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# Uncertainty quantification and sensitivity analysis in electrical machines with stochastically varying machine parameters

Peter Offermann\*, Hung Mac<sup>†</sup>, Thu Trang Nguyen<sup>‡</sup>,  
Stéphane Clénet<sup>†</sup>, Herbert De Gersem<sup>§</sup> and, Kay Hameyer\*

\*Institute of Electrical Machines, RWTH Aachen University, D-52062 Aachen, Germany

<sup>†</sup>L2EP / Arts et Métiers ParisTech, 8. Bd Louis XIV 59046 Lille cedex, France

<sup>‡</sup>L2EP / Université Lille1, 59655 Villeneuve d'Ascq, France

<sup>§</sup>Wave Propagation and Signal Processing Research Group, KU Leuven, 8500 Kortrijk Belgium

**Abstract**—Electrical machines that are produced in mass production suffer from stochastic deviations introduced during the production process. These variations can cause undesired and unanticipated side-effects. Until now, only worst case analysis and Monte-Carlo simulation have been used to predict such stochastic effects and reduce their influence on the machine behavior. However, these methods have proven to be either inaccurate or very slow. This paper presents the application of a polynomial-chaos meta-modeling at the example of stochastically varying stator deformations in a permanent-magnet synchronous machine. The applied methodology allows a faster or more accurate uncertainty propagation with the benefit of a zero-cost calculation of sensitivity indices, eventually enabling an easier creation of stochastic insensitive, hence robust designs.

**Index Terms**—electrical machines, production tolerances, spectral stochastic finite element method, uncertainty quantification

## I. INTRODUCTION

Electrical machines are subjected to stochastic variations introduced by the production process [1]. Subsequently, each produced machine instance may deviate slightly with respect to its ideal and initial design. As a result, parasitic effects – such as for instance undesired harmonic components in the machine's torque – can occur and will negatively influence its overall performance (e.g. by radiating unanticipated and undesired noise). One possibility to elude these problems is the creation of robust machine designs [2]. However, finding a robust design has proven to be difficult until now. This difficulty stems from the fact that the standard finite element method (FEM) neither provides any intrinsic possibilities for sensitivity analysis of its (post-processed) results, nor any suitable way to propagate the occurring stochastic deviations onto the considered output sizes. Accordingly, the available tools to generate robust designs have been limited mostly to worst case analysis, Taguchi-Design of Experiments (DoE) and crude Monte-Carlo (MC) simulation until now. While worst case analysis proves to be imprecise, MC simulations (or DoE) in combination with the FEM result in high computational costs or, when reducing the required sample count, result in inaccurate predictions again.

In order to provide better tools for the creation of robust designs, this paper presents the application of a polynomial-chaos (PC) meta-modeling [3] technique for the simulation of an electrical machine. The choice of a PC approach is motivated by the easy calculation of sensitivity indices within the PC-framework, without being bound to the necessity to model all input deviations as Gaussian (as it is the case with

e.g. kriging). The required PC methodology is briefly recalled in section II.

Afterwards, the technique is applied to take account of stochastic geometry deformation modes that are imposed on an electrical machine's stator by being pressed into a housing. While section III details the chosen modeling, section IV analyses the deformation modes' influence on the cogging torque of the considered permanent-magnet synchronous machine by calculating the Sobol sensitivity indices directly from the PC meta-model. This way, a faster or more accurate tool for uncertainty propagation is provided along with the straightforward possibility to calculate sensitivity indices, hence enabling an easier and faster way to calculate and create robust designs.

## II. METHODOLOGY

In the following, an electrical machine with stochastically deviating dimensions is considered. We assume that the occurring deviations can be expressed as function of a random vector of variables  $\mathbf{x} = (x_1, x_2, \dots, x_n)$ . The probability density function (PDF) of  $\mathbf{x}$  is supposed to be known and stochastically independent<sup>1</sup>. Due to the randomness contained in  $\mathbf{x}$ , all studied machine output quantities  $\mathcal{M}$  also become random variables in dependence of  $x_1, x_2, \dots, x_n$  and cannot be calculated with a single finite element (FE) calculation anymore. Hence, the overall goal is to find a suitable way to quantify the randomness contained in the machine's output quantities  $\mathcal{M}(\mathbf{x})$ .

### A. Polynomial chaos expansion

One way to characterize  $\mathcal{M}$  is to determine an explicit expression or approximation  $\tilde{\mathcal{M}}(\mathbf{x}) \approx \mathcal{M}(\mathbf{x})$ , a so-called meta-model. The polynomial chaos theory [6] enables the

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<sup>1</sup>for stochastic dependent problems the introduction of *copulas* can be used to relax this restriction, see [4] along with [5]

calculation of such an explicit representation

$$\mathcal{M}(\mathbf{x}) \approx \widetilde{\mathcal{M}}(\mathbf{x}) = \sum_{i=0}^P \alpha_i \psi_i(\mathbf{x}) \quad (1)$$

by approximating the analyzed output quantity  $\mathcal{M}(\mathbf{x})$  as a function of a polynomial basis  $\psi_i(\mathbf{x})$ ,  $i \in \mathbb{N}$  and its scalar polynomial coefficients  $\alpha_i$ . For a given PDF, [6] proposes rules for the construction of the polynomials  $\psi_i(\mathbf{x})$ . The remaining task is the determination of the coefficients  $\alpha_i$ .

### B. Coefficient determination

Two different approaches which allow the calculation of the  $\alpha_i$  are *projection* and *regression* [4]. In the beginning, both methods require  $Q$  realization pairs  $(\mathbf{x}_k, \mathcal{M}(\mathbf{x}_k))$  of the input random vector  $\mathbf{x}$  and its corresponding output  $\mathcal{M}(\mathbf{x})$  in order to find  $\alpha_i$  which fulfill

$$\{\alpha_i\} = \arg(\min \left\{ E \left[ \mathcal{M}(\mathbf{x}) - \widetilde{\mathcal{M}}(\mathbf{x}) \right]^2 \right\}) \quad (2)$$

Afterwards, the projection method takes advantage of the polynomial basis' orthogonality by calculating the coefficients as

$$\alpha_i = E[\mathcal{M}(\mathbf{x}) \psi_i(\mathbf{x})] \quad (3)$$

where  $E[\cdot]$  is the expectation. The implicit integral in the right hand side of (3) then can be estimated by

$$E[\mathcal{M}(\mathbf{x}) \psi_i(\mathbf{x})] \approx \sum_{k=1}^Q \mathcal{M}(\mathbf{x}_k) \psi_i(\mathbf{x}_k) \omega_k \quad (4)$$

with  $\omega_k$  representing the associated integration weight to the point  $\mathbf{x}_k$ . The values of  $\omega_k$  depend on the chosen numerical integration scheme, e.g. in MC-integration  $\omega_k = \frac{1}{Q}$ .

In the regression method, the integral  $E[\mathcal{M}(\mathbf{x}) - \widetilde{\mathcal{M}}(\mathbf{x})]^2$  in the right hand side of (2) is approximated by

$$E[\mathcal{M}(\mathbf{x}) - \widetilde{\mathcal{M}}(\mathbf{x})]^2 \approx \frac{1}{Q} \sum_{k=1}^Q [\mathcal{M}(\mathbf{x}_k) - \widetilde{\mathcal{M}}(\mathbf{x}_k)]^2 \quad (5)$$

From (5), (2) and (1), it can be deduced that

$$\{\alpha_i\} = (\Psi \cdot \Psi^T)^{-1} \cdot \Psi \cdot \mathfrak{M} \quad (6)$$

where

$$\mathfrak{M} = \begin{pmatrix} \mathcal{M}(\mathbf{x}_1) \\ \vdots \\ \mathcal{M}(\mathbf{x}_N) \end{pmatrix} \quad (7)$$

and

$$\Psi = \begin{pmatrix} \psi_0(\mathbf{x}_1) & \cdots & \psi_0(\mathbf{x}_N) \\ \vdots & \ddots & \vdots \\ \psi_P(\mathbf{x}_1) & \cdots & \psi_P(\mathbf{x}_N) \end{pmatrix} \quad (8)$$

In a continuous environment, both approaches create identical results for an infinite polynomial base. With the introduction of a discretization,  $(\Psi \cdot \Psi^T)^{-1}$  differs from the identity matrix, yielding more accurate results with use of the regression method. Hence, the regression method is typically preferred in application, even though the inversion of  $(\Psi \cdot \Psi^T)$  is not guaranteed to be numerically stable [4].

For both methods – projection and regression – the choice of the  $Q$  realisations  $\mathbf{x}_k$  influence the methods' efficiency.

Several works have shown that low discrepancy sequences (for example Sobol sequences) are suitable and outperform randomly drawn samples. Nevertheless, the number of realizations  $Q$  has to be chosen adequately in order to obtain good approximations that yield a  $\widetilde{\mathcal{M}}(x)$  close to  $\mathcal{M}(x)$ . If the number of input random variables  $n$  is high, then the number  $Q$  can become excessive large. In case of the regression method, the number  $Q$  must be e.g. at least equal to the number of contributing polynomial chaos terms  $P + 1$  that is then calculated as

$$P + 1 = \frac{(n + p_{max})!}{n! \cdot p_{max}!} \quad (9)$$

where  $p_{max}$  is the maximum degree of the polynomial chaos [6]. In the end, the time required to calculate all realizations  $\mathcal{M}(\mathbf{x}_k)$ ,  $k \in Q$  can become a challenge. To overcome this difficulty, one seeks only polynomial chaos terms whose impact on  $\mathcal{M}(\mathbf{x})$  is significant [7], as has been done in the present work. This way, the number of polynomial chaos terms and hence the number of realizations  $Q$  can be reduced drastically.

### C. Post-processing

Based on the polynomial coefficients, stochastic moments as the model's mean value  $\mu$  and its variance  $\sigma^2$  can be calculated. The definition of the expectancy value provides

$$\mu \approx E[\widetilde{\mathcal{M}}(\mathbf{x})] = \int_{-\infty}^{\infty} \widetilde{\mathcal{M}}(\mathbf{x}) \cdot f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} \quad (10)$$

and after inserting the meta-model's definition

$$\widetilde{\mathcal{M}}(\mathbf{x}) = \sum_i \alpha_i \cdot \psi_i(\mathbf{x}) \quad (11)$$

from equation (11) into equation (10) one can exploit the polynomials' orthonormality to deduce that:

$$\mu \approx \sum_i \alpha_i \cdot \delta_{i0} = \alpha_0 \quad (12)$$

The derivation of the variance  $\sigma^2$  occurs analogously and results in

$$\sigma^2 \approx \sum_i \alpha_i^2 \quad (13)$$

Eventually, a sensitivity analysis can be performed to evaluate the impact of each input's random variable  $x_i$  on the output's variation  $\sigma_{\mathcal{M}}^2$ . To do so, Sobol sensitivity indices [8] are a suitable choice. The calculation of Sobol sensitivity indices yields values in the interval  $[0, 1]$  with  $S_i$  close to 0 representing a weak influence and  $S_i$  close to 1 indicating a high impact of the input  $i$  on the variation of  $\mathcal{M}$ . Once that the approximation  $\widetilde{\mathcal{M}}$  in the polynomial chaos expansion form is available, the Sobol indices can be deduced straightforwardly [9], [10]. Applying the sensitivity indices, design paradigms as robust design and tolerance allocation can finally be implemented.

## III. APPLICATION

The examination's goal is to analyze the effect of randomly occurring static pressures that deform a PMSM's stator. We here investigate the pressures' influence on the machine's cogging torque in particular. Possible sources for such pressure can be e.g. stator welding seams or fixation points of the stator

in its housing. Here it is assumed, that the pressure points occur symmetrically distributed over the stator's circumference, and thus cause stator deformation modes as depicted in figure 1.

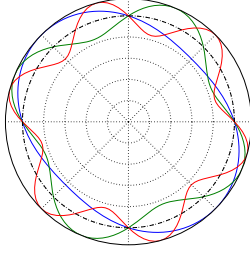


Fig. 1. Stator deformation modes number 2, 4, 6.

Two effects of this deformation are studied at first separately, later on also in combination:

- 1) The applied pressure on the stator eventually results in variations of the air gap. To avoid the need for a complete mesh remodeling, these changes have been simplified and are represented as variations of the stator tooth height (parameter  $l_1$  in figure 2), effectively creating a similar air gap variation.
- 2) The stator's deformation causes variations in the slot opening width between all stator teeth. This effect is modeled with parameter  $l_2$  (compare figures 2 and 3).

Applying conformal mapping theory in an approach comparable to [11], the influence of the first twenty modes has been tested for both parameters. Significant changes in the cogging torque can only be observed for the deformation modes  $D = \{1, 2, 3, 6, 12\}$ . Hence, these modes have been used to model the parameter input variations as

$$l_1(n) = l_1^0 + a_0 + \sum_{k \in D} a_k \cdot \sin\left(k \frac{2\pi n}{36} + a_k^*\right) \quad (14)$$

$$l_2(n) = l_2^0 + b_0 + \sum_{k \in D} b_k \cdot \sin\left(k \frac{2\pi n}{36} + b_k^*\right) \quad (15)$$

with  $n$  representing the stator tooth number,  $k$  as mode number,  $l_i^0$  being the nominal parameter values,  $a_k$  &  $b_k$  as independent uniform random variables for the mode  $k$  defined in the interval  $[-0.02; 0.02]$ , and  $a_k^*$  &  $b_k^*$  as independent uniform random variables for the mode  $k$  defined in the interval  $[0; 2\pi]$ .

#### IV. RESULTS

In order to build the PC meta-models of the presented variations, an A- $\Phi$  formulation FE-model is employed with

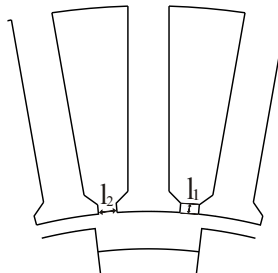


Fig. 2. Stator with varying tooth height ( $l_1$ ) and slot opening width ( $l_2$ ).

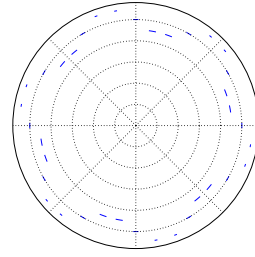


Fig. 3. Variation of the stator's slot opening width (parameter  $l_2$ ). The dashes represent position and deformation of the slot opening width introduced by the occurrence of mode number 6.

a mesh of 103,126 nodes. Introducing a parallelization with 20 calculation nodes, 400 evaluation points are calculated in 4 days. Within these simulations, the analyzed geometry variations are considered by the transformation method presented in [12]. At first, the influence of the parameters  $l_1$  and  $l_2$  on cogging torque is studied separately, afterwards combined simulations with both parameters are executed. Within the meta-models, the cogging torque is directly analyzed in its frequency domain as multiples of one pole pitch of the motor with  $\tau = \pi/3$ . With the help of the Sobol sensitivity indices it can then be determined which cogging torque harmonic is originating from or influenced by which input random variable. Figure 4 summarizes the analysis' work-flow.

The simulations allow the following conclusions for the given geometry:

##### A. Separate variations of $l_1$ and $l_2$ :

For both parameter variations, the 6<sup>th</sup> harmonic of the pole pitch (here  $2p = 6$ ) has got a dominating influence on the mean value. This behavior is conform to expectations due to the 36 stator teeth of the machine. While the effective air gap distortion modeled by the tooth height variations mainly influences the variance of the harmonics 1<sup>st</sup> and 2<sup>nd</sup>, the slot opening width yields a dominating variance again for the 6<sup>th</sup> output harmonic. The accordance with expectations confirms the method's correctness. The sensitivity analysis allows to give the following influence mapping of the relevant output harmonics:

- Harmonic 1 is influenced mainly by  $a_6$  and  $b_6$ .
- Harmonic 2 is influenced mainly by  $a_{12}$  and  $b_{12}$ .
- Harmonic 6 and 12 are influenced mainly by  $a_0$  and  $b_0$ .

##### B. Interaction of both parameters

As the influence of the input modes  $D_{neglect} = \{1, 2, 3\}$  is very small in the separate parameter variations, the corresponding input random variables are discarded for the joint

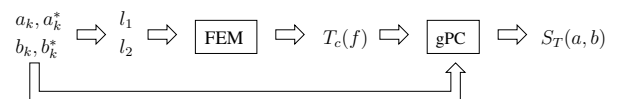


Fig. 4. Simulation flow: Tooth height ( $l_1$ ) and slot opening width ( $l_2$ ) are calculated from the input random variables  $a_k, a_k^*, b_k, b_k^*$  and are used in the FEM. The FEM's results together with the corresponding input random variables are employed to calculate the PC meta-model, which enables a straightforward calculation of the Sobol sensitivity indices  $S_T(a, b)$ .

Harm. of $\tau_p$	Mean	Variance	Sobol	$a_0$	$a_6$	$a_6^*$	$a_{12}$	$a_{12}^*$	$b_0$	$b_6$	$b_6^*$	$b_{12}$	$b_{12}^*$
1	0.0464	0.816e-3	total	0.057	<b>0.869</b>	0.047	0.048	0.037	0.058	0.050	0.048	0.052	0.055
			1 <sup>st</sup> ord.	0.000	<b>0.816</b>	0.001	0.000	0.000	0.000	0.002	0.001	0.003	0.003
2	0.0373	0.539e-3	total	0.028	0.035	0.024	<b>0.918</b>	0.031	0.029	0.032	0.035	0.023	0.043
			1 <sup>st</sup> ord.	0.002	0.001	0.000	<b>0.887</b>	0.001	0.001	0.002	0.000	0.000	0.001
6	0.1822	0.024e-3	total	<b>0.543</b>	0.000	0.000	0.000	0.000	<b>0.456</b>	0.000	0.000	0.000	0.000
			1 <sup>st</sup> ord.	<b>0.543</b>	0.000	0.000	0.000	0.000	<b>0.456</b>	0.000	0.000	0.000	0.000
12	0.0186	0.293e-6	total	<b>0.105</b>	0.001	0.001	0.001	0.001	<b>0.895</b>	0.001	0.001	0.001	0.001
			1 <sup>st</sup> ord.	<b>0.103</b>	0.000	0.000	0.000	0.000	<b>0.893</b>	0.000	0.000	0.000	0.000

TABLE I  
SIMULATION RESULTS VARYING  $l_1$  &  $l_2$ : MEAN, VARIANCE AND SENSITIVITY INDICES CALCULATED FROM THE META-MODEL.

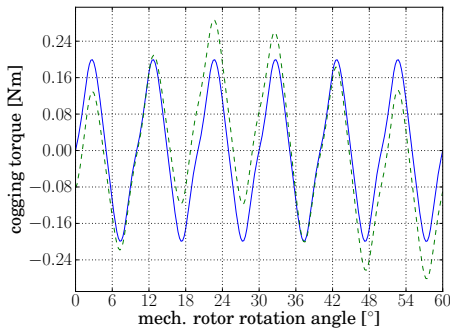


Fig. 5. Comparison of ideal cogging torque compared to a realization that shows a significant low frequency effect (dashed).

influence analysis. The joint simulation's results are given in table I. It can be observed, that

- The influence of all phase shifts ( $a_k^*$ ,  $b_k^*$ ) in the input modes is very weak.
- Total and first order Sobol indices of each random variable are close to equal. It can be deduced, that there are nearly no interactions within the model for the simultaneous occurrence of  $l_1$  and  $l_2$ .
- The variance of the harmonics 1 and 2 is dominant compared to the variance of the harmonics 6 and 12. One can reason that the occurring low frequency effect is significant. This result is also shown in Figure 5.

## V. CONCLUSIONS

This paper presents the application of a non-intrusive polynomial-chaos meta-modeling technique for uncertainty quantification in electrical machines. In particular, the influence of randomly occurring stator deformation modes on the cogging torque of a permanent-magnet synchronous machine is modeled, propagated and assessed with the help of Sobol sensitivity indices. The utilization of the gPC based uncertainty propagation reduces the required computational effort. Furthermore, it allows to derive a direct correlation between the modeled input random variables and the harmonic components of the machine's cogging torque. This step eventually simplifies the creation of robust machine designs by providing a cause-effect mapping.

The presented methodology is universal and can be applied to arbitrary tolerances in electrical machines, given that the

stochastic parameters can be expressed within the FE-model. Future work will investigate the impact of tolerances in the soft magnetic material properties with respect to loss calculations in electrical machines.

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