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Nadim EL HAYEK, Mohamed DAMAK, Hichem NOUIRA, Nabil ANWER, Eric NYIRI, Olivier GIBARU - Fast B-Spline 2D Curve Fitting for unorganized Noisy Datasets - 2014

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Context

- Optical and Tactile Metrology for Absolute Form Characterization (EURAMET project IND10)
- Fast polynomial spline curve reconstruction from very large unstructured datasets

Objective

Curve reconstruction of freeform shapes, specifically turbine blades, from data with unknown topology

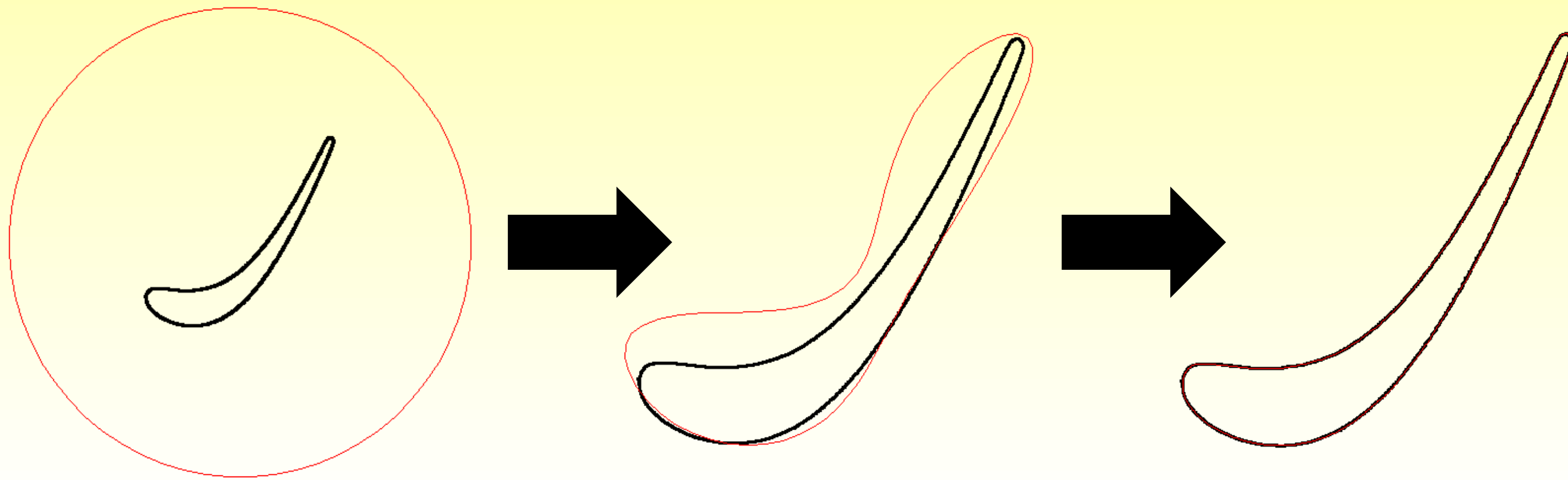
Objective function

$$\min_{t_1, t_2, \dots, t_m} \sum_j ((MT_m)_j - \delta_j)^2$$

M is the subdivision matrix (1)

$T = \{t_1, t_2, \dots, t_m\}$ is the control points translations vector

Discrete B-Spline Convection scheme



- ✓ NO initial parameterization
- ✓ NO differential calculations
- ✓ NO sampling requirements

- ✓ Invariance of final control polygon geometry to initial position and orientation

Methodology

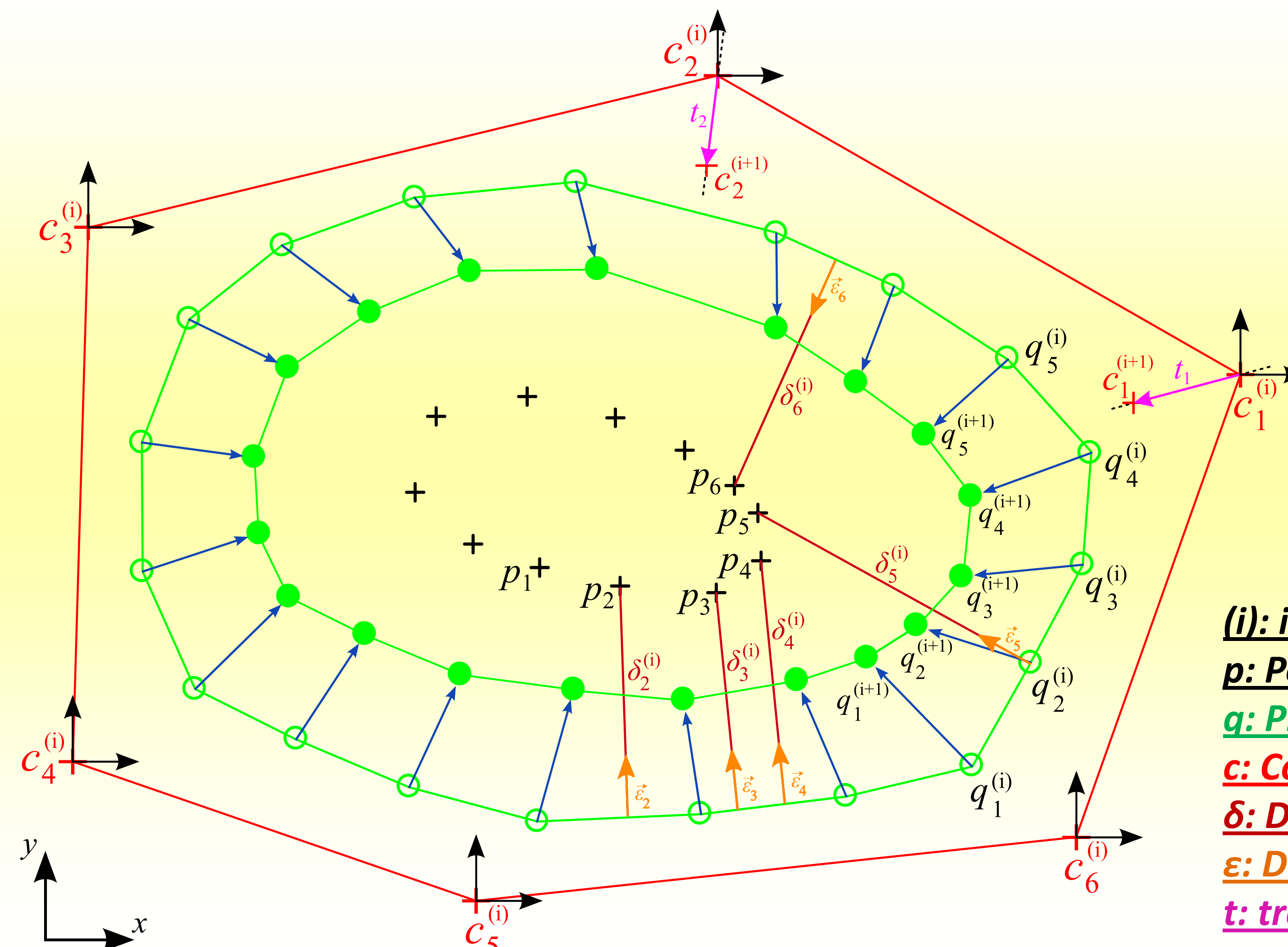
Coincide new B-Spline curve at iteration (i+1) with data points by minimizing the distances

The B-Spline (green) is initialized by a few control points around the data.

Distances δ_i are calculated with geometrical and topological considerations.

L_2 minimization \rightarrow translation vectors $\{t_1, t_2, \dots, t_m\}$ by which control points must move.

If the minimization does not meet the error tolerance, point insertion is applied locally.



(1) Subdivision relation

$$q_j^{(i)} = MC_m^{(i)}$$

(2) Convection equation

$$C_m^{(i+1)} = C_m^{(i)} + T_m^{(i)}$$

(3) Solution equation

$$q_j^{(i+1)} = M(C_m^{(i)} + T_m^{(i)})$$

(i): iteration i

p: Point set

q: Piecewise linear B-Spline curve

c: Control points

delta: Distances

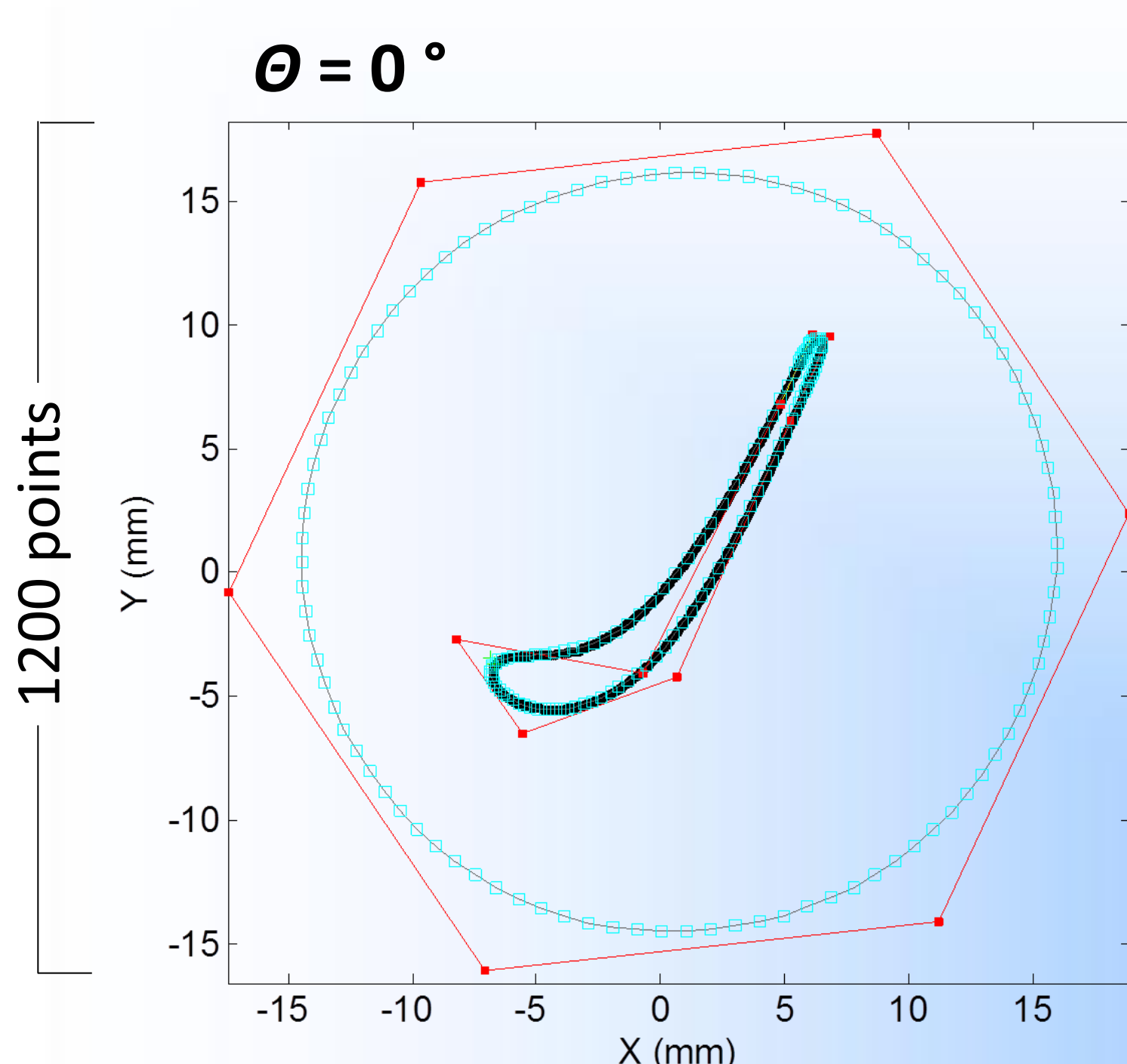
epsilon: Distance vectors

t: translation vectors

Experimental results

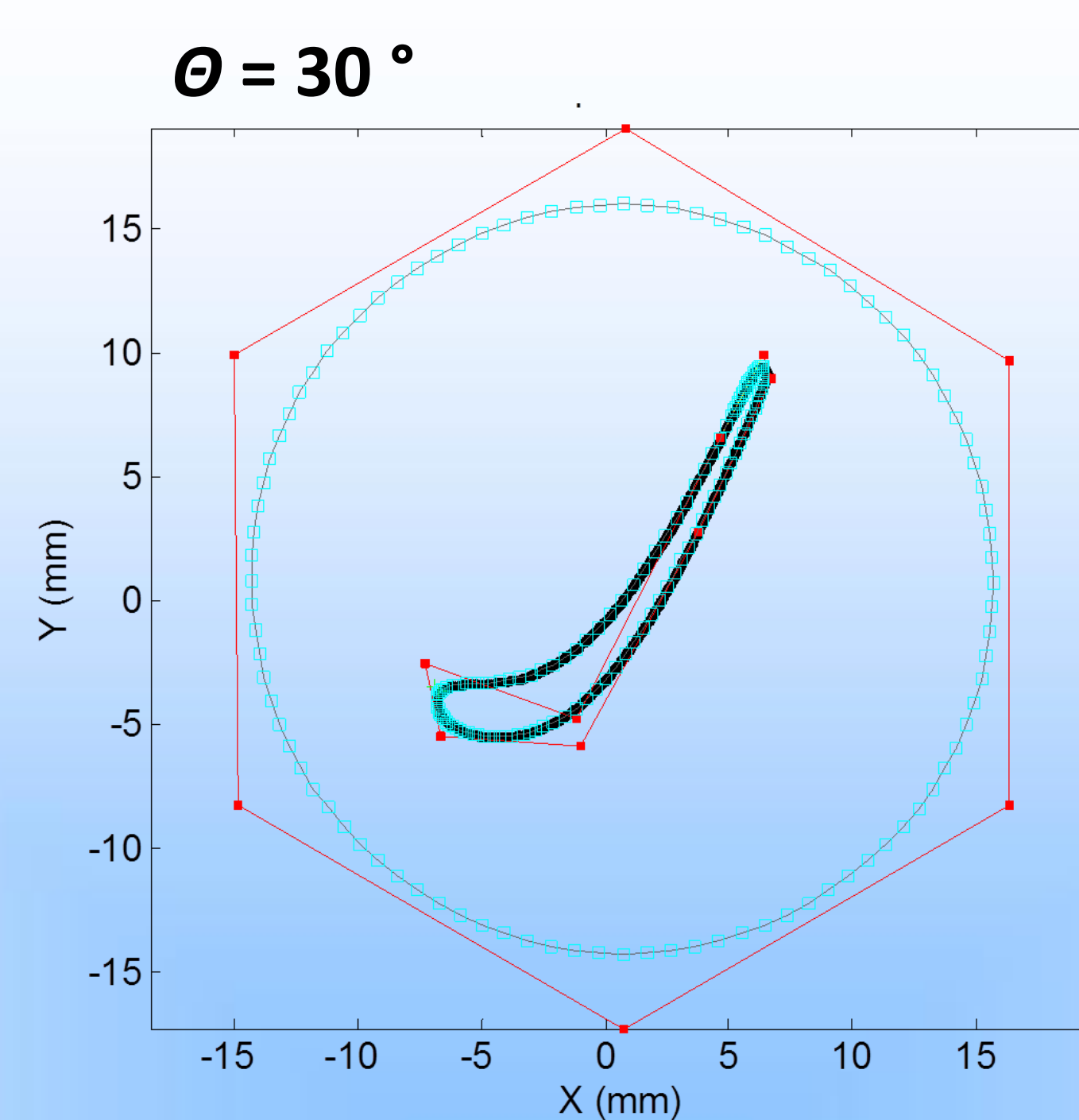
Invariance to point-set orientation

ϵ = mean of residual errors



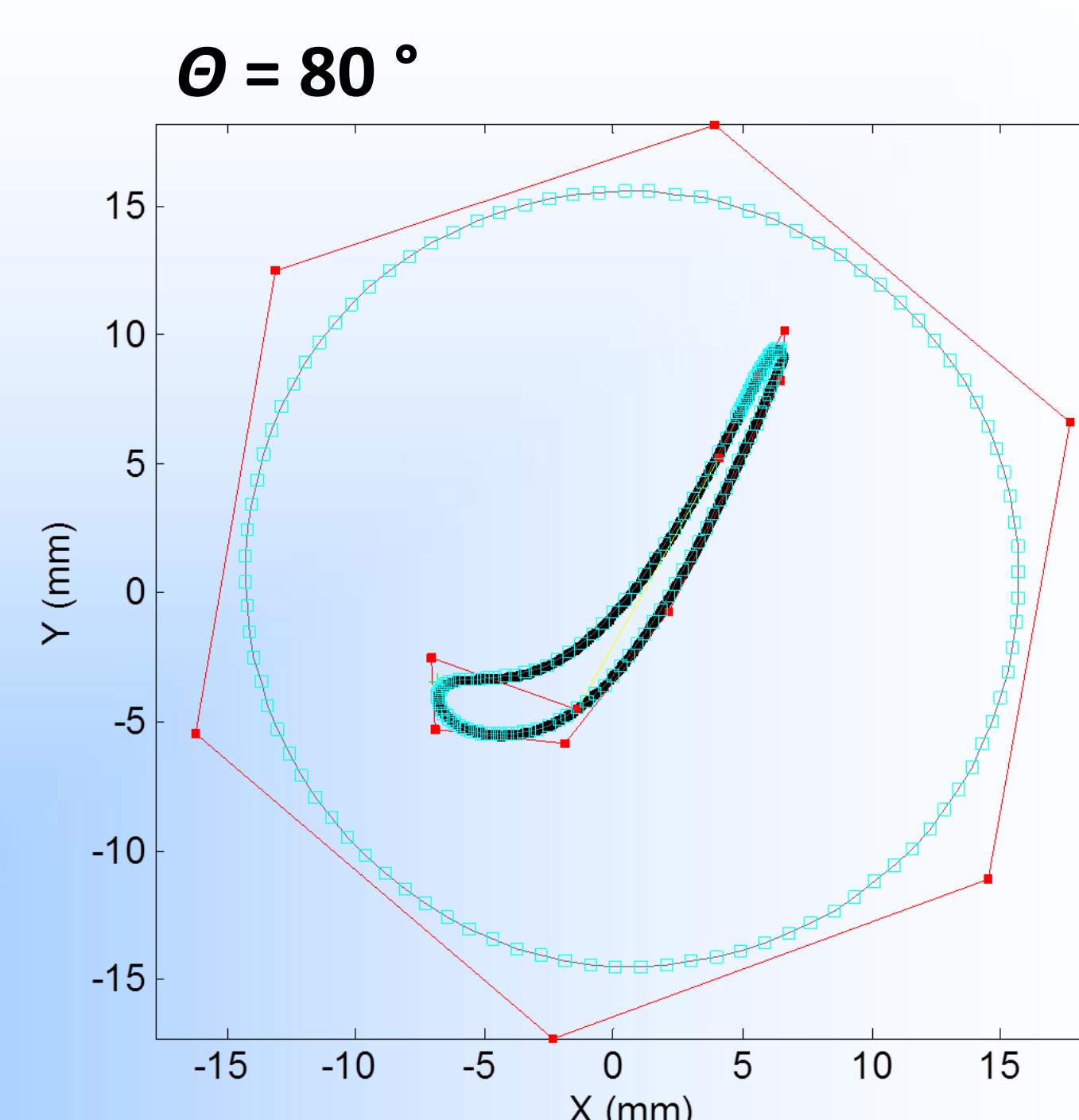
$\theta = 0^\circ$

8 final control points:
 $\epsilon \approx 0.0015$ mm, 140 iterations



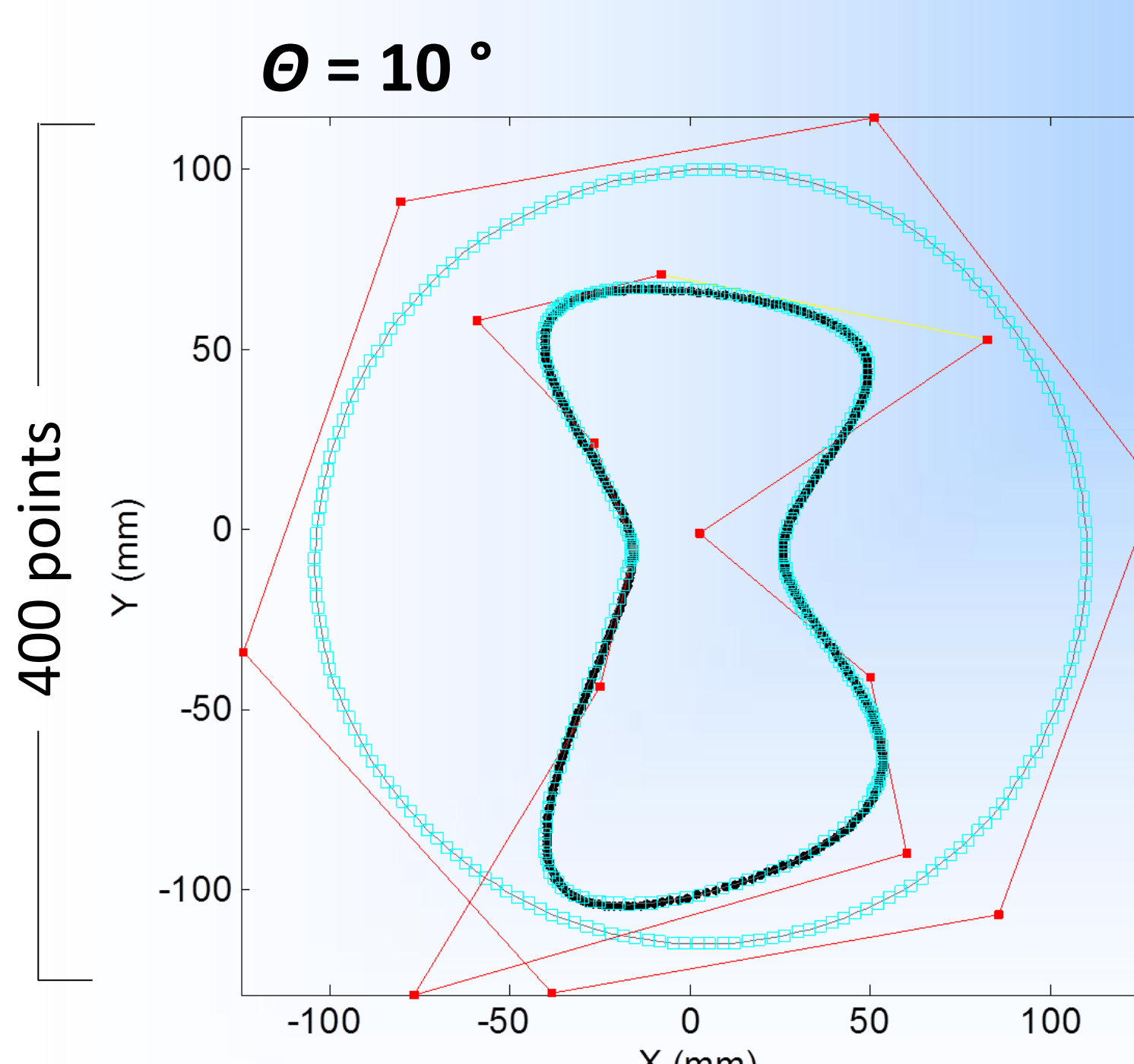
$\theta = 30^\circ$

8 final control points:
 $\epsilon \approx 0.00088$ mm, 140 iterations



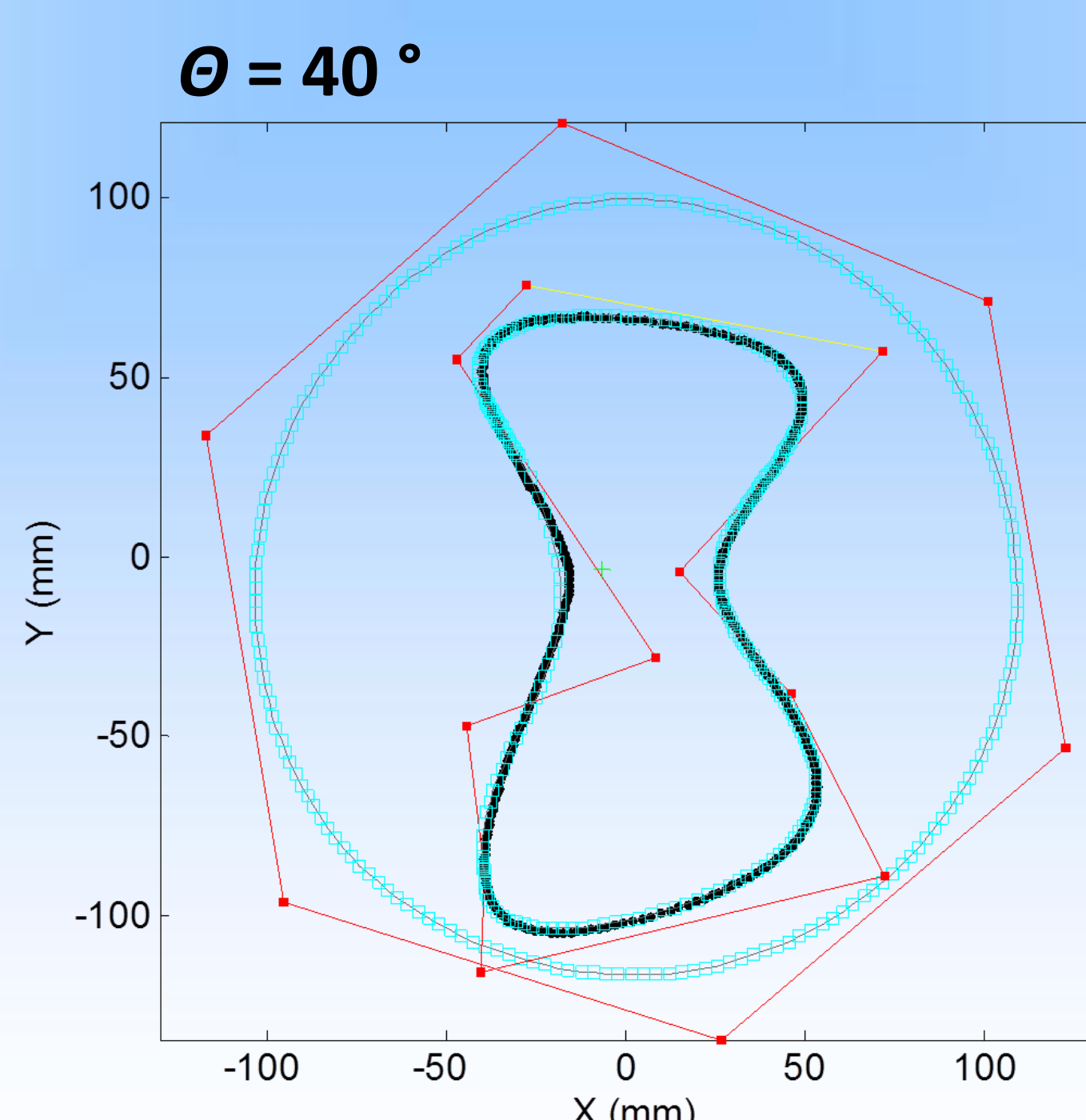
$\theta = 80^\circ$

8 final control points:
 $\epsilon \approx 0.0023$ mm, 140 iterations



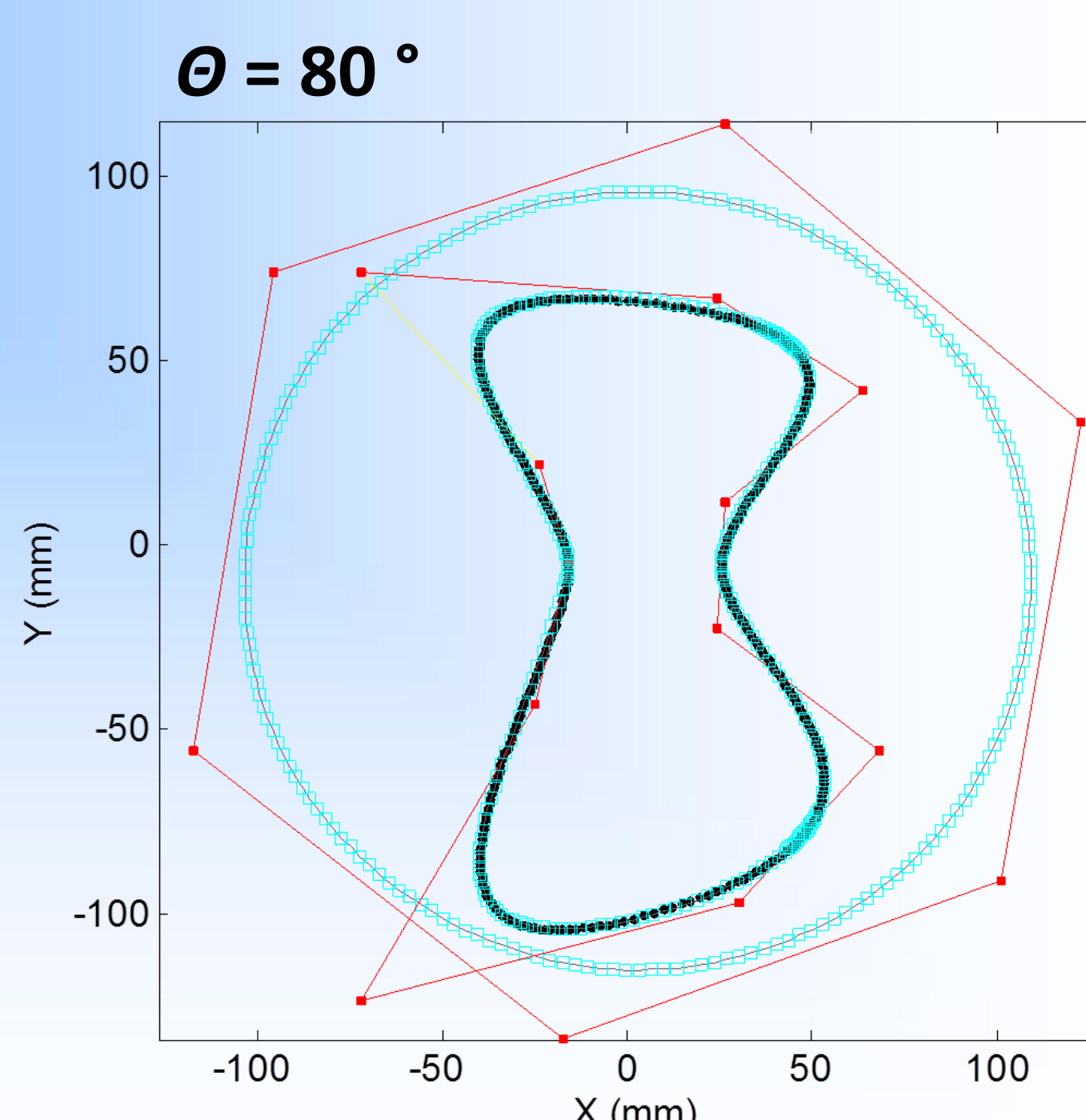
$\theta = 10^\circ$

10 final control points:
 $\epsilon \approx 0.0024$ mm, 140 iterations



$\theta = 40^\circ$

10 final control points:
 $\epsilon \approx 0.0017$ mm, 140 iterations



$\theta = 80^\circ$

11 final control points:
 $\epsilon \approx 0.00091$ mm, 140 iterations

Conclusions

- The B-Spline convection algorithm is founded on discrete computations.
- The algorithm is robust regarding the relative initial position of both the B-Spline and the data.
- The algorithm is tested on several shapes and returns residual errors below threshold if not too small.
- The initial number of control points must be minimal.
- The algorithm can be subject to time complexity improvement.
- Precision is not yet controllably achievable.

- [1] Speer T., M. Kuppe, and J. Hoschek. Global reparametrization for curve approximation, in Computer Aided Geometric Design, 1998.
 [2] Wang W., H. Pottmann, and Y. Liu. Fitting B-Spline Curves to Point Clouds by Curvature-Based Squared Distance Minimization, in ACM Transactions on Graphics, 2006.
 [3] Zheng W., P. Bo, Y. Liu, and W. Wang. Fast B-spline curve fitting by L-BFGS, in Computer Aided Geometric Design, 2012.

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