ON SIGNAL DENOISING BY EMD IN THE FREQUENCY DOMAIN

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ABSTRACT

In this work a new denoising scheme based on the empirical mode decomposition associated with a frequency analysis is introduced. Compared to classical approaches where the extracted modes are thresholded in time domain, in the proposed strategy the thresholding is done in the frequency domain. Each mode is divided into blocks of equal length where the frequency content of each one is analyzed. Relevant modes are identified using an energy and a frequency thresholds obtained by training. The denoised signal is obtained by the superposition of the thresholded modes. The effectiveness of the proposed scheme is illustrated on synthetic and real signals and the results compared to those of methods reported recently.

Index Terms— Empirical Mode Decomposition, Intrinsic Mode Function, Denoising,
use an energy-based threshold, either in the form of a correlation function, amplitude..... Although being a sufficient criterion in a lot of applications, it ignores the fact that in some particular cases, the energy is not the only criterion distinguishing useful signals from noise-only ones. It turned out that in the case of a Doppler signal like in Figure 1, a very interesting part of the signal (highlighted with a red rectangle) has very low amplitude, which means very low energy. This part of the signal can be clearly seen in the EMD decomposition in Figure ??, particularly in the 2nd, 3rd and 4th modes. The horizontal lines represent the thresholds used in [9], one can notice that these thresholds remove a very important part of the signal, because it does not have a sufficient energy to exceed the thresholds in any of the three modes. Hence the need to add another criterion that can eventually lead to a better thresholding strategy. It turned out that adding a bandwidth-based threshold can be useful in cases where signal energy is small. It should be noted that neither the EMD or the EEMD did not succeed in finding the regions of interest in the signal (see Figures ?? and 2). None of the aforementioned methods succeeds in dealing with such cases. Hence the necessity to develop a denoising algorithm that can extract useful information from low energy signal intervals.

A new EMD-based denoising strategy for detecting the IMF’s signal-dominant intervals is presented. In classic situations, when the EMD succeeds in separating noisy modes from signal ones, one can use a hard-thresholding method [6], [10]. However, when signal modes are mixed with noise ones, while the signal ones has significant energy level, any of these methods [8], [9] will do. But in more complicated cases, when the original signal has low energy level, we proved that the new thresholding strategy outperforms the other methods in terms of SNR especially for low energy parts of the signal.

1.1 EMD-FT denoising

The EMD-based methods studied in [6], [10] are very fast and robust when the modes are not mixed, otherwise, we will certainly lose relevant information or add unwanted noise because of the hard thresholding decision applied (i.e., every mode is either selected or rejected). To overcome the mode mixing problem, one should look inside every IMF and make a decision on every interval. In order to do that, we must understand how the noise does effect the EMD. The white noise has a constant power spectral density, i.e., it has equal power within a fixed large enough bandwidth. On the other hand, for many categories of signals (e.g., Doppler), one interval

Fig. 1. (a) from top to bottom: original Doppler signal, –2dB noise and noisy Doppler signal, the red square represents a sliding window of 128 samples (b) from top to bottom: the windowed Doppler, the windowed noise and the windowed noisy Doppler (c) from top to bottom: power spectral density of the original Doppler, PSD of the noise and the PSD of the noisy Doppler

Fig. 2. EMD applied to a Doppler signal. The lines in red represents the thresholds used for CIIT method.
of a certain IMF of an almost noiseless signal is likely to have narrow bandwidth. Thus, one way to discriminate between very noisy intervals and almost noiseless intervals, is to study the frequency domain of each interval of each IMF and decide basing on two criteria: bandwidth and energy. In order to do that, one can start by decomposing each IMF into a number of 50% overlapping blocks and then multiplying each block by a certain analysis window (e.g., a sine window) and then applying the Fourier Transform (FT) on each one of the windowed blocks. Then, we choose two thresholds (one for bandwidth and the other for energy) and we compare the bandwidth and the energy of the block with the chosen thresholds. Based on this comparison, we can estimate the nature of the block (useful signal or noise). For example, we will compare the power spectral densities of two different IMF intervals \( |P_{n,1}^t(f)| \) and \( |P_{n,2}^t(f)| \), the one that has a narrow-band frequency domain will be considered as signal, while the one with a wide-band frequency domain will be considered as noise.

Let \( s(t) \) be a noise free signal contaminated by a white Gaussian noise \( b(t) \). \( x(t) = s(t) + b(t) \). The algorithm described above can be summarized in the following steps:

1. **Decompose the noised signal** \( x(t) \) using EMD.
2. **Decompose each mode** \( p_i(t) \) into \( k \) blocks: \( \{p_i^1(t), \ldots, p_i^k(t)\} \), \( i \in \{1, 2, \ldots, N\} \), where \( N \) is the number of modes, with \( p_i^k(t) = p_i(t).w_a(\frac{t-\tau}{\lambda}) \) where \( w_a(\frac{t-\tau}{\lambda}) \) is a sliding window, \( \lambda \) is its width and \( \tau \) is its delay. Each block has 50% overlapping with the previous one.
3. **Compute the discrete FT on each one of the blocks:** \( \mathcal{F}(p_i^k(t)) = P_i^k(f) \).
4. **Compute the energy** \( E_i^k \) and the bandwidth \( \sigma_i^k \) of each \( P_i^k(f) \) [11].
5. **Find by learning (Figure 3) the frequency threshold** \( \bar{\sigma} \) and the energy threshold \( \bar{E} \).
6. **Select only the intervals having their energy above** \( \bar{E} \) and their bandwidth below \( \bar{\sigma} \). The estimated blocks are expressed as:
   \[
   \hat{P}_i^j(f) = P_i^j(f)\mathcal{H}(E_i^j - \bar{E})\mathcal{H}(\bar{\sigma} - \sigma_i^j) \tag{1}
   \]
   where \( \mathcal{H} \) denotes the function:
   \[
   \mathcal{H}(E_i^j - \bar{E}) = \begin{cases} 
   1 & \text{si } E_i^j \geq \bar{E} \\
   0 & \text{si } E_i^j < \bar{E}
   \end{cases}
   \]
   and
   \[
   \mathcal{H}(\bar{\sigma} - \sigma_i^j) = \begin{cases} 
   1 & \text{si } \sigma_i^j \leq \bar{\sigma} \\
   0 & \text{si } \sigma_i^j > \bar{\sigma}
   \end{cases}
   \]
7. **Compute the inverse FT of each block**, then multiply it by the synthesis window \( w_s \):
   \[
   \tilde{s}(t) = \sum_{i=1}^{N} \sum_{j=1}^{k} \hat{P}_i^j(t) + r(t) \tag{3}
   \]

It turns out that the optimal thresholds depend on the signal. More precisely, they depend on the nature of the signal. But this is not really a problem, since in almost all practical cases, we already know what is the nature of the signal. For example, we can predetermine optimal thresholds for a Doppler signal, and these thresholds would also be good for any other signal having the same nature as the Doppler signal, such as FM signals or any modulated signals having a continuous phase (e.g., CPFSK or MSK).

![Fig. 3](https://example.com/fig3.png)

**Fig. 3.** Learning process: each of the seven grey rectangles corresponds to the maximum SNR region. The red area corresponds to the intersection between the grey regions. The thresholds in this region are considered as optimal thresholds.
1.2 Results and discussions

The denoising method is tested using a synthetic Doppler signal and a real-world signal (ECG). The obtained results are compared with wavelet approach [12], [13], [?2] and EMD-based denoising methods [9]. First we proceed by estimating the threshold values $\sigma$ and $E$ by learning. Figures ?? and 4 show the variations of SNR$_{out}$ of the reconstructed Doppler signal versus a range of possible thresholds values. The frequency threshold range is chosen so as $\sigma \in [0, f_m(N-1)]$ with $f_m(N-1)$ denotes the mean frequency of the $(N-1)^{th}$ IMF, which is considered as narrow-band. The energy threshold $E \in [0, S]$, where $S$ is the maximum energy threshold defined as $S = C \sqrt{E_{1_{ws}} 2 \ln \lambda}$, where $E_{1_{ws}}$ denotes the energy of a window from the first mode of length $'\lambda'$ (it’s worth recalling that the first mode is considered as noise) and $C$ is a constant. Thus, the value of SNR$_{out}$ is calculated for all couples $(\sigma, E)$. It should be noted that for some particular combinations, the energy and bandwidth criterion are not suitable, which leads to a low SNR$_{out}$. In contrast, there are other combinations $(\sigma, E)$ resulting in an optimal SNR$_{out}$, which results in a region of optimal combinations. The retained couple is chosen as the barycenter of this region $(\sigma = 1.8, E = 0.35)$.

As for the mode reconstruction, $\tilde{p}_i(t)$, (Eq. 2), we chose $w_a = w_s = \sin(\pi \frac{t}{L} - \frac{i - 1}{2})$ which is a smooth window. The resulting block is thus guaranteed to be circular smooth. Hence, no irregularities will affect the FT. All simulations are performed using sinusoidal window.

The robustness of the method is proved by calculating the SNR$_{out}$ for the Doppler signal (Figure 5), for a range of SNR$_{in}$ values, using 1024 and 4096 samples. EMD-FT and EMD-CIIT [9] have almost similar performance for small SNR$_{in}$ with a small advantage for the EMD-FT. These two methods offer better performance than the Bayesien and the wavelet methods [13], [12]. However, for higher values of SNR$_{in}$, the EMD-FT gives the best performances in terms of SNR$_{out}$. It is worth noting that for all aforementioned EMD-based methods, the more data points we have (number of samples), the better performance we get.

We have estimated the processing time of these five methods (Table 1). Simulations are conducted on a Intel Core i3 (3rd generation) 3120M/2.1 GHz, with 6 GB of RAM. The results show that our denoising method is faster than the others, which make it more useful for real-time applications.

The proposed method is also tested for a real-world ECG signal. Three signals are presented in Figure 6, the original ECG signal $s(t)$, its noised version $x(t)$ and its denoised version $\tilde{s}(t)$. The results show that the main

![Fig. 4. SNR$_{out}$ of the Doppler signal reconstructed in function of (a) both the frequency threshold $\sigma$ and the energy threshold $E$. (b) in function of the energy threshold $E$ (c) in function of frequency threshold $\sigma$.](image)

![Fig. 5. SNR$_{out}$ versus SNR$_{in}$ for the reconstructed Doppler signal (a) 1024 samples (b) 4096 samples.](image)

<table>
<thead>
<tr>
<th>Methods</th>
<th>Number of samples</th>
<th>Processing time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hard-TI [13]</td>
<td>0.0324</td>
<td>0.0672</td>
</tr>
<tr>
<td>Soft-TI [14]</td>
<td>0.0291</td>
<td>0.0689</td>
</tr>
<tr>
<td>Bayesien [12]</td>
<td>0.0092</td>
<td>0.0248</td>
</tr>
<tr>
<td>EMD-CIIT [9]</td>
<td>1.5817</td>
<td>3.5335</td>
</tr>
<tr>
<td>EMD-FT</td>
<td>0.0067</td>
<td>0.0228</td>
</tr>
</tbody>
</table>
characteristics of the signal are well preserved, while reducing enormously the noise.

![ECG signal](image)

**Fig. 6.** ECG signal (original, noisy and denoised).

### 1.3 Conclusion

This paper presented a new EMD-based method associated with a frequency analysis for denoising. The first results are very promising and show the importance of frequency thresholding. Indeed, the method leads to a SNR of the order of 20 dB for very low SNR. The reduced processing time should be emphasized, which permits the use of this method for real-time applications. This method should be validated using a larger class of real-world signals contaminated with different types of noise, while varying the window length, so we can associate an optimal window length to each case of study. Also, a special attention should be paid to mathematical formulation of this method, especially the learning process.

### References


