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Exponential sine sweeps for the autonomous estimation of nonlinearities and errors assessment by bootstrap
Application to thin vibrating structures

Marc Rébillat¹, Kerem Ege², Maxime Gallo², Jérôme Antoni²
A linear behavior in vibroacoustics?

- Geometrically nonlinear vibrations of plates, shells:

  when the amplitude of the transverse displacement $w$ exceeds the plate/shell thickness $h$

  $\rightarrow$ jump phenomenon, hysteresis, internal resonance

- Boundary conditions, joints, contacts...

Example: Gong

Cyril Touzé, Olivier Thomas
A linear vibroacoustic measurement?

How to extract the linear/nonlinear parts of a typical vibroacoustic measurement?

How to estimate the effects of experimental noise?
Outline

- Estimation of Parallel Hammerstein models
  - Theory
  - Application to a vibrating plate
  - Separation of the intrinsic nonlinear contributions

- Improvements of the method (sweep repetition)
  - Theory
  - Noise estimation by synchronous averaging
  - Uncertainty estimation by bootstrap
  - Autonomous kernel order estimation

- Conclusion and perspectives
Parallel Hammerstein models estimation

⇒ Slightly nonlinear system modeled as parallel Hammerstein models

⇒ Kernels easily estimated using exponential sine sweeps
(see [Farina, AES, 2000], [Rébillat, JSV, 2011] or [Novak, IEEE Instrumentation, 2011] for example)

⇒ Separation of the linear/nonlinear orders
Parallel Hammerstein models estimation

Rébillat et al., JSV, 330, 2011

Separation of the linear/nonlinear orders in time domain
Application to a vibrating plate

- Clamped Steel plate (1mm)
- Shaker + Accelerometer

One sweep of **20 seconds**
[20Hz-1kHz]

After deconvolution:

Importance of the **length of the sweep** for time domain separation

**Identified Kernels of the system**

- **Kernel $h_1$ (linear part)**
- **Kernel $h_2$**
- **Kernel $h_3$**
- **Kernel $h_4$**
Improvement of the method: Sweep repetition & Noise estimation

- Measured signal = system response + experimental noise \[ x(t) = s(t) + n(t) \]

- Repetition of the same sweep \( K \) times

- Estimation of the noise \( \bar{n}(t) = x(t) - \bar{s}(t) \)

using the time synchronous averaging of the system response \( \bar{s}(t) \)

\[
\bar{s}(t) = \frac{1}{K} \sum_{k=0}^{K-1} [x(t - kT)]
\]
Application to a vibrating plate – Noise estimation

- Free-free damped steel plate (1mm)
- Shaker + Accelerometer

Influence of sweep repetitions (number of periods K)

Increase of K

\[ \tilde{s}(t) = \frac{1}{K} \sum_{k=0}^{K-1} [x(t - kT)] \]

Synchronous averaging on more periods

\[ \rightarrow \] Better extraction of the noise

\[ \rightarrow \] More precise estimations of the kernels of high orders
Uncertainty of measurement using Bootstrap

How to study the variability of the measurements? → Bootstrap method

B random sample with replacement (of K sweeps each time)
Synchronous averaging on K repetitions → B different kernel estimations

Identified Kernels of the system for B=10 and K=19

Way to identify and quantify variability (uncertainty) of plate nonlinearities estimations

Kernel $h_1$ (linear part)

Kernel $h_2$

Kernel $h_3$

Kernel $h_4$
Uncertainty of quantification

- **K repetitions** of exponential sine sweeps (ESS)
- **B estimations** of the response
- **Estimation of parallel Hammertstein model** (performed B times)
- **Mean over the B estimations**
- **1 mean estimation of the N Kernels**
- **B estimations of the « estimation noise » on the N kernels**
No averaging (K=1, B=150)
Averaging (K=19, B=150)

Order 1 (SNR=51 dB)

Order 2 (SNR=33 dB)

Order 3 (SNR=33 dB)

Order 4 (SNR=26 dB)

Order 5 (SNR=29 dB)

Order 6 (SNR=24 dB)
Uncertainty of quantification

B estimations of the N kernels

Mean over the B estimations

1 mean estimation of the N Kernels

SNR(K)

B estimations of the «estimation noise» on the N kernels

1 mean estimation of the uncertainty on the N Kernels

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No averaging (K=1, B=150)
Averaging (K=19, B=150)

Order 1 (SNR=51 dB)

Order 2 (SNR=33 dB)

Order 3 (SNR=33 dB)

Order 4 (SNR=26 dB)

Order 5 (SNR=29 dB)

Order 6 (SNR=24 dB)
Effect of repetition number $K$ on SNRs

$$\text{SNR}_{th}(K) = \text{SNR}(1) + 3 \log_2(K)$$
Autonomous kernel order estimation

Example on the suspended damped plate (5 different gains)

Estimation of the proper kernel order $N$ to identify

Threshold defined following a statistical F-test (Fisher)
Example of autonomous Kernel estimation
Conclusion & Perspectives

- Original method to **estimate the nonlinearities** of a vibro-acoustical structure
  - Slightly nonlinear systems modelled as **parallel Hammerstein models**
  - Kernels easily estimated using **exponential sine sweeps**

- **Improvements** of the sine sweep method
  - Repetition of the excitation signal – $K$ sweeps
  - Extraction of the noise through time synchronous averaging
  - Uncertainty estimation by bootstrap
  - Autonomous kernel order estimation

- **Perspectives**
  - Comparisons with other methods (spectral domains): **periodic multisines**...