FIR filter-based online jerk-constrained trajectory generation

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Abstract

In the context of human-robot manipulation interaction for service or industrial robotics, the robot controller must be able to quickly react to unpredictable events in dynamic environments. In this paper, a FIR filter-based trajectory generation methodology is presented, combining the simplicity of the analytic second-order trajectory generation, i.e. acceleration-limited trajectory, with the flexibility and computational efficiency of FIR filtering, to generate on the fly smooth jerk-constrained trajectories. The proposed methodology can generate synchronized (fixed-time) and time-optimal jerk-limited trajectories from arbitrary initial velocity and acceleration conditions within 20 microsecond. Other jerk-constrained trajectories such as jerk-time fixed trajectories, which are particularly suitable for vibration reduction, can be easily generated. Experimental validations carried out on a seven axis Kuka LBR iiwa are presented.

Keywords: Online trajectory generation, time-optimal trajectory, jerk, FIR filter, vibration.

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1. Introduction

Jerk limitation of robotic systems trajectories is a well-established method for smoothing the dynamic behaviour, reducing the vibrations and the system wear and improving the trajectory tracking performances (Macfarlane and Croft (2003)). Initially used in industrial handling and manufacturing robotics, the jerk-bounded trajectories can be advantageously exploited in mobile robotics (Broquere et al. (2008); Yin et al. (2012)), in human-robot collaboration and in human motion reproduction (Johnson et al. (2012); Frisoli et al. (2013); Dinh et al. (2015)). One notes that in these more recent areas, the trajectory generation problem have to be solved with online capability. Indeed robots attempt to instantaneously react to unpredicted events originating from the environment and potentially from the physical human-robot interaction.

Among the significant amount of efficient and elegant solutions to the jerk-bounded trajectory generation that can be found in the literature (Gasparetto and Zanotto (2008); Boryga and Grabos (2009); Macfarlane and Croft (2003); Liu et al. (2013)), jerk-limited profile with piecewise constant jerk function is a widespread solution used by the motion systems of machine-tools and industrial robots (Lange and Albu-Schffer (2016)). A jerk-limited trajectory can be defined as the time-optimal solution to the problem of jerk limited rigid body control with velocity, acceleration and jerk constraints and can be generated using several approaches.

In the first works, jerk-limited trajectories were usually defined analytically by computing optimal profiles offline, and implemented with some a priori information about the systems trajectories (Erkorkmaz and Altintas

(2001)). More recently, analytic solutions to the time-optimal and time-fixed jerk-limited online profile generation problem were proposed (Jeong et al. (2005); Haschke et al. (2008); Knierim et al. (2012)). The computing time of such algorithms is about 350 microseconds for a single trajectory, which can be a significant limitation for multi-axis systems during one control cycle (typically 1 millisecond). In Kroger et al. (2006); Kroger and Wahl (2010), Kroger introduce an elegant framework based on the current state of the system and acceleration-limited trajectories. The time-optimal jerk-limited solution is presented, but not detailed, in Kroger (2011). This algorithm belonging to the well-known Reflexxes Motion Library is efficient for multi-axis system. One notes that this library is no longer freely available for robotics research.

Another interesting approach to naturally comply with the online capability of the trajectory generation consists in using a filter-based approach. In Lu (2008), the author presents an original jerk-limited time-optimal control (JCTOC) scheme, i.e. a dynamic filtering or closed-loop filtering approach. The proposed controller is time-optimal according to the constrained jerk and is, by definition, adapted to real-time use, but the kinematic constraints on the maximum values of the velocity and acceleration are not considered. A similar approach, but with enhanced capabilities, is described in Gerelli and Guarino (2010). The proposed dynamic filter-based trajectory generation can handle freely assignable bounds on the velocity, the acceleration and the jerk, but the time-fixed solution and consequently the multi-axis synchronization problem is not taken into consideration in this approach. A simpler approach consists in using FIR (Finite Impulse Response) filters properties to shape the trajectory according to a given set of kinematic, time and frequency constraints. This open-loop approach conducts to less complex algorithms, which can be naturally adapted to the real-time trajectory generation. In Olabi et al. (2010), a FIR filter-based approach is exploited to generate the tool feedrate of a machining robot with limited jerk. The jerk-limited trajectory is obtained offline by convolving a pre-calculated low complexity acceleration-limited profile with a moving average filter, i.e. a FIR filter with all weight coefficients set equal to one. In Biagiotti and Melchiorri (2012), the FIR filter-based approach is generalized for the online trajectory generation. Starting from a rough step input, a cascade of n FIR filters generates a trajectory of order n respecting the kinematic constraints. The time-fixed trajectory generation problem and multi-axis synchronization can be solved using this approach. However, the proposed online generator can only deal with symmetric kinematic constraints and the time-optimality of the trajectory is not assured between via-point. Moreover, it is not possible to specify on the fly desired values of time-derivative of the position (velocity, acceleration and jerk) different from zero at the via points. On the other hand, the strength of the previous FIR-filter based approach relies on the fact that the smooth online trajectory generation can be efficiently combined with the interesting properties of FIR filters in the frequency domain for vibration reduction.

On this subject, previous works exploiting the FIR filter-based approach (Olabi et al. (2010); Bearee and Olabi (2013)) demonstrated that the FIR filter time, i.e. the constant jerk-time for a jerk-limited trajectory, can be tuned to cancel the motion-induced vibrations of an undamped system. In

Bearee (2014), the author proposes an improvement by using asymmetric jerk profiles, i.e. by using non-equal FIR filter coefficients, to take into account the damping coefficient of the flexible system. This approach was generalized and improved in Biagiotti et al. (2016) by using trajectories based on an exponential jerk profile and by deriving analytically the new FIR filter parameters that guarantee the residual vibration suppression.

In this work, we propose a trajectory generator that exploits the FIR filtering approach to generate on the fly jerk-constrained trajectories, i.e. jerk-limited trajectories or jerk-time fixed trajectories (with constant jerk-time). The trajectories are obtained by the convolution of FIR filters with low computational complexity acceleration-limited trajectories. The analytical formulation of the initial acceleration-limited trajectory takes account of the FIR filter time and the system trajectories can be modified during execution time while ensuring velocity and acceleration continuity. The following properties regarding time-constraints, multi-dimensional case and vibration-constraints can be chosen online:

- time-optimal jerk-limited trajectory;
- time-fixed jerk-limited trajectory and multi-axis synchronization;
- jerk-time fixed solution for systems sensitive to low-frequency vibration.

The next section presents the methodology adopted for the generation of jerk-limited trajectories. Sections 3 and 4 detail the procedure to generate respectively online jerk-time fixed trajectories and time-optimal jerk-limited trajectories. In Section 5, a fixed-time solution for multi-axis motion synchronization is proposed. Experimental results obtained with a collaborative 7 axis robot are presented in Section 6. Conclusions are drawn in Section 7.

2. FIR filter-based Jerk-constrained trajectory generation

In the remainder of this paper, the subscript f stands for "filtered", i.e. denotes the variables of the jerk-limited profile, which is the output of the algorithm. Moreover, for the online trajectory generation, it is assumed that the final velocity and acceleration are zeros, since if no new set-point is given, the goal is reached and the motion stops.

2.1. Proposed methodology for the trajectory generation

The generation of time-optimal acceleration-limited trajectories for a rigid system, i.e. the time-optimal control problem for a double integrator submitted to kinematic limitations on the first and second time derivative of the position, can be easily solved analytically. This low computational complexity approach is well adapted for online trajectory generation, but results in unsmooth profiles with step accelerations (bang-bang or bang-cruise-bang type control) and triangular or trapezoidal shaped velocity. Considering a FIR filter defined for implementation as

$$a_{f,k} = \frac{1}{N} \sum_{i=1}^{N} c_i \cdot a_{k-i+1}, \tag{1}$$

with respectively $a_{f,k}$ and a_k the acceleration output and input value of the filter at time $k.T_e$ (T_e being the sampling time of the signal), N the number of taps of the filter calculated as the integer part of T_j/T_e, T_j the filter time

(T_j is practically chosen as a multiple integer of T_e) and $c_i = \{c_1, \ldots, c_N\}$ the filter coefficients. Convolving an acceleration step function of value a_{max} with a sliding average FIR filter ($c_i = 1, i \in \{1, \ldots, N\}$) produces a ramp function with ending value a_{max} and a slope value, i.e. a jerk value noted j_{max}, given by

$$j_{max} = a_{max}/T_j. \tag{2}$$

Therefore, as illustrated in Fig. 1, an acceleration-limited profile may be turned into a jerk-limited profile by FIR filtering. Unfortunately, FIR filtering of a time-optimal acceleration-limited trajectory does not necessarily produce a time-optimal jerk-limited trajectory. Indeed, the jerk limitation given by (2) could be exceeded and moreover, the initial acceleration and velocity conditions being not explicitly taken into account, there is no guarantee that the desired position will be reached. Fig. 2 shows the trajectories resulting from FIR filtering (averaging filter) of different acceleration-limited profiles and illustrates the need for the adaptation of the initial acceleration-limited profile in view of producing the expected jerk-constrained trajectory (time-optimal or time-fixed or jerk-time fixed).

The methodology detailed in this paper for the online generation of jerk-constrained trajectories is based on the convolution of the adapted version of an acceleration-limited trajectory with a time-varying FIR filter (with coefficients, number of taps and initial state that can change over time) inducing the final jerk-constrained trajectory. The adaptation stage ensures the compliance of the filtered output trajectory with the initial conditions and the selected property, i.e. the time-optimal or the time-fixed or the jerk-time

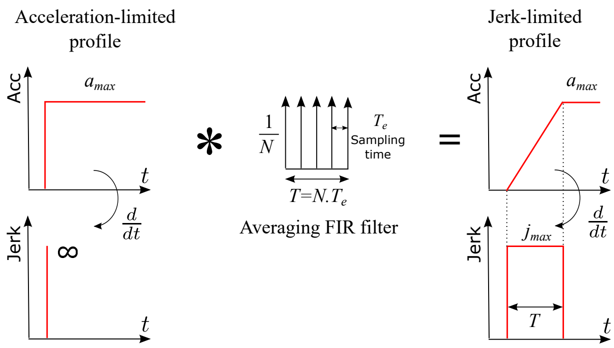

Figure 1: FIR filtering principle for jerk limitation of an acceleration-limited profile.

fixed solution. The principle of the proposed algorithm is described in Fig. 3. An event, which can be triggered by a new reference coming from a sensor, from a manual control or from a given dataset of via points, is defined as a new set of inputs for the algorithm. The inputs are the new set-point q_e, the associated kinematics limits (v_{max}, a_{max}, d_{max}, j_{max}) and the current state of the generator (q_{0f} , v_{0f} , a_{0f}). The algorithm is divided into two stages. The first stage consists in generating an analytically-defined acceleration-limited trajectory, which is adapted for the following FIR filtering stage. The output of the filtering stage is a jerk-constrained trajectory, which fulfills the requirements in terms of final position, initial state and kinematic limitations on the velocity, acceleration and jerk. As detailed in the remainder of this paper, the adaptation of the initial acceleration-limited trajectory is based on simple analytical relations. According to the current state of the trajectory and the selected property for the new trajectory, this adaptation process may be repeated iteratively according to a decision tree. The maximum number of iterations is 2 for the time-optimal trajectory case.

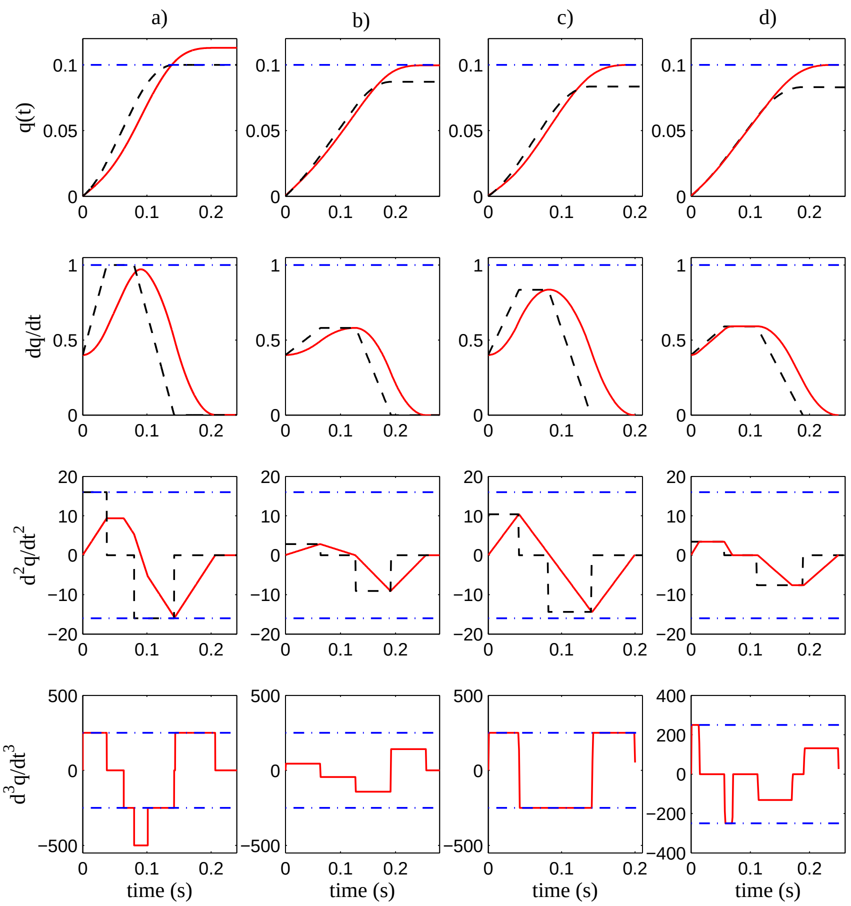

Figure 2: Examples of trajectories resulting from FIR filtering (averaging filter) of different acceleration-limited profiles. Solid and dashed lines indicate respectively the filtered and the initial profiles, dotted lines represent the kinematic constraints (set-point $q_e = 0.1m$, $v_m = 1m/s$, $v_{0f} = 0.4m/s$, $a_m = d_m = 16m/s^2$, $j_m = 250m/s^3$. a) Filtered trajectory resulting from a time-optimal acceleration-limited profile, b) the initial acceleration profile is adapted to generate jerk-time fixed trajectory ($T_j = a_m/j_m = 0.064ms$), c) the initial profile is adapted to generate time-optimal jerk-limited profile, d) the initial profile is adapted to generate time-fixed jerk-limited trajectory ($T = 0.25s$)

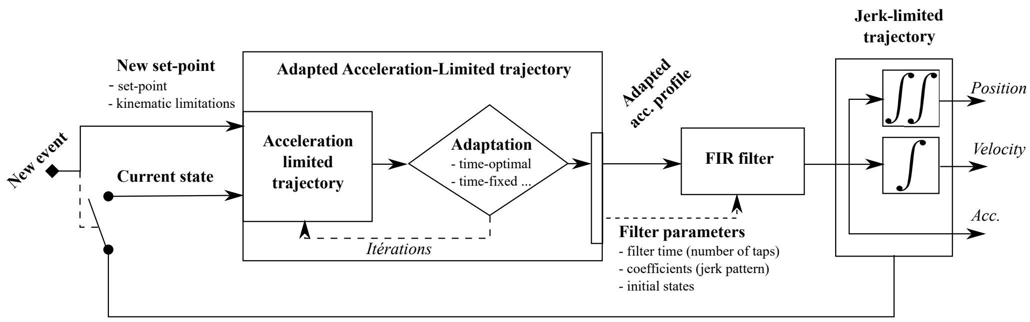

Figure 3: Proposed online trajectory generation algorithm based on FIR filtering of adapted acceleration-limited profile.

The main notations used in this document are defined in the Table 1 below.

2.2. Time-optimal acceleration-limited trajectory generation

The basis of the methodology consists in generating an acceleration-limited trajectory, which respects the following kinematic constraints and boundary conditions:

$$\begin{aligned} |v(t)| &\leq v_{max}, \\ -d_{max} \leq a(t) &\leq a_{max}. \end{aligned} \tag{3}$$

The generic acceleration-limited profile, presented in Fig. 4, can be divided into three stages: first an acceleration stage with constant acceleration value noted a_r and duration time T_a; second a constant velocity stage with duration time T_v and constant velocity value v_r; finally, a deceleration stage with constant acceleration value noted d_r and duration time T_d. The initial values of the position and velocity are noted q_0 and v_0 and the final position is q_e. For time-optimality reasons, the reached value a_r and d_r are set equal to

Symbol	description
T_a, T_v and T_d	Duration of the stages of the AL profile
q_{0f}, v_{0f} and a_{0f}	Initial conditions of the jerk-limited trajectory
q_0 and v_0	Adapted initial conditions for the AL profile
q_e	Position set-point
s	Direction (sign) of the trajectory
v_{max}, a_{max}, d_{max} and j_{max}	Kinematic bounds
v_r, a_r and d_r	Reached values of velocity and acceleration
T_j, T_{ja}, T_{jv} and T_{jd}	Length of the constant jerk stages (FIR filter time)

Table 1: Main notations

their respective bounds a_{max} and d_{max}.

The first step of the trajectory generation consists in expressing the distance noted $\Delta_{q,stop}$, which represents the minimum stopping distance. $\Delta_{q,stop}$ can be calculated as the traveled distance starting from the initial velocity v_0 to full stop with the maximum deceleration value:

$$\Delta_{q,stop} = \frac{v_0^2}{2 \cdot d_{max}}. \tag{4}$$

Posing $\Delta_q = q_e - q_0$ the distance to be traveled, if $|\Delta_q| \leq \Delta_{q,stop}$, then a full stop trajectory is required and the trajectory generation can be repeated with the new starting conditions $v_{0new} = 0$ and $q_{0new} = q_0 + \text{sign}(v_0) \cdot \Delta_{q,stop}$. A second specific case has to be checked if $|v_0| > v_{max}$. In this special case, which may occur if the constraint v_{max} is changed on the fly, the velocity has to be reduced to v_{max} as fast as possible and the trajectory generation can be repeated just as before posing $v_{0new} = \text{sign}(v_0) \cdot v_{max}$ and $q_{0new} = q_0 + \text{sign}(v_0) \cdot \Delta_{q,v_0}$, with $\Delta_{q,v_0} = (|v_0| - v_{max})^2 / (2d_{max})$. One notes that the

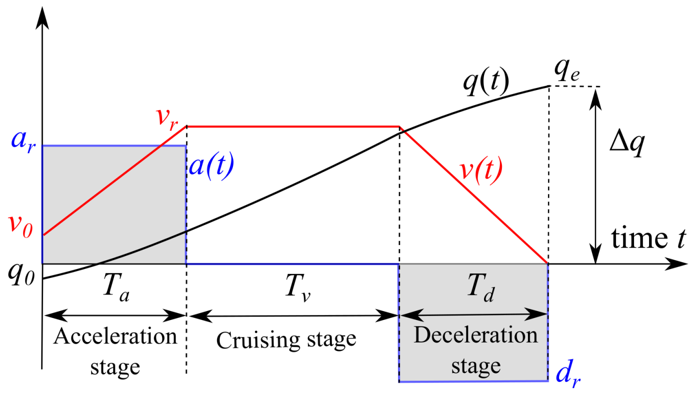

Figure 4: Generic acceleration-limited profile.

resulting trajectory is a double deceleration case. Finally, the last verification consists in defining the shape of the velocity profile (trapezoidal or triangular) according to the fact that the maximum velocity is reached or not. The minimum distance for which the maximum velocity is reached, noted $\Delta_{q,v_{max}}$, can be easily calculated considering $T_v = 0$ (see Fig. 4) and expressed as

$$\begin{aligned} \Delta_{q,v_{max}} &= \frac{(v_{max} + v_0) \cdot T_a + v_{max} \cdot T_d}{2} \\ &= \frac{v_{max}^2 - v_0^2}{2a_{max}} + \frac{v_{max}^2}{2d_{max}}. \end{aligned} \tag{5}$$

Then, if $|\Delta_q| \geq \Delta_{q,v_{max}}$, the maximum velocity is reached and the velocity profile is trapezoidal. Defining $s = \mathrm{sign}\,(\Delta_q)$ the sign of the motion, the resulting trajectory parameters a_r, v_r, d_r, T_a, T_v and T_d are given by

$$a_r = s \cdot a_{max};\ d_r = s \cdot d_{max};\ v_r = s \cdot v_{max}, \tag{6}$$

$$T_a = \frac{v_r - v_0}{a_r};\ T_d = \frac{v_r}{d_r};\ T_v = \frac{\Delta_q - s \cdot \Delta_{q,v_{max}}}{v_r}. \tag{7}$$

Otherwise, if $|\Delta_q| < \Delta_{q,v_{max}}$, the velocity profile is triangular and $T_v = 0$ in the previous equation. The reached velocity v_r can be calculated replacing

$\Delta_{q,v_{max}}$ and v_{max} in (5) by Δ_q and v_r. The trajectory parameters are

$$a_r = s \cdot a_{max};\ d_r = s \cdot d_{max};\ v_r = s \cdot \sqrt{\frac{2 \cdot a_{max} \cdot d_{max} \cdot \Delta_q - d_{max} \cdot v_0^2}{a_{max} + d_{max}}}, \quad (8)$$

$$T_a = \frac{v_r - v_0}{a_r};\ T_d = \frac{v_r}{d_r};\ T_v = 0. \quad (9)$$

3. Adaptations for jerk-time fixed trajectory

This section is concerned with the definition of the initial acceleration-limited profile in view of producing, after FIR filtering, a jerk-time fixed trajectory, i.e. a trajectory with the constant jerk stages equal to a desired value T_j, as described in Fig. 2(b). Hence, for this kind of trajectory the filter time T_j is fixed (constant number of taps of the filter) and the initial condition of the filter is given by the current acceleration a_{0f}.

3.1. Handling initial conditions

As previously seen in Fig. 2(a), if initial velocity (or acceleration) are different from zero, the filtered time-optimal acceleration-limited trajectory does not respect the final position. Hence, the parameters $(q_0; v_0)$ used to generate the initial acceleration-limited profile have to be adjusted according to the real current state $(q_{0f}; v_{0f}; a_{0f})$. Fig. 5 illustrates the definition of the acceleration-limited profile according to the initial conditions. Deriving the difference between the non-filtered and filtered velocity and position profiles, the analytical relationship between the initial parameters can be given as.

$$v_0 = v_{0f} + \delta_v,\ q_0 = q_{0f} + \delta_q, \quad (10)$$

with

$$\delta_v = \int a_f(t)dt - \int a(t)dt, \quad (11)$$

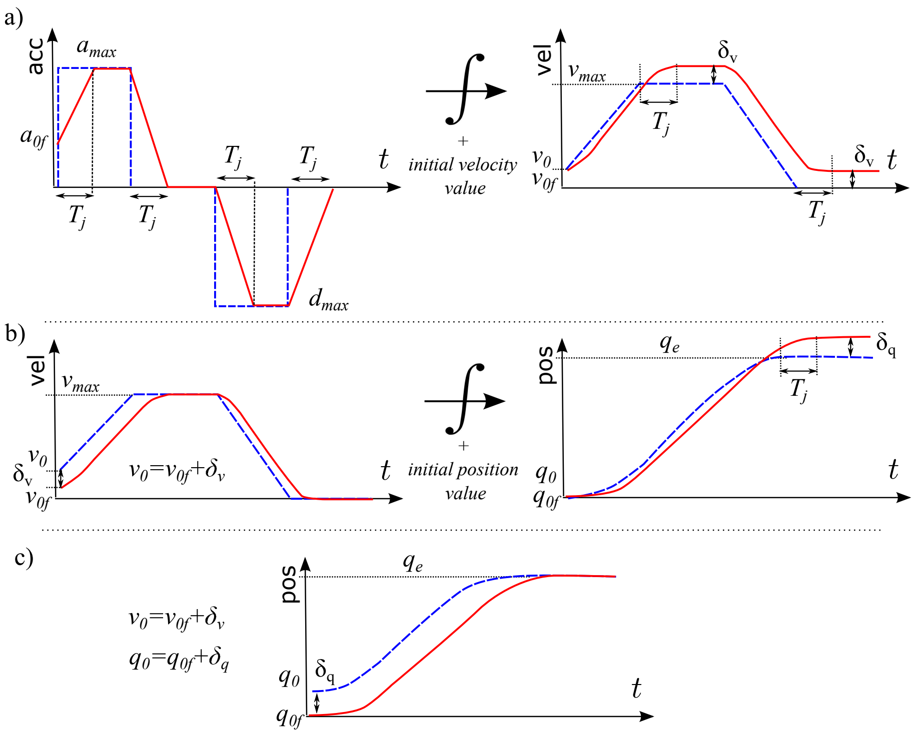

Figure 5: Principle of initial conditions definition for trajectory FIR filtering. a) Influence of the initial acceleration value on the filtered velocity: the maximum and final filtered velocity values present an overshoot of δ_v. b) Adaptation of the initial velocity condition v_0 of the acceleration-limited profile to fulfill the velocity constraints after filtering: the final filtered position presents an overshoot of δ_q. c) adaptation of the initial position condition q_0 of the acceleration-limited profile: the filtered trajectory fulfill the kinematic constraints.

$$\delta_v = a_{0f} \cdot \frac{T_j}{2}, \tag{12}$$

and

$$\delta_q = \int \left(\underbrace{v_{0f} + \int a_f(\tau)d\tau}_{v_f(t)} \right) dt - \int \underbrace{\left(v_0 + \int a(\tau)d\tau \right)}_{v(t)} dt, \tag{13}$$

$$\delta_q = v_{0f} \cdot \frac{T_j}{2} + a_{0f} \cdot \frac{T_j^2}{12}. \tag{14}$$

3.2. Jerk limitation and adaptations

For a given jerk-time T_j, the maximum jerk value resulting from the FIR filtering of an acceleration step of value a_{max} will be given by the relation (2). Now, considering a jerk-time fixed trajectory, the maximum jerk value could depend on the initial acceleration value a_{0f} too (the first acceleration step being $a_r - a_{0f}$). Hence, the maximum jerk value that can be reached for a jerk-time fixed trajectory can be expressed as

$$j_{max} = \max \left\{ \frac{|a_r - a_{0f}|}{T_j}, \frac{|a_r|}{T_j}, \frac{|d_r|}{T_j} \right\}. \tag{15}$$

To generate a jerk-time fixed trajectory, each stage of the initial acceleration-limited trajectory has to be low-bounded by T_j. Hence, if $T_i < Tj$ for $i \in \{a, v, d\}$, the adaptation consists in re-generating the acceleration-limited profile with a lower value of v_r calculated by imposing $T_i = T_j$ in (7) or (9) according to the fact that the maximum velocity value is reached or not.

For instance, Fig. 6 illustrates such adaptation. Based on the desired position, the initial conditions and the kinematic constraints, the initial acceleration-limited trajectory calculated using the relations (5) to (9) leads to T_a, T_v and T_d lower or equal to the FIR filter time T_j. The resulting

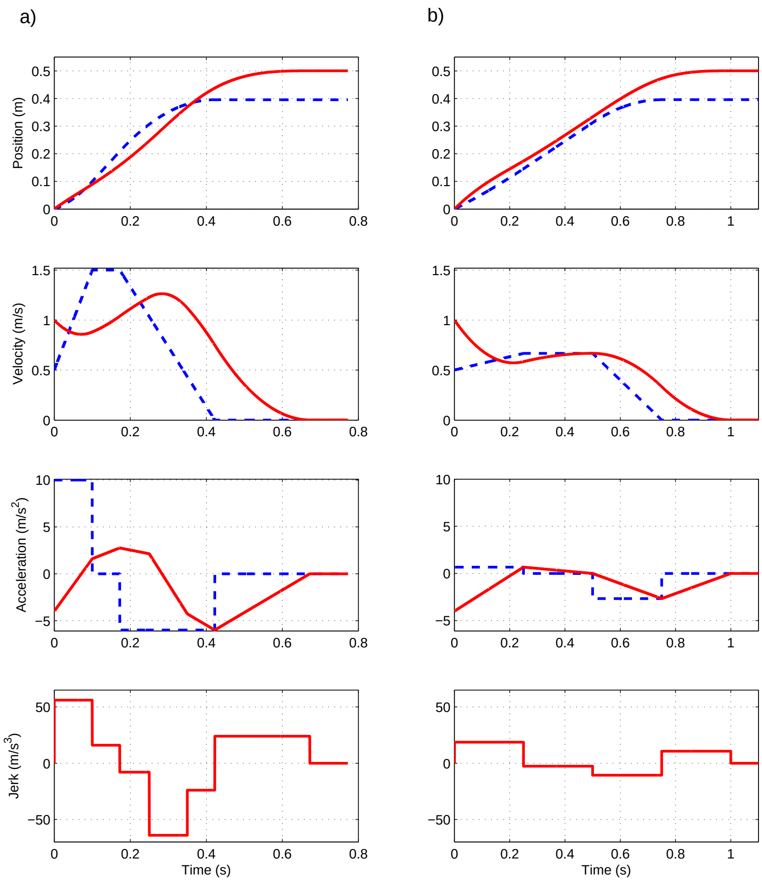

Figure 6: Example of adaptation of the initial acceleration-limited profile (dotted line) for the generation of jerk-time fixed trajectory ($\Delta q_f = 0.5\ m$, $a_{0f} = -4\ m/s^2$, $v_{0f} = 1\ m/s$, $a_{max} = 10\ m/s^2$, $d_{max} = 6\ m/s^2$, $v_{max} = 1.5\ m/s$ and $T_j = 0.25\ s$). a) non-adapted initial profile with T_a, T_v and T_d lower or equal to the FIR filter time T_j. b) adapted profile, $T_v = T_a = T_d = T_j$ and $|v_r| < v_{max}$ consequently.

filtered trajectory reached the maximum velocity, but is not jerk-time fixed (left plot in Fig. 6). Then, imposing each stage of the initial acceleration-limited trajectory to be equal to the desired filter time T_j, i.e. posing $T_v = T_a = T_d = T_j$ in (7), leads to the following new values of the reached velocity $v_r = (\Delta_q - v_0 \cdot T_j)/2T_j$, acceleration $a_r = (v_r - v_0)/T_j$ and deceleration $d_r = v_r/T_j$ for the definition of the adapted initial acceleration-limited profile (right plot in Fig. 6).

3.3. Advantage of fixed Jerk-time for vibration compensation

As mentioned in the introduction, online jerk-time fixed trajectories can be useful for vibration reduction. Indeed, similarly to input shaping methodology, the jerk-time (filter-time) and the jerk-shape (filter coefficients) can be calculated to reduce vibration. Hence, for an undamped dominating vibratory mode with natural period noted T_0, constant jerk stage T_j with time equal to an integer multiple of T_0 will suppress vibration (see Fig. 7). For a dominating low-damped vibration, the damping coefficient ζ can be taken into account by modifying the jerk shape using damped-jerk shape described in Bearee (2014) or exponential jerk shape detailed in Biagiotti et al. (2016). Considering a jerk-time fixed trajectory, the number N and the values of filter coefficients c_i in (1) can be fixed according to the modal parameters T_0 and ζ. For instance, damped-jerk trajectory (see Bearee (2014)) is obtained by setting $T_j = N.T_e$ and $c_i = (N - 1 + \zeta\pi(N + 1 - 2i))/(N - 1)$ with $N = \lceil T_0 \left(1 + 0.083\zeta + 0.047\zeta^2 + 7.1\zeta^3\right)/T_e \rceil$. This kind of online trajectory generation can be advantageously exploited to reduce the vibrations of a flexible robotic arm or the vibrations of the payload during motion.

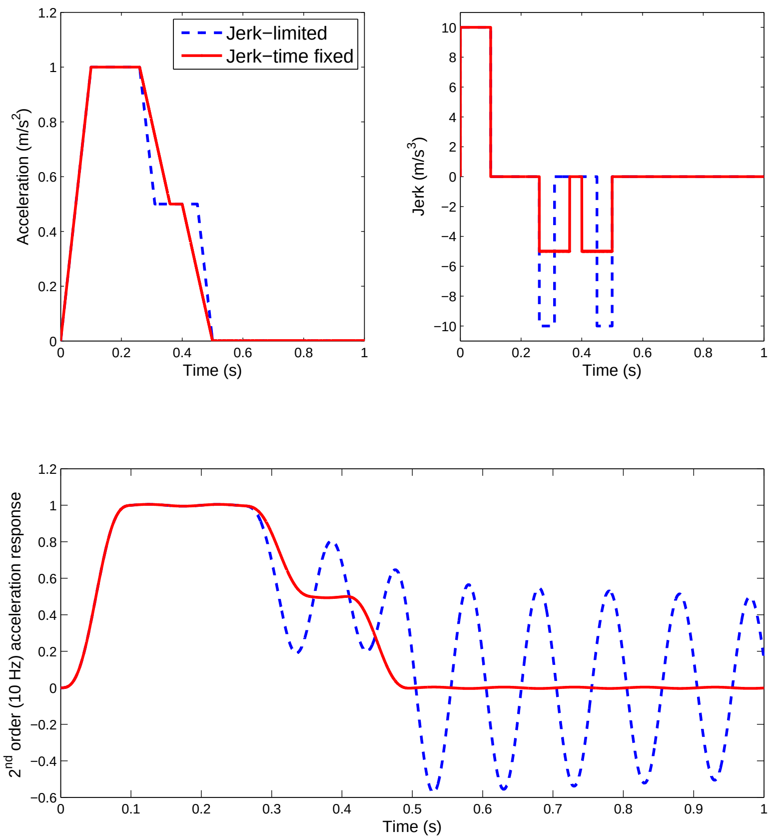

Figure 7: Comparative acceleration responses of a second order system (natural frequency of $f_0 = 10$ Hz and low damping of 5 %) to a jerk-limited acceleration profile (with $j_{max} = 10\ m/s^3$) and a jerk-time fixed acceleration profile (with $T_j = 1/f_0 = 0.1\ s$).

4. Adaptations for time-optimal jerk-limited trajectories

This section is concerned with the definition of the initial acceleration-limited profile in view of producing after FIR filtering a time-optimal jerk-limited trajectory, as described in Fig. 2(c). Fig. 8 shows the definition of the parameters for the jerk-limited profile. One can note that for obtaining this kind of trajectory by FIR filtering, the maximum jerk value should be fixed instead of the filter time. Hence, the filter time has to be dynamically switched. The trajectory starts at $t = t_0$ with a filter initialized at a_{0f} and a filter-time equal to T_{ja}. Then the filter is changed at $t = t_0 + T_a$ to a length T_{jv}, initialized at the value a_r. Finally, at $t = t_0 + T_a + Tv$ the length of the filter is set to T_{jd} with zero initial condition. According to the Fig. 8 notations, the different filter times can be expressed as

$$T_{ja} = \frac{|a_r - a_{0f}|}{j_{max}};\ T_{jv} = \frac{|a_r|}{j_{max}};\ T_{jd} = \frac{|d_r|}{j_{max}}. \tag{16}$$

4.1. Handling initial conditions

The relations (10), (11) and (13) are generic for the filtering of any trajectory, but the expressions (12) and (14) cannot be used for time-optimal generation since the length of the filter will not be constant in general case. The new relations are determined by calculating the difference between initial unfiltered and filtered trajectory with fixed maximum jerk value j_{max} in (11) and (13). The resulting δ_v and δ_q are given by

$$\begin{aligned} \delta_v &= s \cdot a_{0f} \cdot \left(\frac{2a_r - a_{0f}}{j_{max}}\right), \\ \delta_q &= s \cdot \left(\frac{a_r \cdot v_{0f} - (a_r + d_r) \cdot v_r}{2j_{max}}\right) + a_{0f}^2 \cdot \left(\frac{3a_r - 2a_{0f}}{12j_{max}^2}\right). \end{aligned} \tag{17}$$

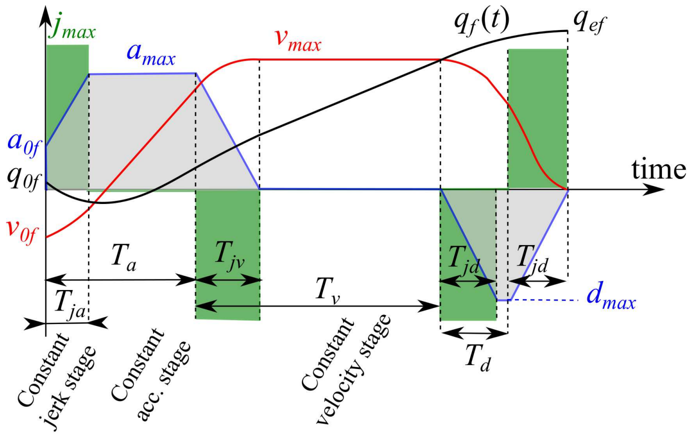

Figure 8: Parameters definition of the time-optimal jerk-limited trajectory.

One notes that the relations (17) depend on the reached velocity, acceleration and deceleration and has to be included in the set of equations (6-9). Practically, v_r, a_r and d_r are initialized to their bounds and the previous set of equations is solved iteratively to find the time-optimal profile according to simple rules described in the following part.

4.2. Decision tree for Time-optimality

As seen for the jerk-time fixed case, the initial constant velocity stage T_v has to verify $T_v \geq T_{jv}$ to ensure that the jerk bound j_{max} is respected. For time-optimality, other iterative adaptations of the initial acceleration-limited profile can be necessary. For instance, if $T_{ja} > T_a$ or $T_{jd} > T_d$ then the acceleration profile after filtering will saturate at a value lower than its bounds, respectively a_{max} and d_{max}, as depicted in Fig. 9 ref fixed. In that case, the initial acceleration-limited profile should be adapted in order to produce the desired time-optimal jerk-limited trajectory. Fig. 10 presents the decision table and the associated decision tree, used to select the adaptation to be applied to the acceleration-limited profile. Hence, for the case $T_{jd} > T_d$ and $T_v > T_{jv}$, a Type III-b adaptation is needed and the initial value of d_r cannot be reached. The adapted value of d_r is calculated by setting T_{jd} equal to T_d in (7) and (16), which leads to the following relation

$$\begin{aligned} T_{jd} = T_d &\Longrightarrow \frac{d_r}{j_{max}} = \frac{v_r}{d_r}, \\ &\Longrightarrow d_r = s \cdot \sqrt{j_{max} \cdot |v_r|}. \end{aligned} \tag{18}$$

One notes that according to the decision tree (see Fig. 10) the adaptations are limited to a maximum of 2 iterations.

5. Adaptations for time-fixed jerk-limited trajectories

In the general case, several Degrees-Of-Freedom (DOF) are controlled simultaneously to reach a targeted system position. Classically, it is wished that the motion of each DOF ends at the same time. Consequently, for a motion involving n DOFs, with $T_{f,i}$ the duration of the time-optimal jerk-limited trajectory for the i-th DOF, $n-1$ trajectories have to be adjusted to the fixed time constraint $T_{f,fix}$ given by the slowest DOF

$$T_{f,fix} = \max_i \left(T_{f,i}\right). \tag{19}$$

This adjustment can be done in several, very different ways. For rest to rest motion, a simple rescaling of the time evolution can be used, but for non-zero initial conditions this approach is not sufficient. Here, we propose one solution to easily compute without any iteration a fixed-time solution for the jerk-limited profile. First, one notes that after filtering, the resulting trajectory time is increased by the last filter time. Therefore, the relation between

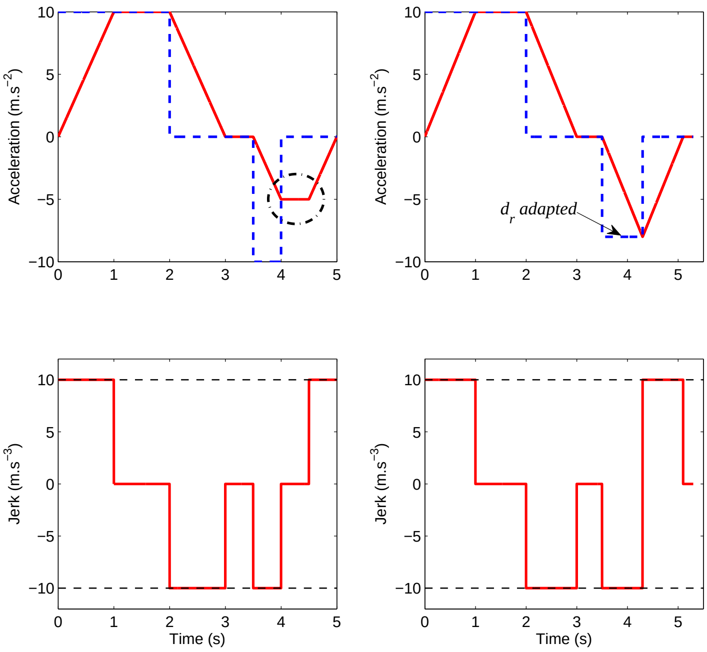

Figure 9: Example of an adaptation: Type III-b (see Fig. 10). To the left: non-adapted initial profile with $T_d < T_{jd}$. To the right, adapted profile with $T_d = T_{jd}$ and $d_r < d_{max}$.

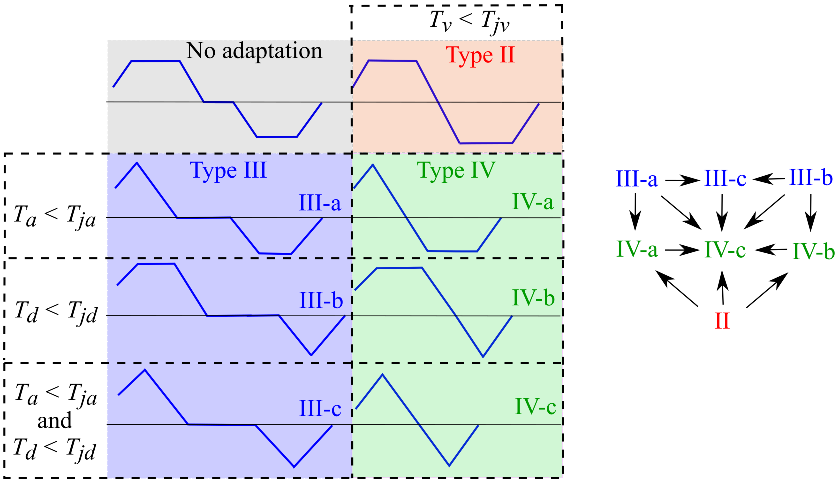

Figure 10: Selection table and associated Decision tree for the iterative selection of the initial acceleration-limited profile.

the duration of the initial acceleration-limited profile for axis i, noted T_i, the duration of the time-optimal jerk-limited trajectory $T_{f,i}$ and the duration of the deceleration stage $T_{jd,i}$ is given by

$$T_i = T_{f,i} - T_{jd,i}. \tag{20}$$

Now, keeping the value of $T_{jd,i}$ given by the time optimal calculation, noted $T_{jd,i}^{opti}$, we can define the rescaling parameter $\alpha_i = T_{fix}/T_i$, with $T_{fix} = T_{f,fix} - T_{jd,i}^{opti}$. The imposed time evolution of the time-fixed acceleration-limited profile is expressed as

$$\tau_{a,i} = \alpha_i T_{a,i};\ \tau_{v,i} = \alpha_i T_{v,i}; \tau_{d,i} = \alpha_i T_{d,i}. \tag{21}$$

Then, the successive filter times are set as follow

$$T_{ja,i} = \frac{|a_{r,i} - a_{0f,i}|}{j_{max,i}};\ T_{jv,i} = \frac{|a_{r,i}|}{j_{max,i}};\ T_{jd,i} = T_{jd,i}^{opti}. \tag{22}$$

Summarizing, the synchronization approach consists in defining a profile with the jerk set at its maximum value j_{max} for the first acceleration stage and preserving the original filter length $T_{jd,i}^{opti}$ during the deceleration stage. Fig. 11 described the proposed approach, the time rescaling leads to peak acceleration, deceleration and velocity values lower than their bounds. To calculate the initial time-fixed acceleration-limited trajectory, the only unknown parameters are the reached acceleration $a_{r,i}$ and deceleration $d_{r,i}$. These two parameters can be analytically defined as

$$d_{r,i} = \frac{s}{\tau_{d,i}} \cdot \left(v_{of,i} + a_{0f,i} \cdot \frac{T_{ja,i}}{2} + a_{r,i} \cdot (\tau_{a,i} + T_{jv,i}/2 - T_{ja,i}/2) \right) \tag{23}$$

with $a_{r,i}$ being solution of

$$\beta_0 + \beta_1 \cdot a_{r,i} + \beta_2 \cdot a_{r,i}^2 = 0, \tag{24}$$

with

$$\begin{aligned}
\beta_0 &= q_{0f,i} - q_{ef,i} + v_{0f,i} \cdot \left(\frac{T_{jd,i}}{2} + \tau_{a,i} + \frac{\tau_{d,i}}{2} + \tau_{v,i} \right) - \frac{a_{0f}^3}{6 j_{max,i}^2} \\
&\quad - s \cdot a_{0f}^3 \cdot \frac{T_{jd,i} + 2\tau_{a,i} + \tau_{d,i} + 2\tau_{v,i}}{4 j_{max,i}}, \\
\beta_1 &= \frac{\tau_{a,i}}{2} \cdot (T_{jd,i} + \tau_{a,i} + \tau_{d,i} + 2\tau_{v,i}) + \frac{a_{0f,i}^2}{2 j_{max,i}^2} \\
&\quad + s \cdot a_{0f} \cdot \frac{T_{jd,i} + 2\tau_{a,i} + \tau_{d,i} + 2\tau_{v,i}}{2 j_{max,i}}, \\
\beta_2 &= -\frac{a_{0f,i} + s \cdot \tau_{a,i} \cdot j_{max,i}}{2 j_{max,i}^2}.
\end{aligned}$$

Finally, based on the calculated time-optimal trajectory parameters, the time-fixed jerk-limited solution can be obtained without iteration by convolution of the acceleration profile given by the time evolution (21) and

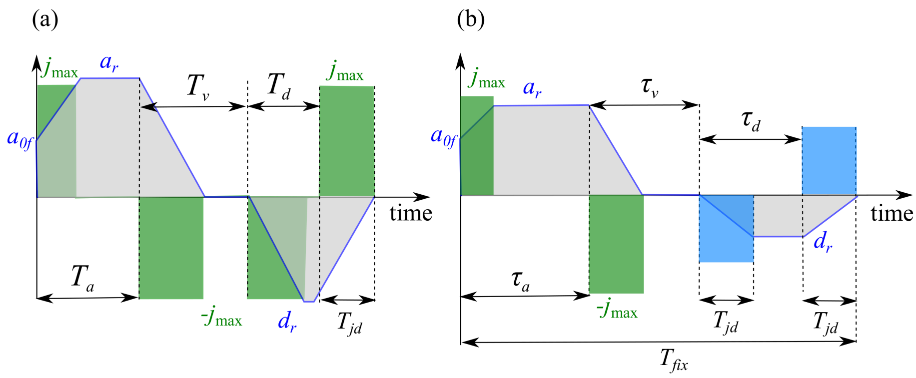

Figure 11: Methodology for time-fixed jerk-limited solution (b) based on the initial calculated time-optimal trajectory (a): the time evolution is rescaled, the value of the jerk is set to j_{max} for the first acceleration stage and the filter length T_{jd} is set equal to T_{jd}^{opti} for the deceleration stage.

the maximum acceleration and deceleration values (23–24), with the time-varying FIR filter following the relations (22). Fig. 13 gives an example of multi-axis synchronisation using the proposed methodology. Axis 2 is here the slowest axis. In this example, the initial value of axis 3 acceleration $a_{0f,3}$ is higher than the calculated reached acceleration value $a_{r,3}$ given by (24) for the time-rescaled solution. For this special case, discussed in section 2.2, the fastest physical solution for the acceleration profile is to reached $a_{r,3}$ as fast as possible, then the trajectory generation is repeated with this new initial condition. For this reason, the resulting time-fixed trajectory of axis 3 (see Fig. 13) includes a double acceleration profile.

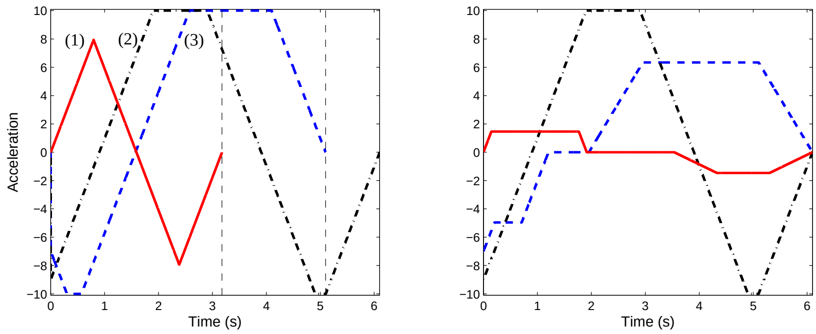

Figure 12: Example of three axis synchronization. Time-optimal trajectories to the left, time-fixed trajectories to the right.

6. Experimental results

The proposed trajectory generation method was implemented in C++. To validate its results, all types of configurations (time-optimal, time-fixed, fixed jerk-time) and initial conditions has been firstly simulated. In any case the algorithm generates the jerk-time fixed solution in less than 15 μs and the time-optimal jerk-limited case in less than 20 μs on a regular laptop i7/2.7GHz running with Windows 7 as operating system. A typical control cycle time being 1 ms, the proposed algorithm can easily handle any multi-axis system and save a significant amount of time to the controller for other tasks, e.g. monitoring, obstacle avoidance, reaction to unforeseen events or any data processing. Fig. 14 shows the result of five trajectories generated online with random set-points. Halfway, the kinematic bounds are lowered. On these trajectories one can observe that the bounds are respected. When lowering the bounds, the trajectories may be outside their limits because this online change can not be anticipated. In other words, the instant compliance

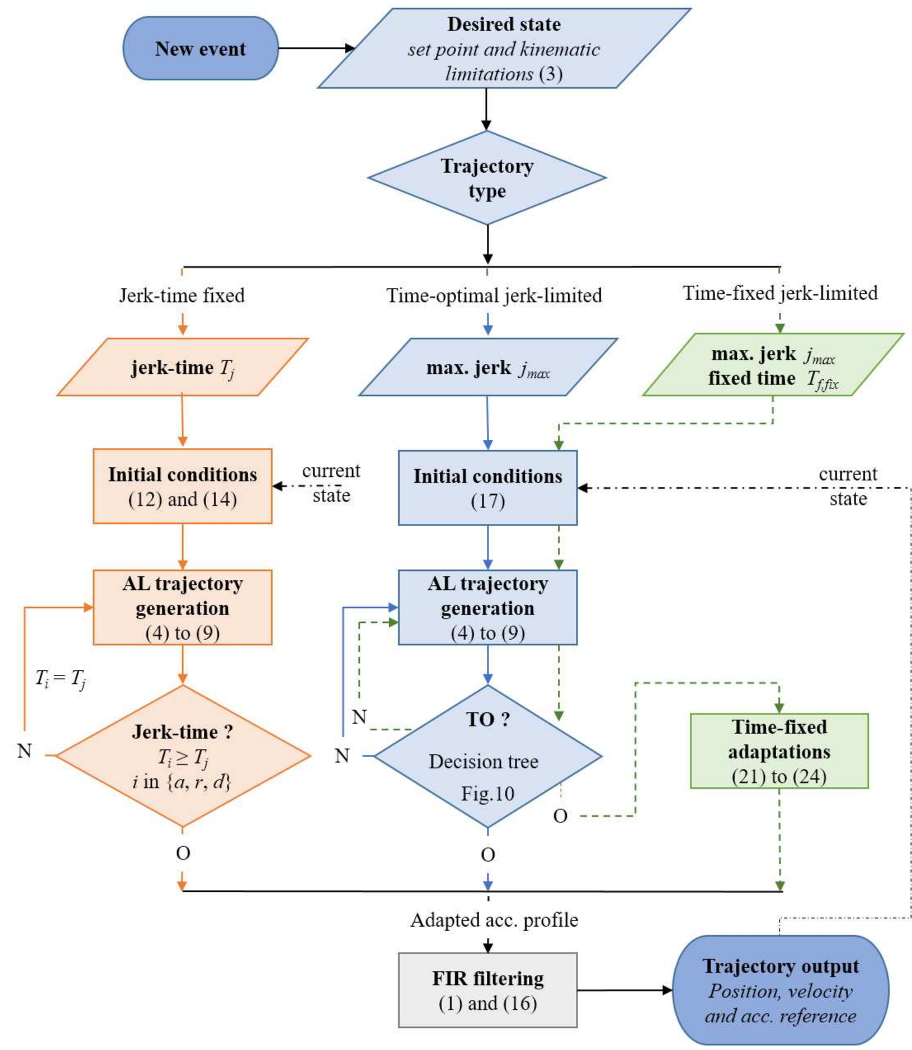

Figure 13: Flowchart summarizing the proposed methodology for the different types of trajectory: Jerk-time fixed, time-optimal jerk-limited and time-fixed jerk-limited.

with the new bound can be physically not allowed regarding the current state of the trajectory. In such case, the generator bring the trajectories back into their new limits in a time-optimal manner (double acceleration or deceleration profile).

Then, to illustrate the effectiveness of the proposed approach, experimental tests have been conducted using the 7 DOF Kuka lightweigth LBR iiwa. The Fast Robot Interface (FRI) of the KUKA *Sunrise.Connectivity* collection of open interfaces was used for real time communication between the robot controller and the trajectory generation algorithm embedded in an external computer, with a rate of 1 ms (see. Fig. 15). Fig. 16 and Fig. 17 present samples of two test cases using the time-optimal jerk-limited solution, respectively with and without multi-axis synchronization. The joint positions plots originate from the joint encoders measurement and the velocity and acceleration plots are obtained by time differentiation. For both acceleration plots, the acceleration output of the trajectory generation algorithm (dash-dotted thin black line) is superimposed to the joint acceleration. For these test cases, the set-points and the associated occurrence times (vertical lines) are predefined and sent as input data to the trajectory generation algorithm. The current set-point being not reached when a new set-point is sent, the initial velocity and/or acceleration conditions are different from zero. In addition, to illustrate the ability to deal with online kinematic constraints change, the vertical solid line indicates a new set-point with a new constraint on the maximum jerk value. As required, the new jerk constraint is immediately recovered (see the change of the acceleration slope after the time indicated by the full line), while acceleration and velocity constraints

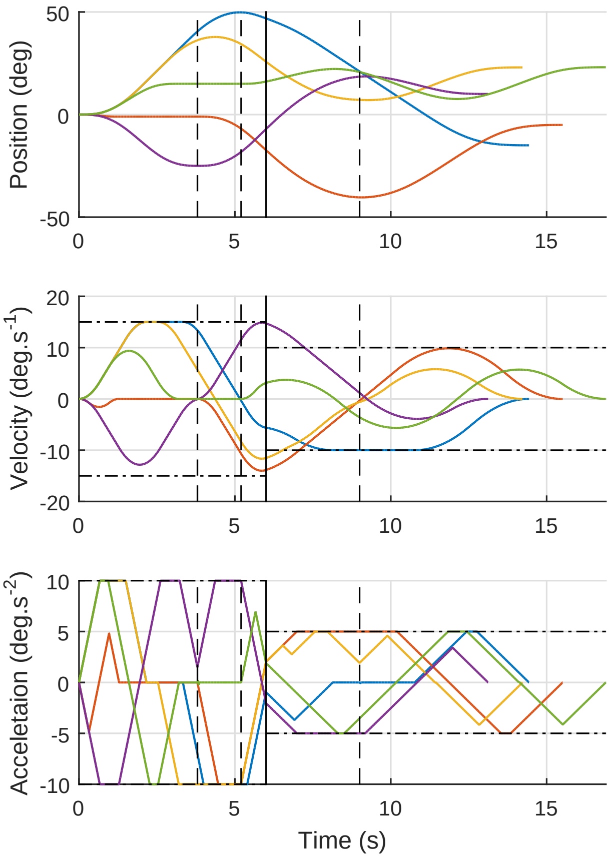

Figure 14: Example of dynamic generation of time-optimal jerk-limited trajectories. New set-points are given at times {3.8, 5.2, 6.0 and 9.0 sec.} (dashed lines) and the bounds are lowered at 5.2 s: from (15 deg.s^{-1}, 10 deg.s^{-2}, 15 deg.s^{-3}) to (10 deg.s^{-1}, 5 deg.s^{-2}, 3 deg.s^{-3}). The acceleration and velocity limits are represented as the horizontal dot-dashed lines.

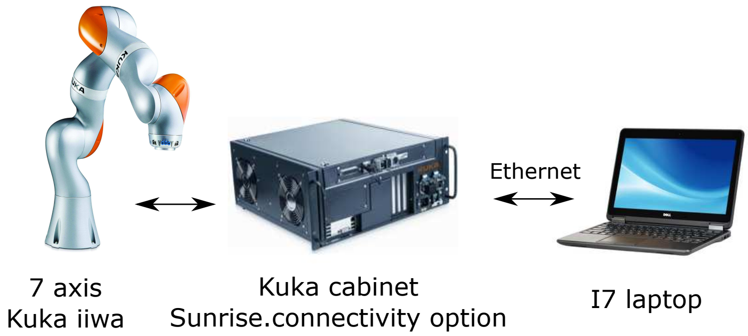

Figure 15: Experimental setup for the validation of the jerk-constrained trajectory generation.

are newly satisfied in minimum-time. The online trajectory generation have also been tested in a third practical test case with the set-points coming from a visual exteroceptive system (Basler ace acA640 CDD monocular camera). Two Kuka iiwa robot were used for this test case. A lemniscate reference path was initially defined for the end-point of the "master" robot. A visual marker put on the "master" robot end-point is tracked by the camera and the measured position is sent to the external computer at a sampling time of 15 ms, i.e. a frequency of almost 66 Hz. Then, the current reference set-point for each joint of the "slave" robot, determined using the inverse kinematics model of the robot, are sent as input to the trajectory generation algorithm. Fig. 17 presents some results from this test case. For a better understanding, a video file is given as supplementary data. It emphasizes that the trajectory generation method is efficient in the context of online trajectory generation, since the slave robot trajectory efficiently tracks the target, here the master robot end-point.

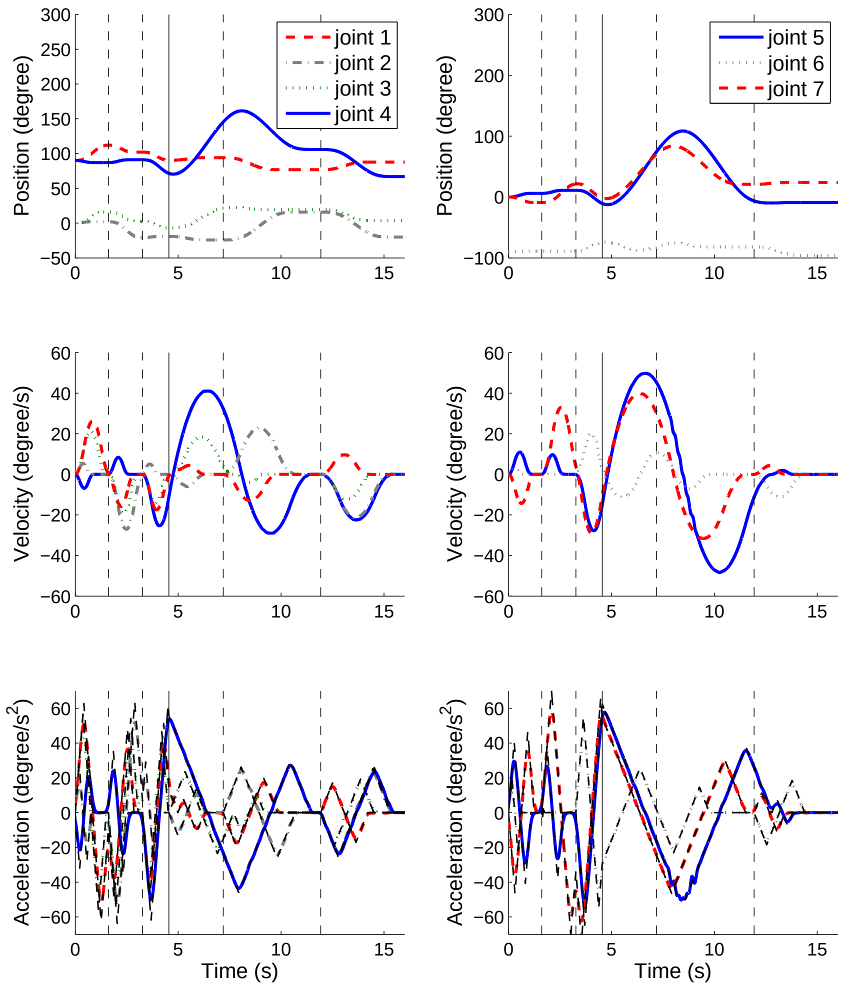

Figure 16: Experimental measurements of the 7 joint positions for the time-optimal jerk-limited trajectories generation. The vertical dotted and solid lines indicate the occurrence of a new target positions. The constraint on j_{max} is modified (lowered) at the vertical solid line.

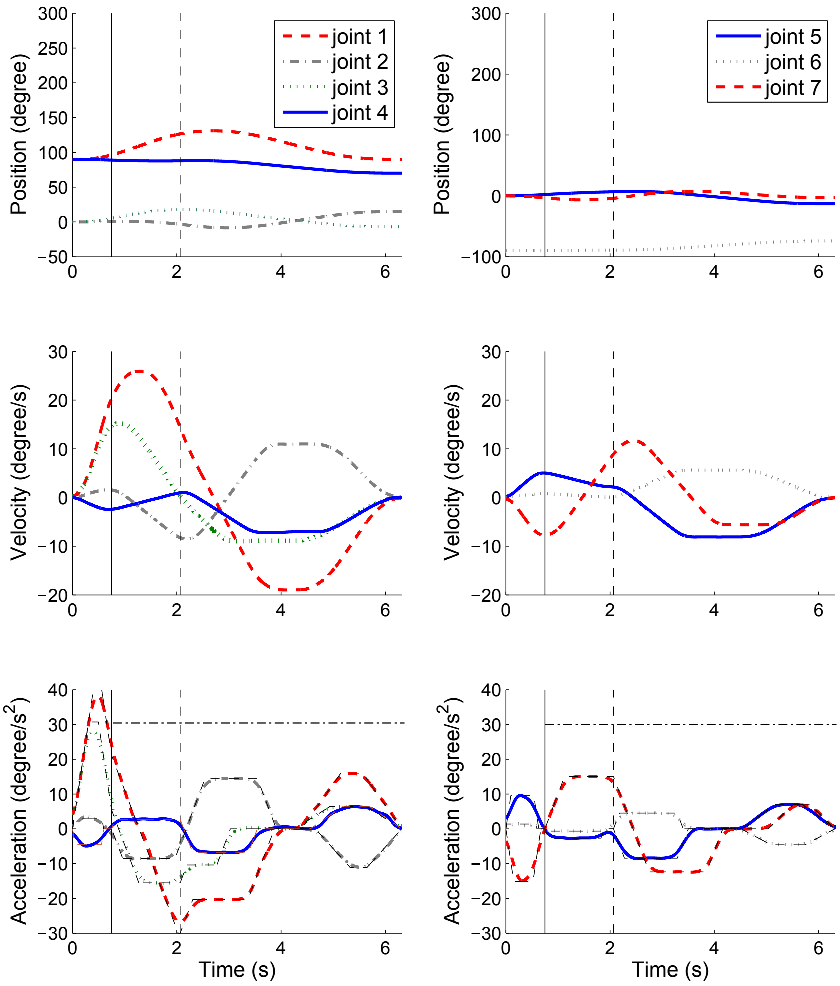

Figure 17: Experimental measurements of the 7 joint positions for the synchronized time-optimal jerk-limited trajectories generation. The vertical dotted and solid lines indicate the occurrence of a new target positions. The constraint on j_{max} is modified (lowered) at the vertical solid line. The kinematic limits are lowered at 0.75 s: from (100 deg.s^{-1}, 100 deg.s^{-2}, 100 deg.s^{-3}) to (100 deg.s^{-1}, 30 deg.s^{-2}, 40 deg.s^{-3})

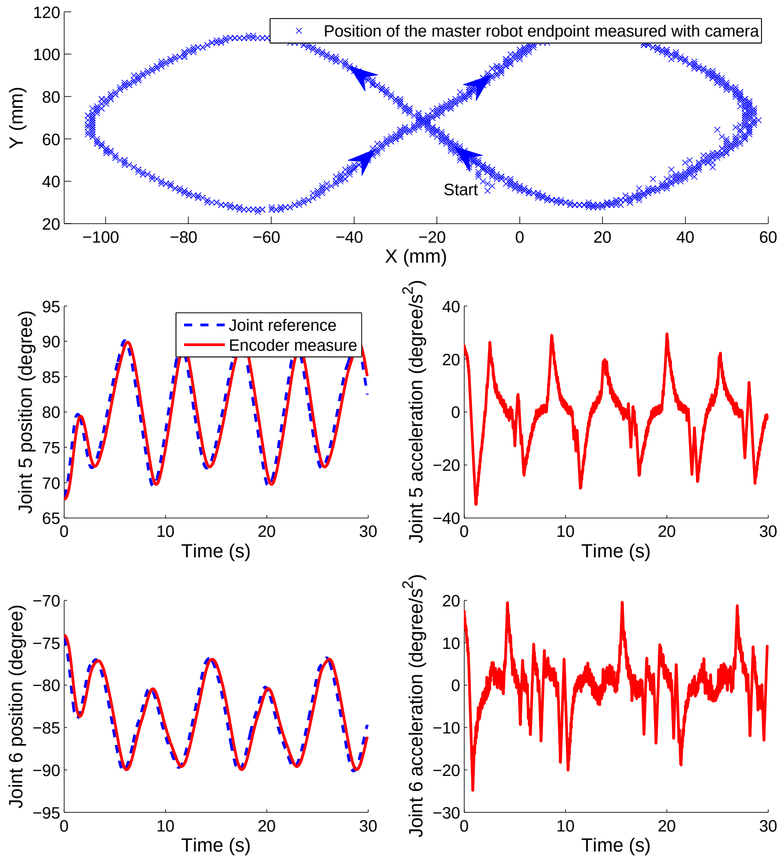

Figure 18: Experimental test case: motion copy between a master and a slave robot. The upper graph shows the position of the master robot end-point measured with an external camera. These signal is sent to the computer and converted into joint set-points for the slave robot using the inverse kinematic model. The lower graphs show the measured position and acceleration of two joints of the slave robot superimposed to the position reference coming from the proposed trajectory generation.

7. Conclusions

In this paper, the proposed trajectory generator is able to generate online time-optimal jerk-limited trajectories from arbitrary initial velocity and acceleration conditions, while respecting the kinematics constraints of position, velocity, acceleration and jerk. The methodology used is a hybrid method which combines the simplicity of the analytic second-order trajectory generation, i.e. the generation of acceleration-limited trajectory, with the computational efficiency of FIR filtering. An iterative algorithm calculates an acceleration-limited profile, which is adapted to the FIR filtering stage in order to output a jerk-constrained trajectory with desired characteristics (time-optimal, time fixed, jerk-time fixed). One notes that the algorithm generating a trajectory in less than 20 microsecond, it is suitable for the online trajectory generation of systems with a large number of DOF. Experimental validations carried out on a seven axis Kuka LBR iiwa demonstrated the efficiency of the proposed online trajectory generator.

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