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## A Framework for Integration of Resource Allocation and Reworking Concept into Design Optimisation Problem

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**Abstract:** The life cycle of an assembled product faces various uncertainties considering the current state of the manufacturing line. Varied of activities are integrated with the manufacturing line including processing, inspection, reworking, assembly, etc. Therefore, any decision taken concerning each activity, will affect the end-product of the manufacturing line. In an early stage, designers define tolerances on parts to ensure the functionality of the end-product. In this regard, this paper integrates resource allocation (as a decision to assign practical resources to parts) and reworking decision (as a decision to improve parts conformity rate) into the tolerance allocation problem. A modular-based cost modelling approach is proposed objecting to minimisation of manufacturing cost concerning resource allocation and reworking decisions. Eventually, a genetic algorithm and Monte-Carlo simulation are adapted to analyse the applicability of the model.

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**Keywords:** Adaptive tolerancing, Tolerance allocation, Resource allocation, Design optimisation

### 1. INTRODUCTION

The need for highly reliable products has broadened the scope of manufacturing technologies. Despite the introduction of new technologies in the industry, geometrical deviations are inevitable. A manufactured part cannot reach exactly the nominal dimensions due to inherent variability in the manufacturing process (Kumar et al., 2009). The geometrical deviations are consequences of uncertainties occurring during the product life cycle. The life cycle of an assembled product includes various activities, e.g., processing, inspection, rework, assembly, inventory, etc. Consequently, the need for reliable and precise parts has impacted the development of tolerancing. Tolerance is an essential part of the design stage and the ubiquitousness of tolerances entails the various stages of a product life cycle.

To determine the effects of tolerance on the system functionality, tolerancing classifies into two distinct categories: tolerance analysis, and tolerance allocation. Tolerance analysis is a method to verify the functionality of a design after tolerances have been specified on each part. On the other hand, tolerance allocation involves the assignment and the distribution of the values of adequate tolerances, therefore, it is the inverse problem of tolerance analysis (Morse et al., 2018). Commonly used tolerance allocation methods are based on specific rules of thumb for the distribution of tolerances such as equal tolerances assumption, same influence, proportional scaling, constant precision factor (Drake, 1999). Then, the necessity for an appropriate holistic methodology for tolerance allocation, taking productions strategies, and the functional fulfilment degree of parts with manufacturing and assembly deviations has emerged.

On these bases, this research aims at developing a methodology for simulation-based optimisation under uncertainties of product tolerances and production strategies such as resource allocation and reworking. Within the research, a new holistic approach of tolerance allocation will be developed, assessing both technical and economic aspects of the product. This research is structured as follows: section 2 provides a state of the art on concurrent tolerance allocation problem. Section 3 describes concurrent tolerance allocation and supports a theoretical model. In Section 4, an overrunning clutch mechanism is illustrated, and the results of the proposed model are analysed. Finally, Section 5 summarises this paper and provides an outlook on prospects.

### 2. STATE OF THE ART

#### 2.1 Tolerance allocation

Tolerance allocation is a key issue in design which distributes values of adequate tolerances through parts of an assembled product while the functionality is ensured. It concerns both quantitative and qualitative aspects in tolerancing and is closely related to design and manufacturing activities. A well-distributed tolerance plan which anticipates manufacturing uncertainties and counteracts risks at the design level can enhance the product life cycle (Morse et al., 2018, Hallmann et al., 2020a). In this context, several research studies have been carried out to perform tolerance allocation. However, the literature can be distinguished into two main categories:

1. The first category is based on rules of thumb and consists in equally distributing tolerance through an assembled product while functionality is ensured (Drake, 1999, Ji et al., 2000)



2. The second category is more complicated and relies on the manufacturing capabilities and production strategies (Jing et al., 2020, Wang et al., 2020, He et al., 2020, Etienne et al., 2017).

The importance of designing complicated engineering products incentivizes this research to study concurrent tolerance allocation.

## 2.2 Concurrent tolerance allocation

Extensive research on the integration of resource capability and the introduction of machine selection into tolerance allocation can be found in the literature. The integration of tolerance allocation and resource allocation on one hand brings two essential challenges in advanced tolerance design. However, on the other hand, it arouses an incontrovertible challenge. The common cost-tolerance models in the literature are parametric models whom structures vary from linear to non-linear (Chase et al., 1990). For instance, several types of manufacturing cost models can find respectively, reciprocal power function (RP) (Sutherland and Roth, 1975), Cubic Polynomial (Cubic-P) (Dong et al., 1994), also, Hybrid models which are adopted from conventional cost models (Dong et al., 1994).

The cost model development relies on an extensive individual study of existing manufacturing resources and tolerance variation sensitive analysis to yield an appropriate cost-tolerance model (Hallmann et al., 2020b, Saravanan et al., 2020, Wu et al., 2021). Tsutsumi et al. (2020) integrated product design, process planning and production planning optimization in multi-product assembly assessing the investment efficiency and reduce the overall production cost. Armillotta (2020) provided a comprehensive review of the parametric cost-tolerance functions and investigated the models' inconsistencies due to parameters variabilities. An alternative to parametric modelling can be seen in Etienne et al. (2009) where authors proposed an activity-based cost modelling. Its main objective is to rationally provide an accurate indicator of the relevance of tolerances values fixed by designers. This model associates the impacts of tolerance allocation on the manufacturing process as well as the production cost.

The impact of reworking in the economy was studied in (Ferrer and Ayres, 2000) which was introduced as a process to repair or substitute parts that are worn out or obsolete. The observations reported a significant reduction in the level of inter-industry transaction, as well as improvement in the manufacturing cost. The traces of reworking in the context of tolerancing can be seen in Lee et al. (2000) where authors proposed a cost-effective means for tolerance allocation. Authors compounded the probabilities of scrap and rework to obtain the expected loss cost. Additionally, Shin and Cho (2007) addressed the reworking concept providing a mean to balance the quality and manufacturing costs.

## 3. CONCURRENT TOLERANCE ALLOCATION DESCRIPTION, FORMULATION, AND OPTIMISATION

At all stages of product development and throughout the product life cycle, uncertainty is ubiquitous and incurs. The risk can impact the product performance(s), process

scheduling, market acceptance, or the whole business. Therefore, a comprehensive engineering design plan which includes key functions of the product using tolerance analysis techniques and mitigating the uncertainties within manufacturing activities to reduce their effects and ensure product functioning is a necessity to the manufacturer risk (as illustrated in Fig. 1). To explain more in detail, each production strategy is associated with consequences in the life cycle of the product and can be clarified as follow:

1. Resource allocation: a tool to assign available practical resources to parts to increase manufacturing line efficiency.
2. Reworking decision: a decision to improve parts conformity rate and decrease the number of scraps.

In the following section, concurrent tolerance allocation problem concerning resource allocation and reworking decision is studied. The section is divided into three main sub-sections. The first section explains statistical definitions of the problem linking production strategies in the context of conformity probabilities. Next, the conformity probabilities are used to formulate manufacturing cost. The last part represents a simulation-based genetic algorithm minimising manufacturing cost developed. The nomenclatures used in this paper are given as follow:

Parameters:	
$N$	Set of parts
$O$	Set of operation
$X_i$	Geometrical deviation on part $i$
$Y$	Functional requirement
$\sigma_{i,j}$	Process deviation of operation $j$ for part $i$
$\sigma_y$	Assembled product deviation
$t_y$	Assembled product tolerance
$\alpha$	Type I failure rate
$\beta$	Type II failure rate
$\gamma \cdot (1 - \alpha)$	Percentage of marketable conformed products
$(1 - \gamma) \cdot \beta$	Percentage of marketable non-conformed
$(1 - \gamma) \cdot (1 - \beta)$	Percentage of detected non-conformed
$\gamma \cdot \alpha$	Percentage of undetected non-conformed
Decision variables:	
$a_{i,j}$	1 if resource $j$ is allocated to the part $i$ , O.W., 0
$rw$	1 if reworking decision is taken, O.W., 0
$t_i$	Allocated tolerance to part $i$
$\gamma_i$	Part $i$ conformity ratio
$\gamma_i^{RW}$	Part $i$ reworking rate
$\gamma_i'$	Part $i$ conformity rate after reworking
$\lambda$	Assembled product conformity ratio
$C_{Total}$	Manufacturing cost

### 3.1 Statistical definition of the concurrent problem

The tolerance of a part can be defined as the permissible variation in measurements deriving from the nominal value. It can be expressed as follow:



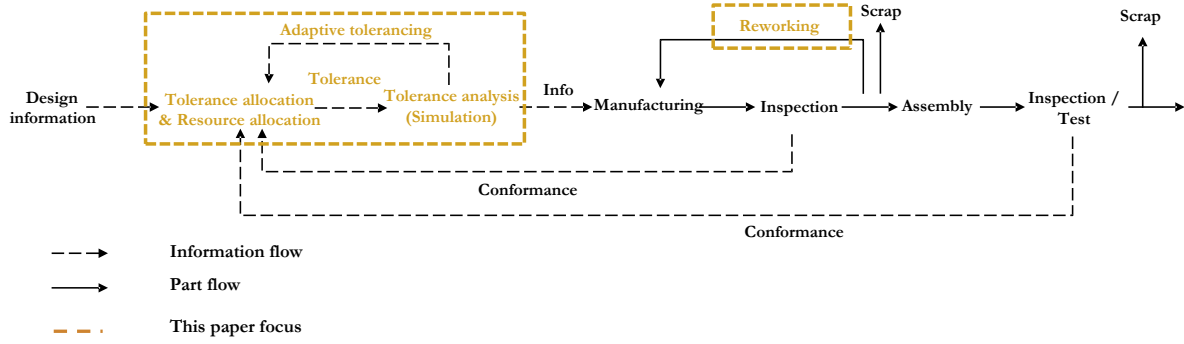


Fig 1. Identification of tolerance's role in an assembled product life

$$t_i = USL_i - \mu_i = \mu_i - LSL_i \quad \forall i \in N \quad (1)$$

where USL and LSL express upper and lower specification limits. Moreover,  $\mu$  denotes dimensional nominal value. This paper supports a statistical-based approach integrating resource allocation and reworking decision into tolerance allocation problem. Within this approach, the consequences of the decisions are associated with probability rates. Therefore, to go further, the model follows several assumptions, also, it is illustrated in Fig. 2:

- (1) The tolerance allocation problem is defined by the dimensional tolerancing of a designed part.
- (2) A generic form of conformity rate estimator is developed based on the normal distribution.
- (3) Dimensions are independent, therefore, the sole dependency in this model is between parts tolerances and functional requirement.

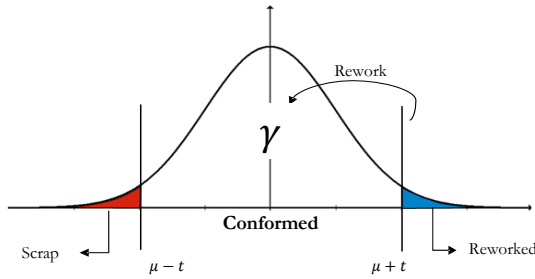


Fig 2. Tolerance design concerning reworking strategy.

Applying the assumptions aforementioned, they lead us to develop an estimation model predicting the conformity rate of the manufactured parts considering resources deviations and reworking impact. The conformity rate can be separated into two states of the manufacturing system, the state without reworking ability, and the state with reworking ability, respectively. On these bases, the conformity rate can be estimated function of three decision variables, namely, allocated tolerance ( $t_i$ ), process variation associated with the assigned resource ( $\sigma_{i,j}$ ), and reworking decision ( $rw$ ). Consequently, the conformity rate without reworking is formulated in Eq. 2. Afterward, the decision on integrating reworking into the manufacturing scheme can be seen in Eq. 3 and Eq. 4 investigating the conformity rate with the reworking concept.

$$\gamma_i = P\left(\frac{\mu_i - t_i}{\sigma_{i,j} \times a_{i,j}} \leq X_i \leq \frac{\mu_i + t_i}{\sigma_{i,j} \times a_{i,j}}\right) \quad \begin{matrix} \forall i \in N \\ \forall j \in O \\ a_{i,j} \neq 0 \end{matrix} \quad (2)$$

$$\gamma_i^{RW} = P(X_i \geq \frac{\mu_i + t_i}{\sigma_{i,j} \times a_{i,j}}) \quad \begin{matrix} \forall i \in N \\ \forall j \in O \\ a_{i,j} \neq 0 \end{matrix} \quad (3)$$

$$\gamma'_i = \gamma_i \times (1 + rw \times \gamma_i^{RW}) \quad \forall i \in N \quad (4)$$

Moreover, the process deviations associated with allocated resources can be used approximating assembled product deviation ( $\sigma_y$ ) and estimating assembly conformity rate (Eq. (5)).

$$\lambda = P\left(\frac{\mu_y - t_y}{\sigma_y} \leq Y \leq \frac{\mu_y + t_y}{\sigma_y}\right) \quad (5)$$

Where

$$\sigma_y \approx f(\sigma_{i,j} | i \in N, j \in O)$$

Ultimately, estimated conformity ratios help developing the cost model in the following section.

### 3.2 Manufacturing cost model

In section 2.2, the existing manufacturing cost models were discussed. In summary, an appropriate cost model which properly represents the manufacturing capabilities relies on extensive study of variation sensitive analysis. Hence, Activity-Based Modelling (ABC) provides an accurate cost assessment tool (Etienne et al., 2009), consequently, the manufacturing cost is proposed. Equation (6) represents the developed cost model where each activity is associated with the relevant decision impacts.

$$C_{Total} = \sum_i^{Part} \sum_j^{Oper} \frac{Cost_{Manu\ i,j} \times a_{i,j}}{\gamma'_i(1 - \alpha) + (1 - \gamma'_i)\beta} + \sum_i^{Part} \frac{Cost_{Inspec\ i}}{\gamma'_i(1 - \alpha) + (1 - \gamma'_i)\beta} + \sum_i^{Part} \frac{Cost_{Scrap\ i}(\gamma'_i\alpha + (1 - \gamma'_i)(1 - \beta))}{\gamma'_i(1 - \alpha) + (1 - \gamma'_i)\beta} + \sum_i^{Part} \frac{Cost_{Rework\ i} * rw}{\gamma'_i(1 - \alpha) + (1 - \gamma'_i)\beta} \quad (6)$$







The nominal value of the parts ( $X_i, i = 1, 2, 3$ ) are 55.3 mm, 22.86 mm, and 101.6 mm, respectively. In this study, a Root Square Sum (RSS) is used which is well-known as an optimistic method to evaluate functional requirement deviation and expresses as follow:

$$\sigma_y = \sqrt{\sum_{i \in N} \sum_{j \in O} \left( \left| \frac{\partial Y}{\partial X_i} \right| \sigma_{i,j} \times a_{i,j} \right)^2} \quad (10)$$

For the sake of simplicity, the derivatives of  $Y$  in respect to  $X_i$  ( $i = 1, 2, 3$ ) are calculated and given:  $\left| \frac{\partial Y}{\partial X_1} \right|_{\mu_{Hub}} = 0.1049$ ,  $\left| \frac{\partial Y}{\partial X_2} \right|_{\mu_{Roller}} = 0.2084$ ,  $\left| \frac{\partial Y}{\partial X_3} \right|_{\mu_{Cage}} = 0.1038$ .

Moreover, the manufacturing cost includes several activities such as processing, inspection, scrapping, and assembly. In table (1), associated costs, process deviations, and inspection errors are provided to model the manufacturing cost.

#### 4.2 Analyses

In section 3.3, a concurrent simulation and optimisation tool was proposed to investigate efficient and optimal decisions regarding the variety of functional requirement tolerances. In this section, a comprehensive result analysis is discussed. In order to shorten this section, the analysis of the hub regarding the tolerances allocated, the resources assigned, and the associated conformity rates are illustrated in Fig. 6. The illustrated analysis concerns three scenarios: 1) Tolerance allocation sole, 2) Tolerance and resource allocation, 3) Scenario 2 including rework.

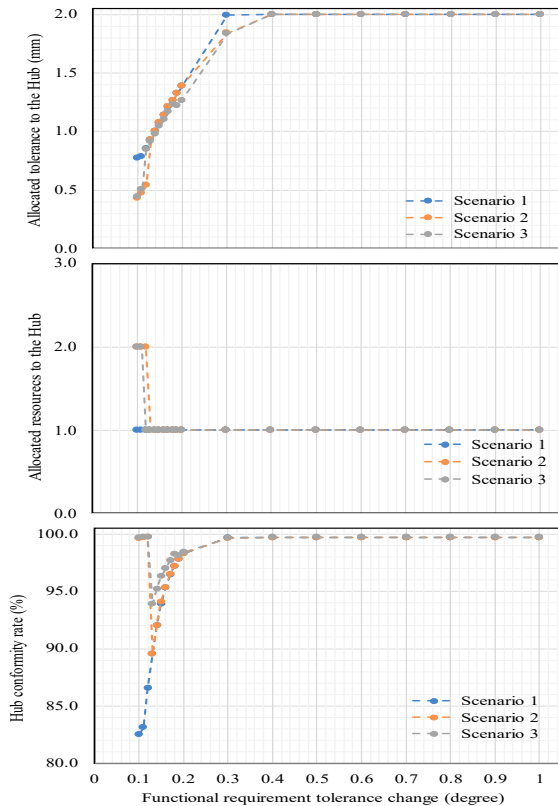


Fig 6. Hub sensitivity analysis

The study of different scenarios on the hub allocated tolerances and resources, besides associated conformity rates, depicts the

impact of different scenarios. Hence, scenario 2 integrates the resource allocation problem, the analysis illustrates the system behaviour which leans toward allocating practical systems to yield a higher conformity rate. Within scenario 3, the reworking strategy was included, and the consequence can be found by the improvement in the conformity rate. The analysis of scenarios on the assembled product can also be realized in Fig. 7. The application of simulation on this case study enabled designers to locate the critical assembled product tolerance range which is in the range of PLB and 0.4 degree. In this range, deployment of different scenarios helped the manufacturing system to improve assembled product conformity rate and reduces the manufacturing cost.

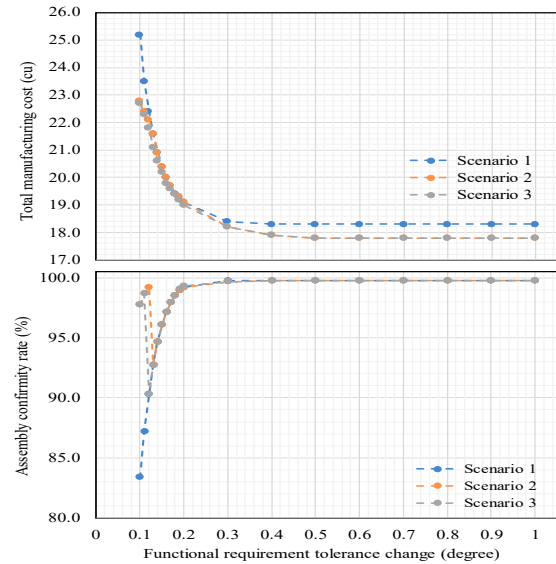


Fig 7. Assembled product total cost and conformity

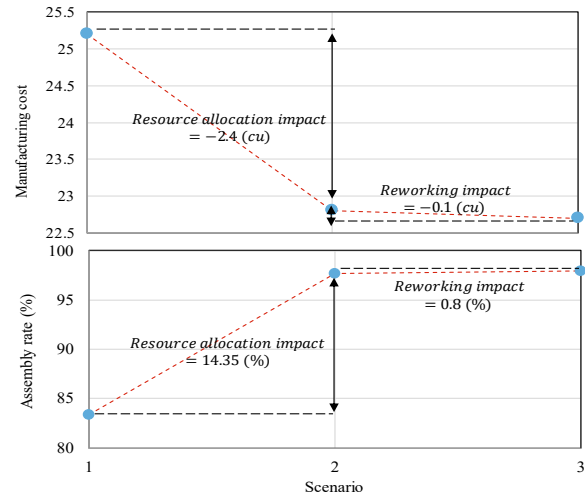


Fig 8. Precise assembled product ( $t_y = 0.1$ ) cost and conformity analyses

As aforementioned, simulation was proposed to locate the critical tolerance range on the case study. To this fact, the analyses of a tighter functional requirement tolerance for instance  $t_y = 0.1$  are depicted in Fig. 8. From the results, it can be deduced integrating resource allocation and reworking for more precise design can improve end-product quality as well as manufacturing cost.



## 5. CONCLUSIONS

The need for highly reliable products has driven manufacturing enterprises to improve their manufacturing capability. It has also impacted the development of tolerancing. The consequences of system improvement can be realized whether the end-product is reliable in the context of quality and cost or not. In this paper, different production strategies including reworking and resource allocation were integrated within tolerance optimisation. An appropriate cost model was proposed where each activity is associated with the relevant decision impacts. A concurrent simulation and optimisation tool were developed to yield optimal solution objects to minimizing manufacturing cost. The study of the obtained results concerning the strategies illustrates improvements in the end-product quality and cost. The complexity of the proposed mathematical model requires a more efficient optimisation algorithm to be developed. Moreover, the model can be extended by introducing assembly activity impacts on the end-product.

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