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# FULLY COUPLED THERMOMECHANICAL MODEL FOR SHAPE MEMORY ALLOYS ACCOUNTING FOR PHASE TRANSFORMATION, MARTENSITIC REORIENTATION, TRANSFORMATION-INDUCED PLASTICITY AND FATIGUE DAMAGE

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## ABSTRACT

The present work proposes a 3D model, based on the thermodynamical coupling of different strain mechanisms such as the forward and reverse phase transformation, the martensitic reorientation, the transformation-introduced plasticity and the fatigue damage. To achieve this goal, all the above mentioned mechanisms are described through the martensitic volume fraction as the coupling parameter. A recently developed, thermomechanically coupled, SMA constitutive law, including both the phase transformation and martensitic reorientation mechanisms, has been validated under non-proportional loading conditions through a series of comparisons between numerical and experimental results. This model is extended further in order to capture the accumulated TRIP residual strain induced by the martensitic transformation, accounting in addition for the accumulated fatigue damage, which evolves during the cyclic loading. The fatigue damage is incorporated into the constitutive law through the concepts of continuum damage theory. Numerical investigation under strongly non-proportional thermomechanical loading conditions demonstrate the capabilities of the new framework.

**KEYWORDS:** shape memory alloy, cyclic loading, thermo-mechanical, plastic strain, fatigue damage.

## INTRODUCTION

Shape memory alloys (SMAs) present the capability of undergoing significant amount of reversible inelastic deformation upon mechanical loading, and retrieving their original shape upon temperature increase. This unique characteristic is due to the phase transformation undergoing at the scale of crystals between the two solid phases that these alloys adopt, the austenitic and the martensitic. The difference between the two phases has its origin on the architecture of the crystalline structure, which varies between a cubic-like configuration in austenite and a less symmetric configuration in martensite. Under cyclic loading the phase transformation mechanism is sometimes accompanied by plastic deformation and damage mechanisms. The present paper proposes a new 3D fully coupled thermomechanical phenomenological model for shape memory alloys,

incorporating the following irreversible mechanisms: the transformation-induced plasticity (TRIP) and the fatigue damage upon multiple cyclic loading. A physical interpretation of the processes occurring inside a SMA grain is provided by redefining the principles of the reorientation, the forward and the reverse transformation. This leads to the introduction of independent scalar variables that drive each of the three reversible deformation mechanisms of SMAs. Both TRIP and damage are assumed in this work to evolve during the forward or the reverse transformation. Accounting for these strongly nonlinear effects, a robust formalism is proposed based on a thermodynamical framework, in which a suitable Gibbs free energy potential is defined. The developed model has been implemented in an open-source numerical simulation library and the simulation of a complex thermomechanical loading path is conducted to illustrate the modeling capabilities of this novel approach.

## DESCRIPTION OF VARIOUS MECHANISMS IN THE MODEL

The existing models aimed at describing the phase transformation response of SMAs introduce, at least, two important internal variables: The martensitic volume fraction (MVF)  $\xi$  and the transformation strain  $\varepsilon^T$ . The model proposed here takes into account the three different SMA mechanisms (forward - reverse transformation and reorientation) and splits these two initial variables in the following manner:

- 1) The rate of the total MVF  $\dot{\xi}$  is assumed to be decomposed in two parts, the rate of change of the MVF induced by forward and by reverse transformation,  $\dot{\xi}^F$  and  $\dot{\xi}^R$  respectively,

$$\dot{\xi} = \dot{\xi}^F - \dot{\xi}^R. \quad (1)$$

The minus in the right hand side of the last expression accounts for the fact that the MVF is reduced during reverse transformation. The scalar  $\xi$  is restricted between the values of 0 and 1.

- 2) The rate of the total MVF transformation strain  $\dot{\varepsilon}^T$  is the sum of three terms, the forward  $\dot{\varepsilon}^{TF}$ , the reverse  $\dot{\varepsilon}^{TR}$  and the reorientation strain rate  $\dot{\varepsilon}^{RE}$ ,

$$\dot{\varepsilon}^T = \dot{\varepsilon}^{TF} + \dot{\varepsilon}^{TR} + \dot{\varepsilon}^{RE}. \quad (2)$$

- 3) The reorientation is assumed as a mechanism similar to the kinematic hardening in plasticity. An additional variable  $v^{RE}$  is defined, which describes the hardening strain for reorientation.

Introducing these volume fractions and strains permits significant freedom for the model to capture the various states that a macroscopic SMA material point can obtain: pure forward transformation, reverse transformation and reorientation or different combinations [Chatziathanasiou, 2016].

The appearance of TRIP and damage during the phase transformation mechanism (forward or reverse) cause permanent irrecoverable deformation and stiffness reduction [Chemisky, 2018].

- The strain due to TRIP is accounted for by splitting the rates of  $\dot{\epsilon}^{TF}$  and  $\dot{\epsilon}^{TR}$  in the following manner:

$$\dot{\epsilon}^{TF} = \dot{\epsilon}^{TFT} + \dot{\epsilon}^{TFP}, \quad \dot{\epsilon}^{TR} = \dot{\epsilon}^{TRT} + \dot{\epsilon}^{TRP}. \quad (3)$$

In the above expressions,  $\epsilon^{TFT}$  is the reversible transformation strain and  $\epsilon^{TFP}$  is the TRIP strain during forward transformation. The terms  $\epsilon^{TRT}$  and  $\epsilon^{TRP}$  represent similar strains for the reverse transformation.

- The stiffness reduction is accounted for through the introduction of a damage variable  $d$ . To separate the damage accumulated during forward and reverse transformation, two damage scalars,  $d^F$  and  $d^R$  with  $\dot{d} = \dot{d}^F + \dot{d}^R$  are considered.

## THERMODYNAMICS FRAMEWORK

The constitutive law of the SMA material is described using a thermodynamic framework. A properly designed thermodynamic potential requires the definition of appropriate observable and internal variables. For the SMA model proposed here, the Gibbs free energy potential is assumed to be a function of the stress  $\sigma$ , the temperature  $\theta$ , the MVF  $\xi$ , the total transformation strain  $\epsilon^T$ , the damage variable  $d$ , the hardening strain for reorientation  $v^{RE}$ , the transformation hardening function  $g^{TT}$  and the TRIP hardening function  $g^{TP}$ :

$$G = G_r + G_{ir}, \quad \text{with} \quad G_r = (1 - \xi)G^A + \xi G^M + G^{mix} \\ \text{and} \quad G_{ir} = g^{TP}, \quad (4)$$

where each term of the reversible part  $G_r$  respectively reads

$$G_r^i = U_0^i - s_0^i \theta + C^i \left[ (\theta - \theta_0) - \theta \ln \left( \frac{\theta}{\theta_0} \right) \right] \\ - \frac{1}{2(1-d)} \sigma : S^i : \sigma - \sigma : \alpha^i (\theta - \theta_0), \\ G^{mix} = -\sigma : \epsilon^T - \sigma : (1 + \lambda^{RE}) X : v^{RE} + g^{TT}. \quad (5)$$

In the above expressions,  $i$  stands for martensitic ( $M$ ) or austenitic ( $A$ ) phase. For each phase,  $C^i$  is the specific heat,  $S^i$  is the compliance tensor,  $\alpha^i$  is the thermal expansion coefficients tensor,  $U_0^i$  is the initial internal energy and  $s_0^i$  is the initial entropy. Additionally,  $X$  denotes the back stress of reorientation,  $\lambda^{RE}$  is a limiting cofactor for reorientation and  $\theta_0$  is a reference temperature.

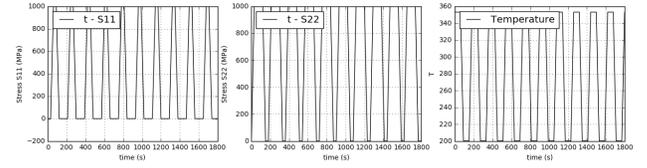


Figure 1: Complex cyclic thermomechanical loading path for a SMA material.

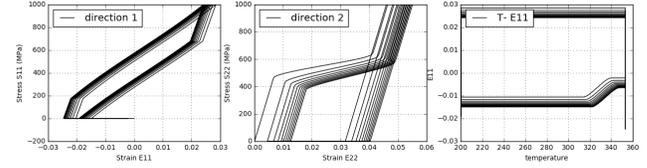


Figure 2: Stress-strain and strain-temperature responses for the complex thermomechanical path.

## NUMERICAL EXAMPLE

The model capabilities are illustrated through a numerical analysis, performed on a material point. The complex thermomechanical, cyclic loading path of Figure 1 is considered. The normal stresses in the directions 1 and 2 and the temperature are varying in a cyclic, non-synchronized manner for a total time of 1800 s.

The obtained results (Figure 2) demonstrate the complicated thermomechanical response of the SMA, which progressively through the cycles generate irrecoverable strain. In addition, a slight degradation of the elasticity modulus appears at each cycle.

## CONCLUSION

The developed, fully coupled, thermomechanical model for SMAs is capable of taking into account the forward, the reverse transformation, the reorientation, the TRIP and the damage mechanism. The chosen Gibbs free energy follows a consistent thermodynamic framework and allows to identify the constitutive law and the evolution equations for all the nonlinear mechanisms. The proposed model is very useful for understanding and capturing the SMA behavior under cyclic or even complicated thermomechanical loading conditions.

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