p_{max}} & (b) \end{cases} \quad (18)$$

and the variation rate of the density is given by

$$\begin{cases} \frac{\partial \rho}{\partial t} = (\rho_v - \rho_l) \delta(x - L_p(t)) \frac{\partial L_p(t)}{\partial t}, \\ 0 \leq x \leq c, \quad (x, y) \in \Gamma_{FS} \quad \text{and} \quad L_{p_{min}} \leq L_p(t) \leq L_{p_{max}} \end{cases} \quad (19)$$

100 where δ is a Dirac function.

101 There are different development phases of sheet cavitation. Firstly, the closing point M
 102 (Figure 3) has a small variation and the cavity could be considered as a steady. Secondly,
 103 the cavity length L_p increases and the closing point oscillates between $L_{p_{min}}$ and $L_{p_{max}}$.
 104 The cavitation development phases may continue to the destabilization of the cavity,
 105 followed by a vapor cloud detachment [16,17]. In this paper we focus on the first two
 106 phases. During the second phase, the cavity length follows a periodic variation [14,18].
 Let us consider the following simplified model of unsteady cavity

$$L_p(t) = L_{p_{min}} + \frac{L_{p_{max}} - L_{p_{min}}}{2} (1 - \cos(2\pi f_c t)) \quad (20)$$

where f_c is the oscillation frequency of the closing point M . The variation rate of cavity length is given by

$$\frac{dL_p}{dt} = (L_{p_{max}} - L_{p_{min}}) \pi f_c \sin(2\pi f_c t) \quad (21)$$

107 5. Numerical resolution

108 A Stainless Hydrofoil (NACA0012) with mass m equal to $14.505\text{Kg}\cdot\text{m}^{-1}$ is consid-
 109 ered. Its natural frequency f_N in the air is 58.52Hz and the chord length c is equal to
 110 0.15m . The stiffness is deduced from the previous values.

Newmark scheme [19,20] presented in Equation (22) is used to discretize the structure dynamics Equation (17). The latter is given by :

$$\begin{cases} \zeta_y^n = \zeta_y^{n-1} + \Delta t \dot{\zeta}_y^{n-1} + \frac{\Delta t^2}{4} (\ddot{\zeta}_y^{n-1} + \ddot{\zeta}_y^n) & (a) \\ \dot{\zeta}_y^n = \dot{\zeta}_y^{n-1} + \frac{\Delta t}{2} (\ddot{\zeta}_y^{n-1} + \ddot{\zeta}_y^n) & (b) \end{cases} \quad (22)$$

111 where Δt is the time step and ζ_y^n is the value of the displacement ζ_y at time $t_n = n\Delta t$.
 112 In this study, the time step is taken equal to $10^{-3}s$, which is a good time sampling of
 113 both the hydrofoil harmonic displacements (with a period of about $17 \cdot 10^{-3}s$) and the
 114 harmonic variations of the cavity (with a period of about $44 \cdot 10^{-3}s$).

115 The problem (fluid and structure) is solved by using the finite elements code CASTEM
 116 [21]. Triangular quadratic elements are used. The computational domain is subdivided
 117 to 34360 elements, which is corresponding to 131720 nodes. As shown in Figure 4, the
 118 mesh of the subdomain around the hydrofoil is refined in order to improve the accuracy
 119 of the numerical results.

120

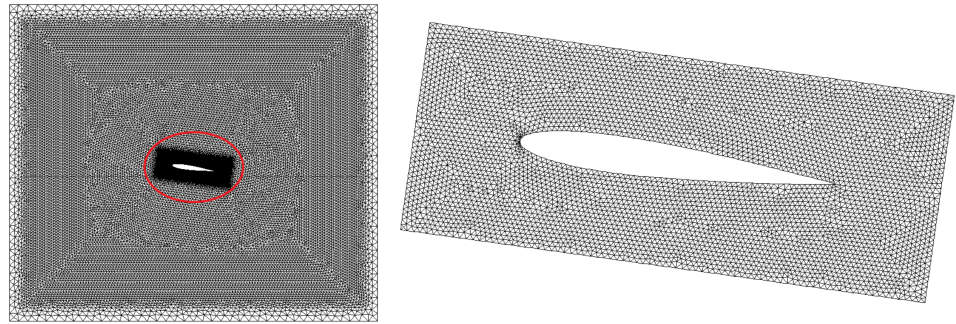


Figure 4. Computational domain and mesh : 131720 elements and 34360 nodes (left). Mesh subdomain around the hydrofoil (right)

121 The same mesh sensitivity study performed in [11] is used here. The Figure 5 show the
 122 mesh dependence of the numerical added mass value obtained by solving the equation
 123 14. The chosen mesh corresponds to a relative error of about 1.56%, compared to the
 124 analytical value of the added mass obtained for a rectangle of the same dimensions as
 125 the used hydrofoil [11].

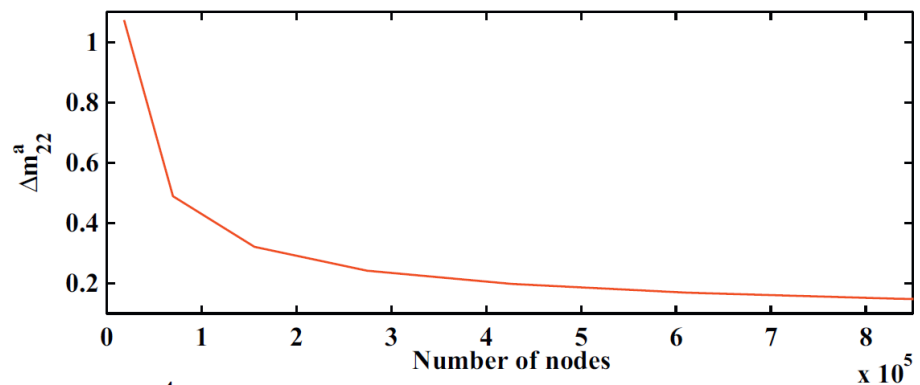


Figure 5. Mesh sensitivity [11]

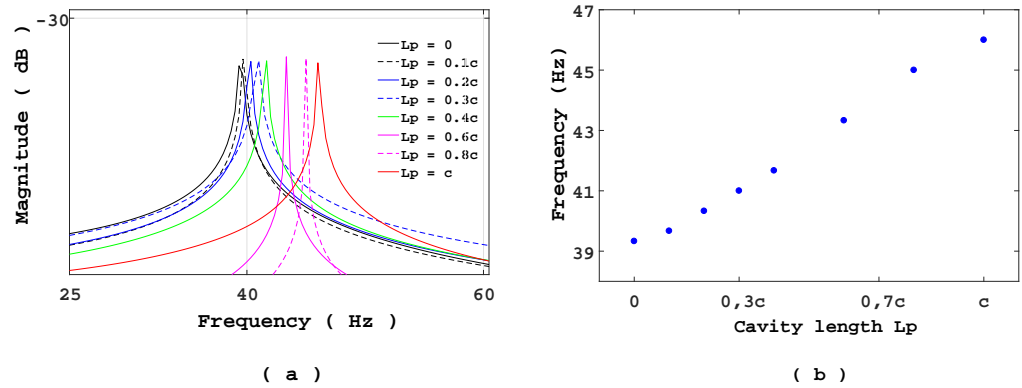
126 5.1. Steady cavity length

127 Steady cavity length is firstly studied in order to understand the effect of cavity
 128 length L_p on the structure dynamics. In this case, the cavity length L_p is considered as
 129 constant. Hence, the fluid density is only space dependent and its variation rate is zero.

130 Hence, $\frac{\partial \rho}{\partial t}$ and p'_2 are equal to zero. It follows that the added mass m_a is constant and the
 131 added damping d_a is zero.

132

133 The simulation is performed for one value of steady L_p equal to $0.4c$. Only Equations (
 134 14) and (17) are solved for the coupled problem. It follows that the induced movement
 135 ζ_y of the hydrofoil is periodic. Thus, it can be defined by the induced frequency f_I and
 136 the corresponding amplitude. The induced frequency f_I of the structure oscillations into
 the fluid flow can be obtained by Fast Fourier Transform (FFT).



137 **Figure 6.** (a) Spectrum of hydrofoil movement ζ_y for different values of L_p . (b) Frequency of ζ_y versus L_p

137 The same study was reproduced for different values of L_p ($L_p = 0$ to $L_p = c$). The induced frequencies versus cavity length are presented in Figure 6. It can be shown that the value of L_p does not affect the amplitude of the hydrofoil displacement (Figure 6-a). However, the frequency increases according to the cavity length (Figure 6-b). This is expected because the surface covered by vapor expands as L_p increases. Furthermore, the added damping d_a is zero because of the steady cavity length. So the induced frequency f_I can be deduced from Equation (17) as following :

$$f_I = \frac{1}{2\pi} \sqrt{\frac{k_y}{m + m_a}} \quad (23)$$

The frequency f_I can be approximated by using the formula (24) given in [22]

$$\frac{f_I}{f_N} = \frac{1}{\left(1 + \frac{m_a}{m}\right)^{\frac{1}{2}}} \quad (24)$$

138 5.2. Unsteady cavity length

139 The simulation of the coupled problem is now performed with unsteady cav-
 140 ity length. The same values of mass, natural frequency (f_N) and chord length used
 141 previously are applied. Equation (20) is applied for cavity length oscillation; where
 142 $L_{p_{max}} = 0.4c$, $L_{p_{min}} = 0$ and $f_c = 22.5\text{Hz}$. The value of f_c is chosen to be close to the
 143 experimental observation [10].

144

145 Solutions p'_1 of Equation (14) in a fluid domain at three different moments, corresponding
 146 respectively to $L_p \approx L_{p_{min}}$, $L_p \approx \frac{L_{p_{max}}}{2}$ and $L_p \approx L_{p_{max}}$, are shown in figure 7 (left). It is
 147 easy to see that the values of p'_1 at the upper surface are smaller than those of the lower
 148 surface. Indeed, p'_1 is proportional to the fluid density and the hydrofoil is surrounded
 149 by the vapor at the upper surface.

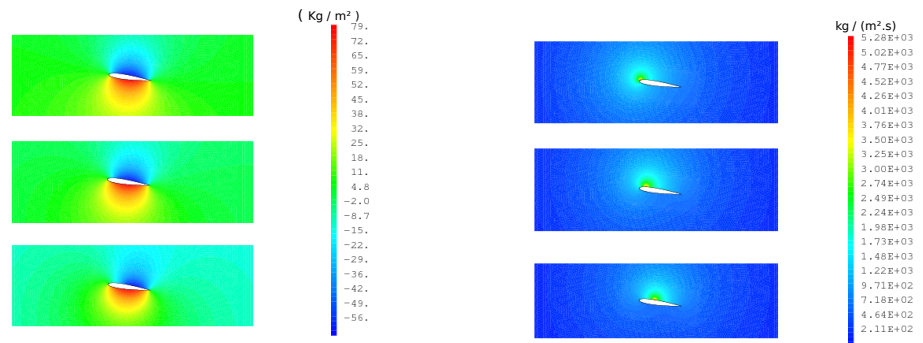


Figure 7. Solutions p'_1 (left) and p'_2 (right) for three times corresponding respectively to $L_p \approx L_{p_{min}}$, $L_p \approx \frac{L_{p_{max}}}{2}$ and $L_p \approx L_{p_{max}}$.

150 Values of p'_2 at the same three moments, corresponding to the three values of L_p , are
 151 shown in Figure 7 (right). High values match with the closure points M where the
 152 density changes. Indeed, p'_2 is proportional to the variations rate of the density as
 153 shown in Equations (15) and (19), and that formulation includes Dirac function. **So,**
the variations of L_p can be observed within the solution p'_2 . It follows that the added

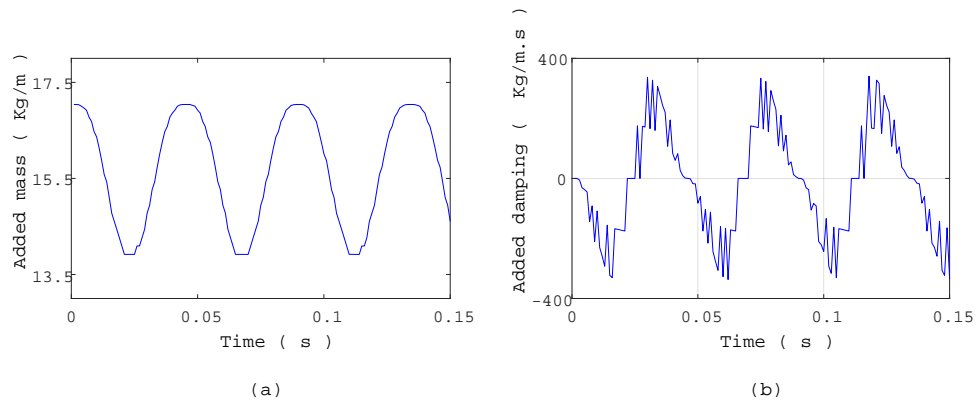


Figure 8. (a) Added mass variation versus time. (b) Added damping variation versus time

154 mass m_a is time dependent. Its variation are shown in Figure 8-a. It oscillates between
 155 13.92 Kg.m⁻¹ and 17.04 Kg.m⁻¹. These values correspond respectively to values of $L_{p_{max}}$
 156 and $L_{p_{min}}$. The maximum value corresponds to that obtained in the homogeneous fluid
 157 case. Hence, it is assumed that the added mass variations is periodic and has the same
 158 frequency as the cavity length variation. It can be conclude that a frequency modulation
 159 of the structure is expected in this case.

160 The added damping variation is shown in Figure 8-b. It is periodic with frequency equal
 161 to f_c and it can take negative values. It may cause structure instabilities or amplitude
 162 modulation.
 163

164
 165 Hydrofoil motions in both homogeneous and non-homogeneous cases are shown in
 166 Figure 9-a. The dynamics of the structure are modified by the cavity length oscillation
 167 and the phase shift between the two motions increases over time. The spectrum analysis
 168 obtained by FFT shows one fundamental frequency centered between two harmonics
 169 (Figure 9-b). These harmonics specifically characterize an amplitude modulation. The
 170 study is reproduced for different values of maximum cavity length ($L_{p_{max}}$). It is shown

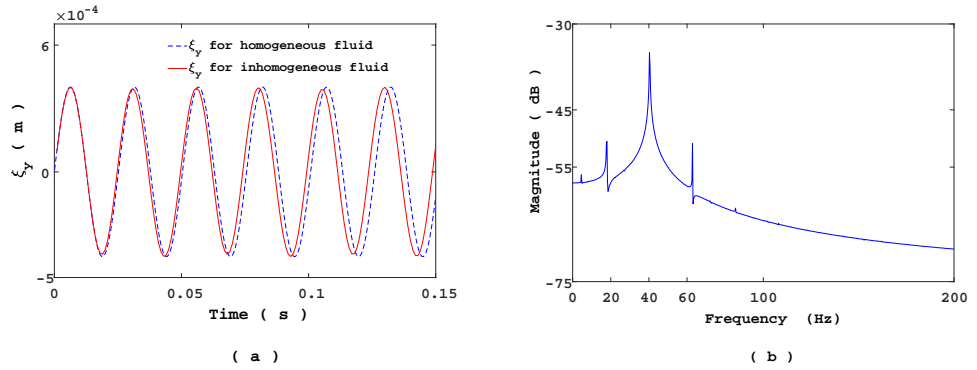


Figure 9. (a) Comparison of hydrofoil displacements in homogeneous and non-homogeneous cases. (b) Frequency of hydrofoil displacements in non-homogeneous case.

171 that the frequency spectrum are still composed by the fundamental frequency and the
 172 two harmonics (Table 1). It is noted that the fundamental frequency increases with
 173 the cavity length. Variation of the added damping from positive to negative sign and
 174 vice versa is observed. This can induce an amplitude modulation of the hydrofoil
 175 displacements.

Table 1. Frequencies spectrum of the hydrofoil motion for different $L_{p_{max}}$

$L_{p_{max}}$	1 st Harmonic (Hz)	Fundamental (Hz)	2 nd Harmonic (Hz)
0.2c	17.54	39.47	61.4
0.6c	19.37	41.16	62.95
0.8c	19.22	41.26	63.61
c	17.47	43.67	69.87

176 5.3. Frequency analysis

177 In the previous section, the natural frequency f_N in air, the cavity length $L_{p_{max}}$ and
 178 the cavity length frequency f_c were fixed to be close to the experimental observation.
 179 However, in this case the effects of variations in added mass and added damping on the
 180 hydrofoil dynamics are difficult to highlight. Indeed, for one period of the hydrofoil
 181 oscillation (ξ_y), the cavity length changes from 0 to $\approx L_{p_{max}}/2.6$ and at the same time
 182 added mass and added damping vary with the same frequency as the cavity.

183 So, in order to highlight these effects, a smaller cavity length frequency $f_c = 11$ Hz and
 184 a larger cavity length $L_{p_{max}} = 0.8c$ are used. It consists of increasing the gap between
 185 f_N and f_c . The values of c , m , f_N and $L_{p_{min}}$ are the same as in previous section. The
 186 solution of the coupled problem (Equations (14), (15) and (17)) is shown in Figure 10-a.
 187 An extended analysis performed over a long period of time is reported in Figure 10-b.
 188 Upper and lower envelopes of the signal ξ_y is represented in black curve. This represents
 189 the amplitude modulation of the structure dynamics.

190
 191 In order to highlight the expected frequency modulation of the structure dynamics,
 192 a spectrogram analysis of the signal ξ_y is performed. However, an accuracy on the
 193 frequency induces automatically a less clearly observable frequency time variation, as
 194 shown by Figure 11. This corresponds to the best spectrogram obtained, according to
 195 the characteristics presented in Table 2. Indeed, a frequency range is observed and it
 196 oscillates with a frequency close to the cavity length variations one. Thus, the classical
 197 frequency analysis methods can not take into account the frequency modulation phe-
 198 nomena. Hence, application of EMD method followed by the Hilbert spectral analysis
 199

200 are used for the estimation of the instantaneous frequency (IF).
 201

Table 2. Spectrogram parameters.

Block length	Frequency discretization	Time lapse between blocks	Sampling frequency (Hz)
64	1024	8	1000

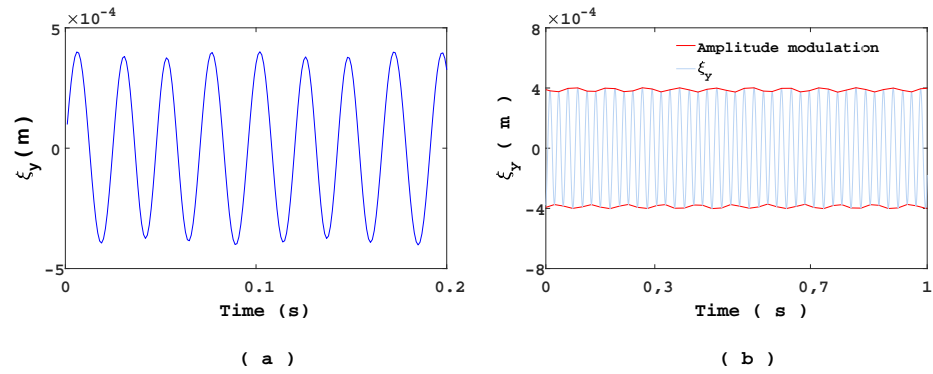


Figure 10. (a) Heave displacement ξ_y of the hydrofoil, $t \in [0, 0.2]$. (b) Heave displacement ξ_y of the hydrofoil over a long period time, $t \in [0, 1]$.

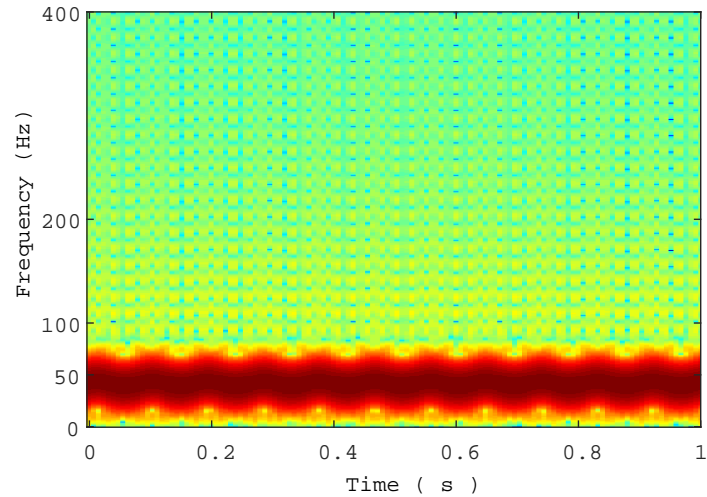


Figure 11. Spectrogram of $\xi_y(t)$ in non-homogeneous case.

202 5.3.1. Empirical Mode Decomposition

203 EMD, introduced by Huang et al., is an adaptive and data-driven decomposition well
 204 suited to decompose non-stationary signals derived or not from linear systems [12].
 205 More precisely, no *a priori* basis functions are required for the decomposition. The
 206 algorithm decomposes the multi-component signal into a linear combination of set of
 207 reduced number of additive oscillatory components termed as IMFs (Intrinsic Mode

208 Functions). Each extracted IMF, a mono-component signal, must satisfy the following
209 conditions:

- 210 (i) The number of local extrema and the number of zero-crossings must either equal
211 or differ at most by one.
212 (ii) The local trend value (mean) of the envelope defined by local maxima and the
213 envelope defined by the local minima is zero

This requirement ensures that the IMFs have no positive local minima and no negative maxima [12]. Furthermore, these conditions allow us to obtain physically meaningful IF estimates from the extracted IMFs. The core of the EMD is called the sifting process and the resulting adaptive expansion can be seen as a type of wavelet decomposition, whose sub-bands are built up as needed to separate the different components of the signal. To be successfully decomposed into IMFs, a signal $s(t)$ must have at least two extrema: one minimum and one maximum [12],[23]. At the end of the sifting, the signal $s(t)$ can be expanded as the sum of mode time series $\text{IMF}_i(t)$ and a residual $r_K(t)$:

$$s(t) = \sum_{i=1}^K \text{IMF}_i(t) + r_K(t) \quad (25)$$

214 where K is the number of modes determined automatically. Based on a dyadic filter bank
215 conjecture of the EMD algorithm, the number of sifted modes K is usually limited to
216 $K \leq \log_2(L)$, where L is the number of samples of the signal $s(t)$ [24]. The signal $r_K(t)$,
217 called residual, is a monotonic function that represents the trend within $s(t)$.

218 5.3.2. Hilbert spectral analysis

219 With the extracted modes $\text{IMF}_i(t)$, Hilbert spectral analysis can be applied to each
220 mode in order to estimate the associated IF $f_i(t)$. To compute the IF, the analytic signal
221 (also called Gabor's complex signal) associated to a real signal $\text{IMF}_i(t)$ is calculated, as
222 follows

$$\begin{aligned} z_i(t) &= \text{IMF}_i(t) + j\mathcal{H}[\text{IMF}_i(t)] \\ &= a_i(t)e^{j\phi_i(t)} \end{aligned} \quad (26)$$

where $a_i(t)$ and $\phi_i(t)$ are the instantaneous amplitude and phase of $\text{IMF}_i(t)$. $\mathcal{H}[\text{IMF}_i(t)]$ is the Hilbert transform of $\text{IMF}_i(t)$ and it is given by

$$\mathcal{H}[\text{IMF}_i(t)] = \frac{1}{\pi} \text{PV} \int_{-\infty}^{+\infty} \frac{\text{IMF}_i(\tau)}{t - \tau} d\tau \quad (27)$$

223 where PV is the Cauchy principal value of the integral. Finally, the IF $f_i(t)$ of $\text{IMF}_i(t)$ is
224 calculated as follows [25]:

$$\begin{aligned} f_i(t) &= \frac{1}{2\pi} \frac{d\phi_i(t)}{dt} \\ \phi_i(t) &= \tan^{-1} \left(\frac{\mathcal{H}[\text{IMF}_i(t)]}{\text{IMF}_i(t)} \right) \end{aligned} \quad (28)$$

225 5.3.3. IMFs and IFs of the signal ζ_y

EMD is applied to the signal given by the hydrofoil displacement ζ_y and ten IMFs are extracted (Figures 12 and 13). Following EMD definition,

$$\zeta_y(t) = \sum_{i=1}^{10} \text{IMF}_i(t) + r_{10}(t) \quad (29)$$

226 Here K is set to 10. In our case, two classes of IMFs can be defined: the high frequency
227 class composed by the three first modes and the low frequency composed by the re-

228 maining modes. Note that the first mode, $IMF_1(t)$, corresponds to the highest frequency
 229 component of the signal. In our case, it has the highest amplitude for the high frequency
 230 class. Zoom of the signal is shown in Figure 14-a. Overall, the hydrofoil movement is
 231 mainly composed by $IMF_1(t)$ and the remaining low frequency mode.

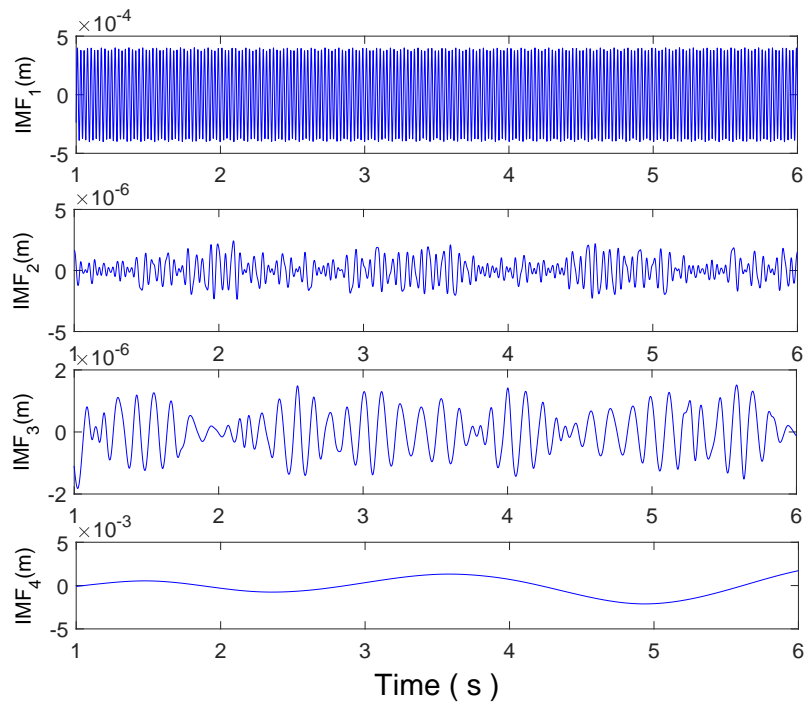


Figure 12. High frequency class: IMF_1 , IMF_2 , IMF_3 and low frequency class IMF_4 extracted from $\tilde{\zeta}_y$.

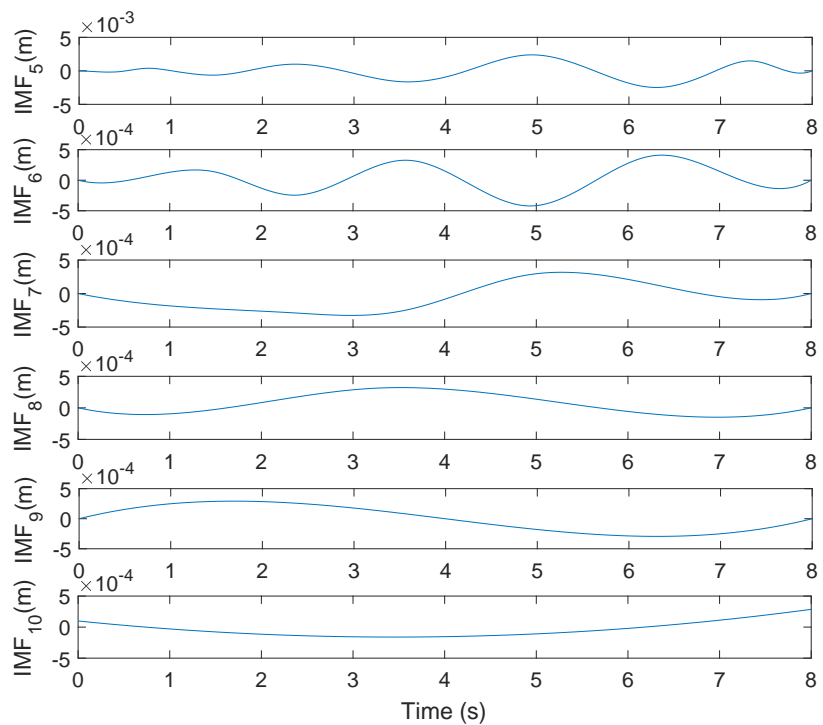


Figure 13. Low frequency class: IMF_5 to IMF_{10} extracted from $\tilde{\zeta}_y(t)$.

232 Hilbert spectral analysis of the modes has been performed. The IF $f_1(t)$ of the first mode
 233 ($IMF_1(t)$), is shown in Figure 14-b. The frequency modulation is explicitly shown. These
 234 oscillations are attributed to the variation of the cavity length. Both variations ($f_1(t)$ and
 235 L_p) have the same frequency. The component $f_1(t)$ oscillates from 39.38 Hz to 44.98 Hz
 236 except at the beginning of the simulation.

237
 238 The IFs of the modes 2 and 3 are shown in Figure 15. They show many peaks (or spikes)
 239 which are similar to Dirac functions. If the peaks are omitted, complex oscillation of the
 240 IFs $f_2(t)$ and $f_3(t)$ are observed. For the low frequency class, the average of IF variations
 241 is in the order of 10^{-2} Hz (Figure 16). It can be concluded that the frequency modulations
 242 of the signal ζ_y come from the first three IMFs.

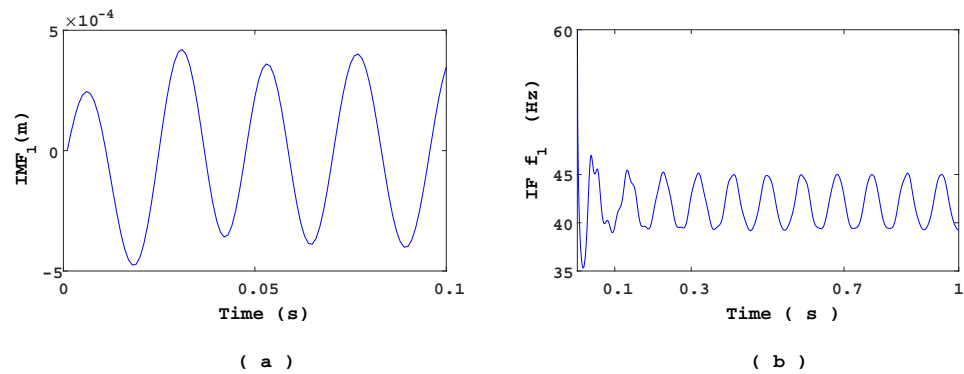


Figure 14. (a) $IMF_1(t)$ mode. (b) IF $f_1(t)$.

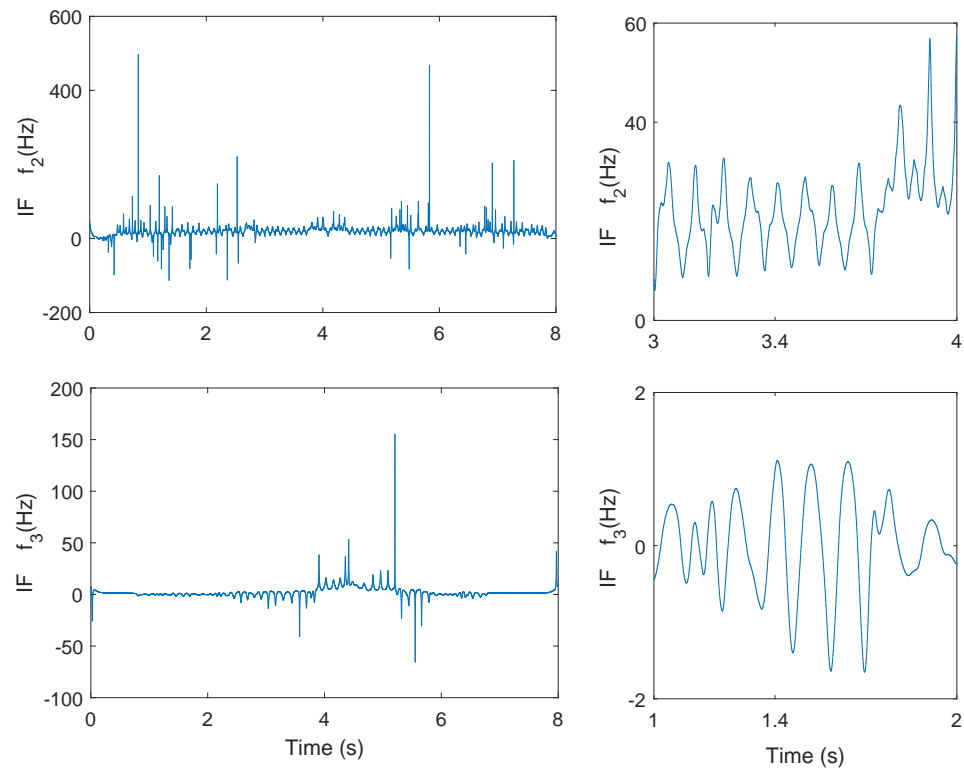


Figure 15. IFs $f_2(t)$ and $f_3(t)$ (left), zoom of IFs $f_2(t)$ and $f_3(t)$ (right).

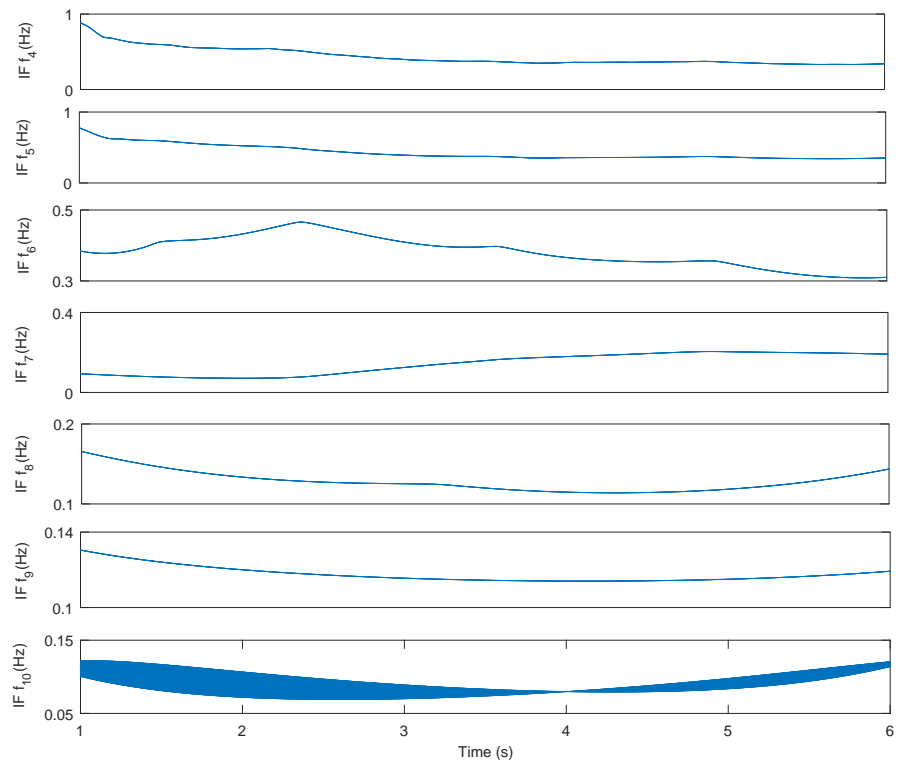


Figure 16. IFs from $f_4(t)$ to $f_{10}(t)$.

243 Some experimental evidence can be found in [15] from experiments conducted
 244 on cavitation induced vibration, performed on a hydrofoil in a hydrodynamic tunnel.
 245 Typical vibration spectra and the corresponding cavity snapshots on the suction side
 246 are shown on Figure 17 for various cavity lengths, according to the cavitation number
 247 σ obtained on a hydrofoil. The smaller σ , the larger the maximum cavity length. σ
 248 is defined as $(P_o - P_v) / (0.5\rho U_\infty^2)$, where P_o is the pressure in the test section and P_v is the
 249 vapor pressure [4,10,15,26].

250 On Figure 17.a, $\sigma = 2.42$ corresponds to cavitation inception with small spots of
 251 vapor attached to the leading edge (bottom of the picture). The corresponding vibration
 252 spectrum exhibits a rather large peak corresponding to the structural bending mode. For
 253 $\sigma = 2.08$, a sheet cavitation was attached at the leading edge and oscillated periodically
 254 between about 30% and 40% of the chord length. This leads to an increase of the vibration
 255 level over several peaks ranging from about 25Hz slightly below the bending structural
 256 mode frequency at 32Hz up to the cavity frequency close to 65Hz. That is the sign of a
 257 complex response including frequency modulation probably. As the cavitation number
 258 decreases again (Figure 17.b, $\sigma = 1.81$), the maximum cavity length increases up to 60%
 259 of the chord length and oscillates at about 35Hz close to the structural frequency. By
 260 decreasing again the cavitation number ($\sigma = 1.63$), the cavity frequency and the bending
 261 frequency merge inducing a strong coupling resulting in a very high level resonant peak
 262 of vibration at the bending/cavity frequency and harmonics.

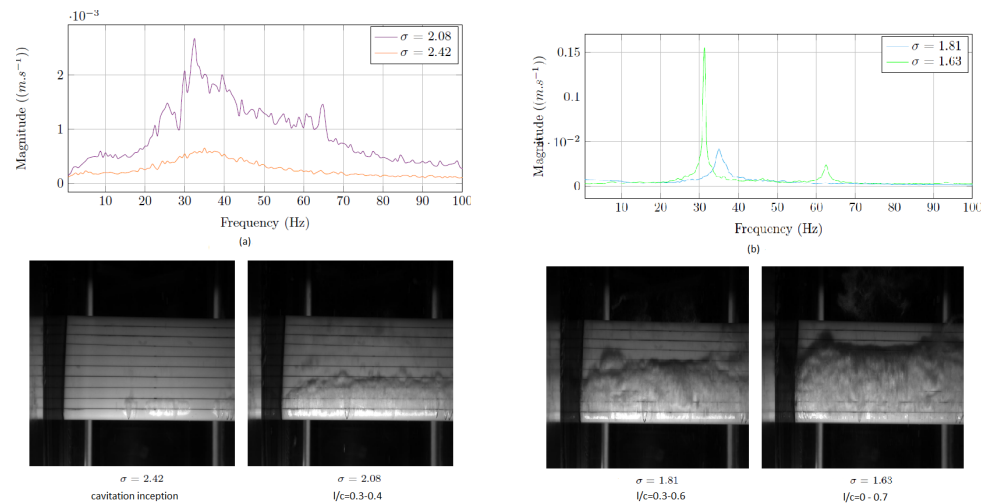


Figure 17. Vibration spectra in cavitating flow and corresponding cavity snapshots on a hydrofoil for various cavitation number ($\theta = 8$, $U_\infty = 6$ m/s) [15].

263 6. Conclusions

264 The effect of the fluid density variations, at the fluid-structure interface, on the
 265 structure dynamics is studied and analysed. A decomposition method is used to linearize
 266 the fluid-structure coupled problem, which is separated into two components. The first
 267 one describes the fluid flow around the fixed hydrofoil while the second one is related
 268 to the flow induced by the structure vibrations. A model of the fluid density variation
 269 along the upper interface of the hydrofoil, based on the sheet cavitation behaviour, is
 270 used. The governing equations are solved numerically using Finite Element Method. In
 271 this study, the hydrofoil is considered to be animated by a free heave motion. For steady
 272 cavity length, the added mass remains constant and the added damping (induced by
 273 the fluid density rate of change) is zero. The study was reproduced for different values
 274 of cavity length. It was highlighted that the frequency increases according to the cavity
 275 length. However, the amplitude of the displacement is kept at the same value.

276 For unsteady cavity length, its oscillations along the fluid-structure interface in-
 277 duces variations in the added mass values. In addition, the fluid density rate of change
 278 generates a fluid load acting as an added damping on the structure dynamics, which
 279 can be negative and thus at the origin of instabilities of the structure. Although clas-
 280 sical methods, such as spectral analysis, make it possible to highlight both amplitude
 281 modulation (AM) and frequency modulation (FM) phenomena, in structural dynamics
 282 requires the use of suitable tools to handle such AM-FM signals. Thus, empirical mode
 283 decomposition (EMD) method, well suited to analyse AM-FM components, was applied
 284 to the signal obtained from the hydrofoil displacement. Such a decomposition makes it
 285 possible to obtain the instantaneous frequencies (IFs) of the signal from the extracted
 286 Intrinsic Mode Functions (IMFs). Therefore, FM is explicitly given through the time
 287 variations of the frequency, obtained from EMD method. It is shown that the IF derived
 288 from the first IMF, sifted by EMD decomposition of the hydrofoil displacement signal ζ_y ,
 289 corresponds to the cavity frequency.

290 This signal processing method allows us to highlight the FM phenomenon which
 291 occurs in the dynamics of a structure immersed in a fluid flow with unsteady non-
 292 homogeneous density. In this study, only the effects of the added mass and added
 293 damping (induced by the fluid density rate of change) on the structure dynamics are
 294 analysed. As future work, we plan to extend this study in order to investigate the
 295 potential of the EMD method in this case, by analysing the information and the related
 296 physics, which could be extracted from all the sifted IMFs and the associated IFs.

297 7. Author Contributions

298 Conceptualization, T. Emerry RAJAOMAZAVA III and Mustapha BENAOUICHA;
 299 Formal analysis, T. Emerry RAJAOMAZAVA III and Abdel-Ouahab BOUDRAA; Investi-
 300 gation, T. Emerry RAJAOMAZAVA III; Methodology, Mustapha BENAOUICHA and
 301 Abdel-Ouahab BOUDRAA; Project administration, Jacques-André ASTOLFI; Software,
 302 T. Emerry RAJAOMAZAVA III; Supervision, Mustapha BENAOUICHA and Jacques-
 303 André ASTOLFI; Validation, Mustapha BENAOUICHA, Jacques-André ASTOLFI and
 304 Abdel-Ouahab BOUDRAA; Visualization, T. Emerry RAJAOMAZAVA III; Writing –
 305 review editing, Mustapha BENAOUICHA, Jacques-André ASTOLFI and Abdel-Ouahab
 306 BOUDRAA.

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