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# An acoustic multi-touch sensing method using amplitude disturbed ultrasonic wave diffraction patterns

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## Abstract

This paper proposes an acoustic multi-touch tactile sensing method. The proposed method is based on an amplitude disturbed ultrasonic wave diffraction pattern. An  $A_0$  Lamb wave transmitted in a thin finite copper plate is processed to provide tactile information, for one or two fingers. A touch event is localized by identifying the diffraction signals among a data base of diffracted Lamb wave references. Statistic models are used to improve the localization reliability. An artificial silicone finger is used in the calibration procedure. This touch interface is evaluated as a 2-touch interface.

*Keywords: Lamb Wave; Tactile; Multi-Touch; Diffraction; Pattern Recognition.*

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## 1. Introduction

Tactile interfaces are largely used for the communication between human and machine through the sense of touch. Their purpose is to localize and track points of contact. Most of the current tactile interfaces can only allow single point localization. The need of more sophisticated interaction forms between a user and a machine drive

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progressing interests in developing multi-touch sensing systems, which allow the user to interact with more than one finger or hand, such as bi-manual operations or multi-user collaborations.

Many laboratory prototypes or commercial products of multi-touch sensing system are already available. Some of them use a matrix of discrete sensors [1]; another approach uses a passive matrix of force-sensitive-resistors [2] or capacitive electrodes [3-4]. Those technologies have pros and cons, and alternative cost competitive methods with video cameras for touch sensing [5-6] or with frustrated total internal reflection (FTIR) [7] are also developed.

Compared to other technologies in the field of tactile sensing, acoustic wave methods have many advantages, as they don't require an extra layer. They can be implanted on different bulk materials or on composite surfaces such as LCD glass panels.

Lamb wave methods have often been used for the localization of contact between an index (pinpoint or finger) and a plate [8]. Measurements may rely on the time delay of an acoustic wave propagating in a plate [9-10] or on a cross correlation with a predefined acoustic signature associated with the position [11-12]. However, these methods can only localize a signal source. Indeed, some modern interaction modes such as "multi-touch" are difficult to achieve with single "impacts", since it would require localize and interpret simultaneously close successive impacts on the surface in a tactile gesture.

This paper proposes a new method using Lamb wave diffraction pattern for multi-point tactile applications. A copper plate with propagating Lamb waves includes plural discrete points ( $X \times Y$ ) on the surface. For one index localization, a predefined acoustic signature associated with one discrete point ( $x, y$ ) is considered as one pattern. When an index is in contact with this plate, which is considered as an object, we find an absorption signal (or attenuated signal) of the propagating Lamb wave. Then the localization involves a pattern recognition step. In this attempt, this paper suggests to change the measurement method from cross correlation with an acoustic signature [11-12] to pattern recognition. In the multi-touch case, the pattern is not associated with one discrete position, but with two or several simultaneous contact positions.

## 2. Experimental set-up

### 2.1. Measurement System

Fig.1. describes the test bench. It contains a function generator Tektronix 3012 and a data acquisition board Pico ADC 212/50. USB and parallel ports are used for the connection between the instruments and a laptop PC. Matlab is used for the signal processing and demonstration of results.

Four piezo-ceramic transducers (PZT, thickness = 0.5 mm, type Pz27 by Ferroperm, Denmark) are bonded to the edges of the plate with a conductive glue Circuitworks CW2400. The transducers are used as transmitters and receivers of Lamb wave.

Some features of Lamb waves' propagation are analytically known, such as group velocity and phase velocity. Following some approximations in [13], corresponding to the case  $k \times h \ll 0.5$ , where  $k$  is the angular wave number and  $h$  the half thickness of the plate, the phase velocity is given by the equation:

$$V_{\varphi} = \frac{\omega}{k} = \frac{V_p}{\sqrt{3}} hk \quad (1)$$

Where  $V_p$  is the plate velocity which depends on Young's modulus  $E$ , the Poisson's ratio  $\sigma$ , and the density  $\rho$  of the material. For the copper plate, we have  $V_p = 4273$  m/s.

The group velocity is twice the phase velocity and similarly given by the equation:

$$V_G = \frac{\partial \omega}{\partial k} = \frac{2}{\sqrt{3}} V_p hk \quad (2)$$

Fig.2. shows the group and phase velocities between 20 kHz and 250 kHz for a 350  $\mu\text{m}$  thick copper plate.

At a frequency  $f$ , the wavelength  $\lambda$  is:

$$\lambda = \frac{V_{\varphi}}{f} = \sqrt{\frac{2\pi}{\sqrt{3}}} V_p \frac{h}{f} \quad (3)$$

For the first asymmetric Lamb mode  $A_0$ , the ratio between the normal displacement component  $S_z$  and the shear displacement component  $S_x$  is equal to:

$$\left| \frac{S_z}{S_x} \right| (h) = \frac{1}{\left[ 1 - \left( \frac{V_\phi}{V_T} \right)^2 \right]} \times \frac{1}{kh} \quad (4)$$

Where  $V_T$  is the shear wave velocity, it depends only on  $E$ ,  $\sigma$  and  $\rho$  of the material. For the copper plate, we have  $V_T = 2270$  m/s.

As we can see from [equation \(3\)](#), the wavelength decreases with the square root of frequency. For the  $A_0$  Lamb mode, the ratio between the normal and the shear components is calculated quantitatively according to [equation \(4\)](#) and ranges from 11 (20 kHz) to 4 times (250 kHz). Therefore, most energy carried out by the Lamb wave is the shear wave energy, which can be attenuated by the human finger contact, and provide position associated information for localization.

According to these analytical results, below 100 kHz, an  $A_0$  mode Lamb wave packet can reflect up to 10 times at the boundaries in 1 ms, when its propagating group velocity is equal to 1000 m/s and the dimensions of the plate are 75 mm  $\times$  100 mm  $\times$  0.35 mm. It means that a 1 ms excitation signal is enough to establish a stable interference figure.

The test bench operates as described in [Fig.1.\(b\)](#). The excitation signal is transmitted from the function generator to the PZT transducers, which generate Lamb waves. After a short-time of interference building, say 1 ms, the superposition of transmitted waves generated by the PZT transmitters with reflected waves at the plate boundaries produces a stable Lamb wave interference figure. The PZT Receivers sample the acoustic field at their locations and transfer it to the computer for processing.

A sinusoidal drive signal creates amplitude attenuation information that changes according to different contact positions. A single frequency is not enough to produce reliable position identification because multiple positions may correspond to the same attenuation. By using several frequency components, Lamb waves exhibit different propagating velocities and ratios between the normal and the shear components, which involve distinct attenuation

values when a finger comes in contact with the plate. The tactile localization by Lamb waves is an extraction of position information from different disturbed Lamb wave signals at various frequencies. To obtain a precise localization, well spaced frequencies are required. The excitation signal is then composed of 30 frequencies distributed between 20 kHz and 80 kHz, with an interval of 2 kHz. The choice of excitation frequencies is important and complex, and a more detailed study is presented by the same authors in another paper [14].

More precisely, the sampling frequency of the data acquisition board is set at 195.31 kHz. A 2 ms time window acquisition collects amplitudes at different frequency components. The FFT provides information of signals amplitudes from 0 to 97.7 kHz. In the localization process, we take into account not only the amplitude of signals at 30 selected carrier frequencies chosen in the FFT domain, but also all the frequencies addressed by the FFT.

Based on a simulation study on modal analysis [8] that established strong variations of resonant frequencies of a plate according to boundary conditions, a support was designed to minimize disturbances coming from boundary conditions. The 75 mm × 100 mm rectangular thin finite copper plate (thickness = 450 μm) is maintained at the four vertexes by the support, as shown in Fig.3.

## 2.2. Localization Principle

The pixilation step is described in Fig.4. An artificial finger made with silicone is used for the calibration process. A detailed study of silicone properties and difference from human finger is presented in [9]. The silicone finger is displaced and controlled by a 3-D robot arm with a 2 μm precision. 36×32 contact points are predefined on the tactile surface, with a spatial grid of 2 mm × 2.5 mm. The tactile surface is 64×90 mm<sup>2</sup>. The contact surface, roughly 1 cm<sup>2</sup>, is assured by applying the same contact pressure.

At each reference position, the artificial finger comes into contact with the tactile surface. Then we measure the disturbed signals from two PZT receivers independently. The FFT signal of this disturbed signal is then considered as a pattern, as shown in Fig.5. It could be described as:

$$P_{mref} = (A_{1mref}, A_{2mref}, \dots, A_{Nmref}) \quad (5)$$

Here  $m$  is the index of Lamb wave receiver,  $A_{imref}$  is the amplitude of one frequency element of the FFT figure,  $N$  is one half of the acquisition points acquired by the data acquisition board.

All  $36 \times 32$  predefined positions are numbered and arranged in an array indexed from  $(x_1, y_1)$  to  $(x_{36}, y_{32})$ .  $36 \times 32$  reference FFT figures are collected by the calibration process. This provides  $36 \times 32$  patterns associated with predefined contact positions.

In the second step, the user touches the plate. The new FFT of the measured signal contains position information and is described as an amplitude vector object:

$$p_m = (A_{1m}, A_{2m}, \dots, A_{Nm}) \quad (6)$$

Analytically, this amplitude vector  $p_m$  can be expressed as a function of emitted signal, the object property and geometry, the contact position, and contact surface at a given pressure. As we maintain the same conditions during the training step, the emitted signal and geometry are constants, so we consider that this amplitude vector is only a function of contact position i.e. an amplitude disturbed diffraction phenomenon. Localization of the contact position consists in finding out the closest pattern to the amplitude vector object. The nearest neighborhood search is used with different definitions of distance ( $d_m$ ), such as Manhattan Distance:

$$d_m \left( P_m, P_{mref} \right) = \sum_{i=1}^n |A_i - A_{iref}| \quad (7)$$

The contact position can be visualized on a figure representing the normalized distances with a grey amplitude scale, as in Fig.6. By using a recognition process of one measured signal among all the reference signals, we are able to find the position of contact. The contact position appears in black. Neighboring points exhibit small distances which implies a limit to the spatial resolution of the learning process.

This method enhances an active acoustic plate featuring a process that continuously generates Lamb wave packets, 1000 per second. Excitation signals and signal processing are fundamental to its reliability.

We can observe from Fig.7 the performance of localizing 10000 consecutive measurements, while a touch is made at the center of the tactile plate. When using a single Lamb wave receiver (R1), 94.63% measurements

identify the contact point with the same reference point  $(x_{18}, y_{20})$  or the position  $(x=36\text{mm}, y=50\text{mm})$ . The two receivers have different accuracies in according with the contact positions. For example, a touch is made at position  $(x=16\text{mm}, y=67.5\text{mm})$ , the measurements using the receiver R2 has 89% accuracy as identifying the contact position with the corresponding reference point  $(x_8, y_{27})$ , and the rest 11% measurements identifying a touch event at a neighboring reference point  $(x_8, y_{28})$ . Indeed, due to the fact that the diffraction signals depend on the placement of PZT transducers and the contact position, it is normal that the two receivers at different positions will have different sensibilities for a given contact point. As well, it is because contact events at some points may have difficulty to provide enough distinct diffraction information for a single Lamb wave receiver that additional receivers are required. Hence, the double-validation check is like a triangulation process to find a point, we need at least two or three PZT transducers to ensure a measurement.

Identification errors may come from the surrounding noise. The accuracy of identification by single Lamb wave receiver varies from 89% to 95%. To improve the accuracy of localization, we propose a double-validation criterion: a touch-localization is trust worthy and validated if and only if the closest patterns found by both receivers are independently identical. Following this process, the localization procedure was found to be 100% reliable in experiments. We can estimate this accuracy mathematically. Supposing that each Lamb wave receiver has at least 89% accuracy and the errors occur only at the 8 neighboring reference points of the correct point, the localization process has a 79.21% probability to provide correct results with the double-validation process. A false localization happens only when the PZT receivers find the same false contact points at the same time, which means a probability less than  $11\%/8 \times 11\%/8$  or 2 per 10 000 measurements. As the diffraction signals get more symmetrical, we found out the error measurements may consist not only in confusing neighboring points from right ones, but also from other different positions. The double-validation process can ensure a high reliability of our localization process; however, it will slow down the system. In the previous example, the response time is prolonged by 1.25 times.

The performance of localization process depends on the reliability of the worst Lamb wave receiver. For example, if one receiver has 20% accuracy for a given contact point, even if the second receiver has 100% accuracy, the localization process could only validate 20% of overall measurements. Furthermore, if one receiver doesn't work properly in case of failure, the localization system is then incapable to output an answer. The double-validation



process is seen as “a reason to reject” scenario. A different approach could be to implement “a reason to accept” scenario.

In this regard, we provide a method based on the extended vector of received Lamb wave signals. The amplitude signals from  $m$  receivers are combined to form an extended vector:

$$p_e = (p_1, p_2 \cdots p_m) = (A_{11}, A_{21}, \cdots A_{12}, A_{22}, \cdots A_{Nm}) \quad (8)$$

The localization of the contact position consists in finding out the closest pattern to the extended amplitude vector object, which has a smallest Manhattan distance from the extended references vectors. With this definition, a failure on one receiver does not stop the identification process.

$$d(P, P_{ref}) = \sum_{j=1}^m \sum_{i=1}^n |A_{ij} - A_{ijref}| \quad (9)$$

With the experimental set-up presented in this paper, the extended vector is a combination of amplitude vectors from the two Lamb wave receivers. When a touch is made at the center of the tactile plate, the localization performance shows 69.3% accuracy with 10000 consecutive measurements. The errors are found to be 20.6% at reference point  $(x_{16}, y_{20})$ , 7.4% at  $(x_{18}, y_{19})$ , 1.3% at  $(x_{18}, y_{21})$ , 0.9% at  $(x_{27}, y_{21})$ , and the rest 0.5% at  $(x_9, y_{28})$ . The first three errors can be considered as neighboring points to the correct one, as they have a distance less than 4 mm.

To further increase the accuracy with the extended vector method, we add a likelihood statistic criterion. As one localization process is quite fast compared to the movement of finger on the tactile surface, we suppose that trustworthy localizations must remain spatially close in a short time window, thus a new touch-localization is trustworthy and validated if and only if its localization remains in the same vicinity of previous ones.

With that criterion, only 13.3% of 10000 consecutive measurements are validated, without a single false identification. The response time of localization is prolonged by 7.5 times to about 15 ms.

### 3. Multi-Touch Localization

The tactile surface presented in this paper can be used for multi-point tactile localization by simply applying the same analysis on disturbed signals. For one point localization, each reference signal is associated with a predefined contact point; in a multi-point case, the reference signals are associated with a combination of plural contact points. For example, in the case of localization of 2 simultaneous contacts, a reference pattern could signify a 4-D point  $(x,y,x',y')$  where  $(x,y)$  is the position of the first contact point and  $(x',y')$  is the position of the second.

In the single-point localization, the identified position is easily illustrated by a contrast figure of the Manhattan distance. In this paper, we arranged all the reference signals in an array. Let us take an example where the plate is divided in a  $5 \times 5$  contact grid. We call  $P_1$  a first configuration with contact positions  $(x_1,y_1,x'_2,y'_2)$  where 2 fingers come in touch with the plate at the position  $(x_1,y_1)$  and  $(x_2,y_2)$ , respectively.

We consider the two fingers used in the calibration procedure to be similar, which means  $(x_1,y_1,x'_2,y'_2)$  and  $(x_2,y_2,x'_1,y'_1)$  represent the same contact situations. We impose in our system that the index of the second position is always higher than the first one, to simplify calculations. All the possible two-point combinations of contact position are then sequentially touched and calibrated, to collect a 300 length array. The configurations are numbered  $P_2$   $(x_1,y_1,x'_2,y'_3)$ ,  $P_{24}$   $(x_1,y_1,x'_5,y'_5)$  and  $P_{300}$   $(x_5,y_4,x'_5,y'_5)$ .

In some current methods of multi-touch applications [2], thumbs are considered as different from fingers. This can also be taken into account in the calibration process by modifying the indexation of reference signals.

Fig.8 shows a measurement with both Lamb wave receivers. The acquired signal has minimum distance to the reference signals numbered 254 by both receivers, which is the index for  $(x_3,y_5,x'_5,y'_4)$ . Two fingers are detected at position  $(x=45 \text{ mm}, y=100 \text{ mm})$  and  $(x=75 \text{ mm}, y=80 \text{ mm})$ , respectively.

The number of reference signals depends on the subdivision degree of the plate. For a plate with a  $5 \times 5$  mesh, the number of potential combinations is  $C_{25}^2$  or 300. In extension, with  $M$  predefined points on the surface and  $N$  multiple contact fingers, this number is  $C_M^N$ .

Fig.9 indicates the distances from each reference signal to all reference vectors. There are several “shadow” lines. The reference vectors in a “shadow” line have a common contact point for their two-point contact combinations. This shows the correlation between the reference signals and the positions.

The accuracy of localization in the two-point case is similar as in the one-point case. We have a 100% reliable localization performance with the double-validation process, or nearly 100% with the extended vector method completed with the likelihood criterion. Not surprisingly, less than 10% of the measurements are validated as a localization error at one of the two-point will make the two-point identification rejected. Thus, the optimization processes generate higher rejection ratio in multi-point cases. The response time increases to 30 ms.

#### **4. Conclusions and Perspectives**

This paper introduces a tactile surface with simple architecture and design. With only four piezo-ceramic transducers, the tactile plate exhibits a spatial resolution down to 2 mm with a response time in the millisecond range. With a double-validation process, simple point localization has 100% accuracy. Multi-point localization by this technology is also possible and a two-point case is described and tested.

This technology is already a good alternative for many commercial tactile interfaces such as ticket distributors' screens. Its low-cost and low consumption features make it also a good alternative for mobile applications.

In the next future, the number of reference points will be increased to develop a quasi-continue tactile plate, which requires solving the problem of ambiguity between adjacent points.

Though the algorithm used in this work is already efficient in pattern recognition, another algorithm such as on-line neural network optimization is also being studied with the expectation to reduce computation and response time. A circuit board for mobile application, which should replace the function generator and data acquisition board, is also in progress.

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## Vitae

Yuan Liu received his Master's degree from Arts et Métiers ParisTech in 2006. He is currently pursuing his Ph.D. degree in the same institute. His research interests include acoustic wave sensors and their applications in robotics or human-computer interactions.

Jean-Pierre Nikolovski is a research engineer at CEA LIST, where he specializes in acoustic actuators and devices in human-computer interactions. He received his Ph.D. degree in Physics in 1995 from University Pierre and Marie Curie. He was formerly at Ecole Supérieure de Physique et Chimie Industrielles de Paris where he studied and designed several ultrasonic transducers and instruments such as wideband longitudinal and shear wave transducers, adaptive surface acoustic wave filters, ultrasonic gas flow meters, ultrasonic digitizing pen for computer graphics and large area touch screens. In 1997, he raised capital and co-founded Intelligent Vibrations SA where as a chief technical officer he designed products such as interactive tables and multimodal touch and vocal shop windows based on large area tap screens. He has authored 20 patents and is the recipient of the Montgolfier Medal from SPI.

Nazih Mechbal is an associate professor in the Laboratory of Mechanical Systems and Processes (LMSP) at the engineering school Arts et Métiers ParisTech of Paris, where he is member of the control team. He received his Ph.D. degree in robotics from the ENSAM Paris in 1999. His research interests include robotics, robust fault detection and diagnosis, active Control and structural health monitoring.

Moustapha Hafez received his M.S. degree in robotics from École Nationale Supérieure d'Arts et Métiers, Paris, France, in 1995, and the Ph.D. degree from the Institute of Applied Optics, Swiss Federal Institute of Technology, Lausanne, Switzerland, in 2000.

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Michel Vergé is professor on control at the Laboratory of Mechanical Systems and Processes (LMSP) of ENSAM ParisTech in Paris (France). He obtained HDR at Nancy University (France) in 1991. His research interests focus on the fault detection methods and Structural Health Monitoring.

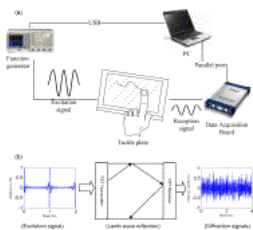


Fig. 1. (a) Test bench with a Lamb wave plate; (b) Signals flux.

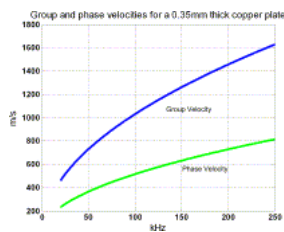


Fig.2. Group and phase velocities of  $A_0$  Lamb modes.

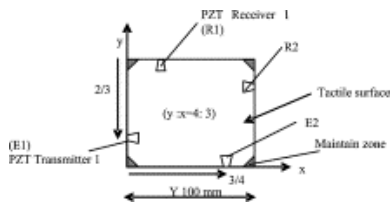


Fig.3. Placement of the PZT transducers.

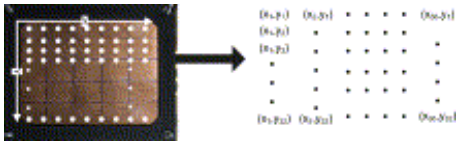


Fig.4. Pixilation of the plate.

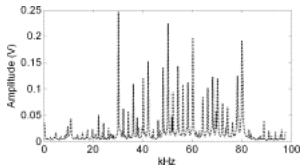


Fig.5. FFT figure of acquired signals from Lamb wave receiver R1, while a human finger is in contact at the position  $(x_1, y_1)$ .

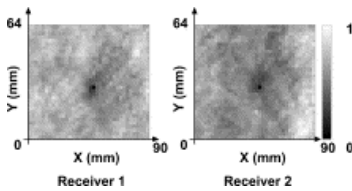


Fig.6. Contrast images based on the normalized distances from one reference signal to all others.

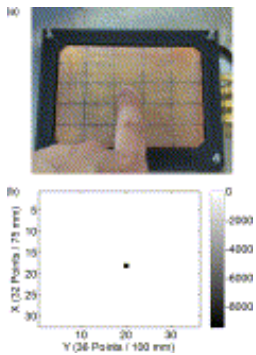


Fig.7. (a) Picture showing touch event; (b) Contrast image showing 10000 consecutive localization results with R1, achieving 94.63% localization accuracy.

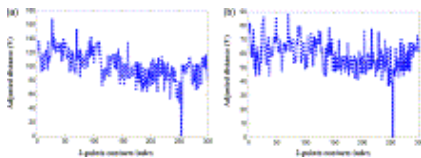


Fig.8. (a) Adjusted distance from one measured signal to all reference signals, by Lamb wave receiver R1; (b) Adjusted distance by Lamb wave receiver R2.

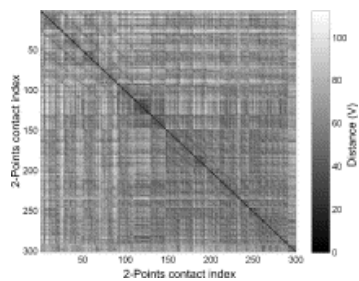


Fig.9. Distance from each pattern to all other references. The shadow lines indicate one common contact point between two-point contacts.