



### **Science Arts & Métiers (SAM)**

is an open access repository that collects the work of Arts et Métiers Institute of Technology researchers and makes it freely available over the web where possible.

This is an author-deposited version published in: <https://sam.ensam.eu>  
Handle ID: <http://hdl.handle.net/10985/26582>

#### **To cite this version :**

Mikhail GUSKOV - Ploughing-limited post-critical dynamics under chatter in turning. Harmonic balance based investigation - 2022

Any correspondence concerning this service should be sent to the repository

Administrator : [scienceouverte@ensam.eu](mailto:scienceouverte@ensam.eu)



# Ploughing-limited post-critical dynamics under chatter in turning. Harmonic balance based investigation

Mikhail Guskov\*

\*PIMM, Arts et Metiers Institute of Technology, CNRS, Cnam, HESAM University,  
151 boulevard de l'Hopital, 75013 Paris (France)

*Summary.* At chatter onset, as many works show, vibrations magnitudes grow until the tool exits the continuous cutting. Another magnitude limitation mechanism, namely ploughing is investigated in the present contribution, in a harmonic balance + continuation framework

## Chatter problematics

Regenerative chatter instability in metal cutting processes such as turning is known to possess subcritical behaviour [1], inducing bistable regions in the vicinity of the bifurcation. Investigations of post-critical vibrations have shown growing magnitude until the tool exit [2]. Once the cut becomes intermittent due to tool exit, the dynamics are much more complex due to the interrupted surface regeneration. Nevertheless, in some cases magnitude is limited by other mechanisms, such as ploughing [3]. In the present work we investigate post-critical dynamics of a 1 degree of freedom (DOF) with a piecewise linear ploughing model via a harmonic balance method (HBM) framework coupled with arclength continuation.

## 1DOF cutting system

The investigation is carried out on a system schematized on Fig. 1, giving place to the following equation of motion:

$$m\ddot{X} + c\dot{X} + kX = F_c(h) + F_p(\dot{X}), \quad (1)$$

$$F_c = -F_{c0} \left(\frac{h}{f}\right)^\alpha, \quad h = f + X - X_D, \quad X_D = X(t - D), \quad (2)$$

$$F_p = -H(\Delta\Gamma) F_{p0} \Delta\Gamma, \quad \Delta\Gamma = \frac{\dot{X}}{V_c} - \Gamma, \quad (3)$$

with  $m$  mass,  $c$  damping,  $k$  stiffness,  $f$  feed,  $V_c$  cutting velocity,  $D$  delay period,  $F_c$  and  $F_p$  cutting and ploughing force,  $F_{c0}$ ,  $F_{p0}$ ,  $\alpha$ ,  $\Gamma$  respective tool-workpiece interaction parameters,  $H$  Heaviside step function.

Cutting force definition (2) is based on the assumption of uninterrupted cut. The ploughing force model (3) features a unilateral penalty term corresponding to a closure of effective clearance angle  $\Delta\Gamma$  at high magnitude oscillations.

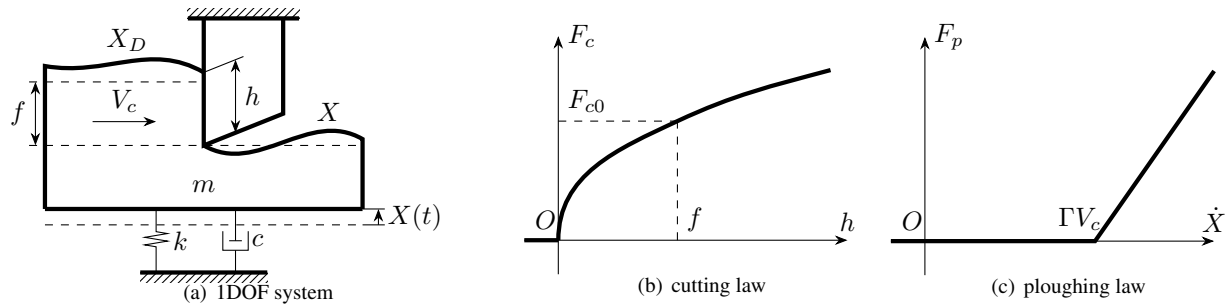


Figure 1: Cutting system model illustration

By introducing, for a given oscillation frequency  $\omega$ , time and length scaling  $\tau = \omega t$  and  $x = X/f$ , the equation (1) can be rewritten in adimensional form:

$$\Omega^2 x'' + 2\Omega\zeta x' + x + \kappa(x - x_D + 1)^\alpha + Z_p \Pi_p(\Omega x' - \phi) = 0, \quad \Pi_p = H(\Omega x' - \phi) \quad (4)$$

In the present work, the system is based on a case described in literature [4], with chatter onset parameters  $\zeta = 0.01$ ,  $\Omega = \omega/\sqrt{k/m} = \sqrt{1 - 2\zeta}$ ,  $\omega D = 3\pi/2$ ,  $\alpha\kappa_0 = 0.02$ , the cutting law being a power law with exponent  $\alpha = 3/4$ .

## HBM approach with continuation

The HBM approach used in this work is a variant of trigonometric collocation [5]. The solution is sought in the form of a trigonometric polynomial or order  $N$  ( $N$ -truncated Fourier series):

$$x(\tau) = \mathbf{T}(\tau)\mathbf{x}, \quad \mathbf{T} = \left[ \frac{1}{2}, \cos \tau, \sin \tau, \dots, \cos N\tau, \sin N\tau \right], \quad \mathbf{x} = [a_0, a_1, b_1, \dots, a_N, b_N]^T \quad (5)$$

In the case of chatter, constituting an auto-oscillation of a priori unknown frequency  $\Omega$ , a phase condition is used ( $b_1 = 0$ ). For variation of  $\kappa$  in the vicinity the chatter onset configuration at a given delay  $D$ , the following continuation condition ("arclength") is used for neighbouring points:

$$\|\Delta \mathbf{x}\|^2 + \Delta \Omega^2 + \Delta \kappa^2 = s^2 \quad (6)$$

with  $s$  a given (small) step distance.

### Investigation of ploughing parameters

The postcritical oscillations were analyzed in a range  $\kappa \in [0, 2\kappa_0]$  for a wide set of ploughing parameters  $Z_p$  and  $\phi$  was simulated, with  $N = 7$  and time discretization of 1024 points. The response is essentially mono-harmonic and thus are shown as the magnitude of the fundamental component on fig. 2. As for other variables, it can be noticed that  $\Omega$  remains very close to 1 and the static component  $a_0$  shows a slight variation with  $\kappa$ .

The upper limits of the plots are due to the cut interruption. The left branch of the plots, especially in cases of high ploughing thresholds  $\phi \geq 0.7$ , corresponds to a "no ploughing" behavior. The ploughing term comes in at  $a_1 \approx \phi/\Omega$ . As a general trend, the oscillation magnitudes tend to be lower for higher ploughing coefficient  $Z_p$  and for small  $\phi$ . In particular, for the lowest values of  $Z_p$  combined with big  $\phi$ , the tool gets to exit the cutting process in the given range of  $\kappa$  (top limits of the plots). At the extreme opposite, one can notice for  $Z_p \geq 3$  a quasi-constant oscillation magnitude is maintained.

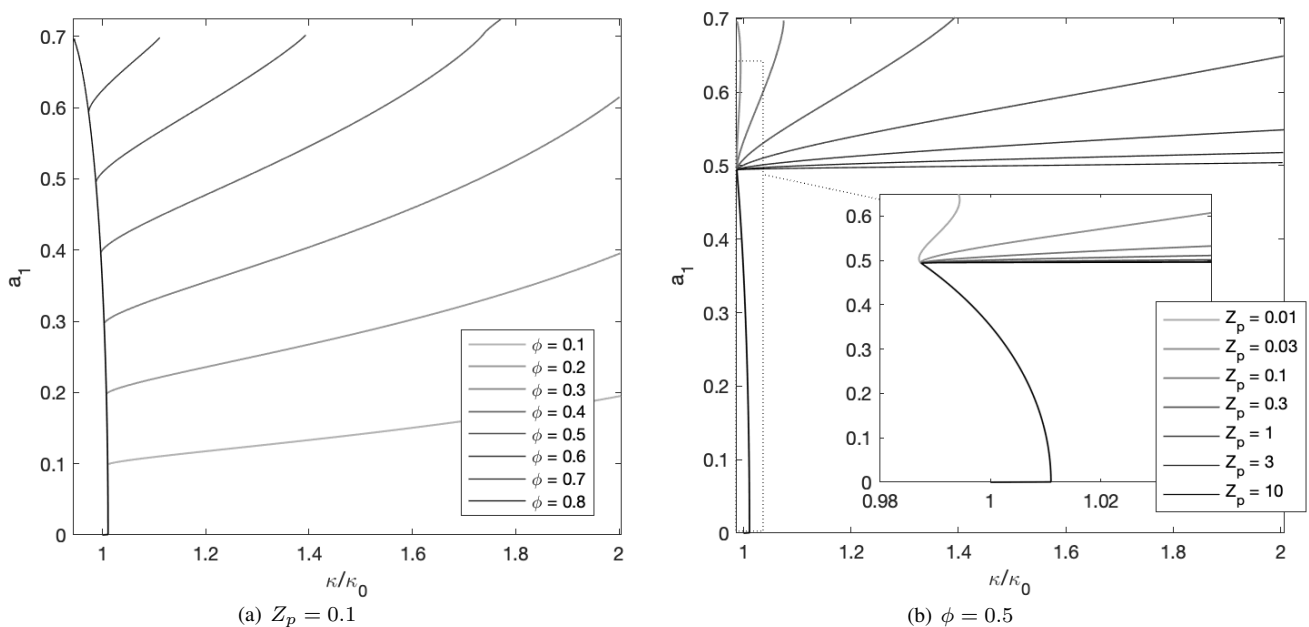


Figure 2: Postcritical response of (4) for  $\kappa \in [0, 2\kappa_0]$

### References

- [1] Hanna, NH, and SA Tobias. (1969) The Non-Linear Dynamic Behaviour of a Machine Tool Structure. *International Journal of Machine Tool Design and Research* **9**, no. 3: 293–307.
- [2] Dombovari, Zoltan, David A. W. Barton, R. Eddie Wilson, and Gabor Stepan (2011) On the Global Dynamics of Chatter in the Orthogonal Cutting model. *International Journal of Non-Linear Mechanics* **46**, no. 1: 330–38.
- [3] Gerasimenko, A., M. Guskov, P. Lorong, J. Duchemin, and A. Gousov (2016) Experimental Investigation of Chatter Dynamics in Thin-Walled Tubular Parts Turning, *HSM16 conference, Metz, France*.
- [4] Altintas, Yusuf (2012) Manufacturing Automation: Metal Cutting Mechanics, Machine Tool Vibrations, and CNC Design. *Cambridge University Press*.
- [5] Nataraj, C., and H. D. Nelson (1989) Periodic Solutions in Rotor Dynamic Systems With Nonlinear Supports: A General Approach. *Journal of Vibration, Acoustics, Stress, and Reliability in Design* **111**, no. 2: 187–93.