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Meriem HAYANI, Ali CHAABA, George MORARU, Philippe VERON - PREDICTION OF MULTI-DIMENSIONAL MILLING BEHAVIOR - International Journal of Engineering Sciences and Research Technology - Vol. 10, p.329-339 - 2017

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# PREDICTION OF MULTI-DIMENSIONAL MILLING BEHAVIOR

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DOI: 10.5281/zenodo.1012535

## ABSTRACT

Milling process is a multi-dimensional cutting process accompanied by chatter vibrations because of material removal discontinuity. Also, chatter phenomenon has a great impact on final milled products quality. In this paper, an adequate criterion derived from mathematical literature is used to predict stability of multi-dimensional chatter milling. For that, firstly, the chatter milling system is represented by a system of three Degrees Of Freedom (DOF) and its dynamics is modeled by a system of Retarded Differential Equations (RDEs). Afterward, stability in Lyapunov sense is computed on the basis of the quasi-polynomial characteristic function in frequency domain. Finally, this method is used to predict stability of chatter milling process under different machining conditions and the output related to each case is verified by resolving the system of RDEs and visualizing the displacements in time domain using Matlab software.

**KEYWORDS:** multi-dimensional, milling process, behavior, Chatter Vibrations, RDEs, Matlab software.

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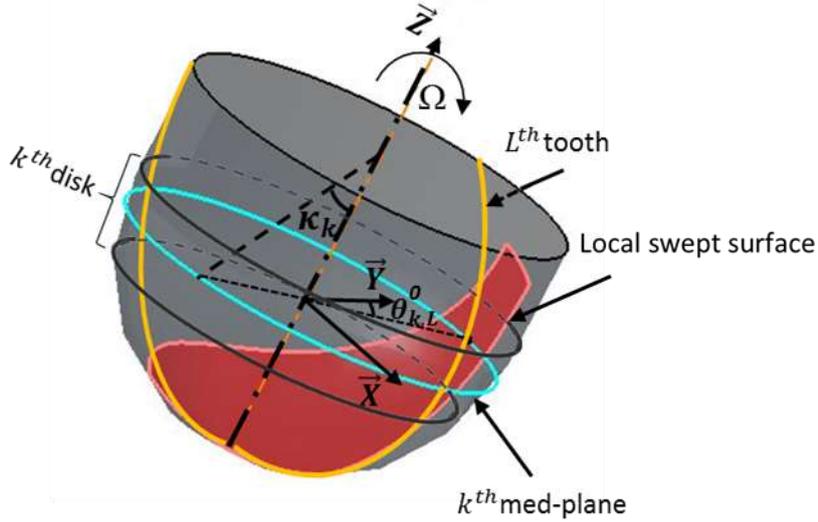
## I. INTRODUCTION

Milling is a manufacturing process where material is removed by chip which is affected by vibrations. Chatter is one of the three vibration types [1], [2] and consists on remanufacturing of the manufactured surface. Firstly, the importance of chatter in vibrations behavior has been demonstrated experimentally by Nicolson [3] then Taylor warned industrials about its lower effect on machined piece quality in 1907 [4]. Effectively, based on mechanistic method, Tlustý proved the direct impact of chatter on chip thickness [5]. Afterwards, he developed a basic non-linear system of cutting force in machining chatter [6] which underlies the strong relationship between cutting force and chip thickness [7]. Whereas many works have been developed to avoid chatter [8] or to suppress it [9]-[10] by varying tool spindle speed, for example, others have been conducted to predict it in time [11] and frequency [12] domains. Since computing stability via simulation milling process in time domain is time consuming, researchers are putting greater focus on predicting stability in frequency domain. The most popular criterion used to achieve this target is the tau-decomposition criterion which stability lobes diagram is based on [13], [14]. Stability lobes diagram is still used to predict chatter in manufacturing by drilling [15], turning [16] and milling [17], [18] systems thanks to its ability to give general overview idea about process stability in constant machining conditions by considering only one degree of freedom at a time. Nevertheless, general milling is a multi-dimensional cutting process. Hence the importance of substituting the Tau-decomposition by a suitable criterion. Therefore, this work aims to analyze stability in the Lyapunov sense [19] of nonlinear differential equations describing chatter dynamical system in multi-dimensional milling process. The second section aims to demonstrate the crossing from milling kinematics to cutting forces. So, milling tool is discretized into elemental cutting edges where differential cutting forces are modeled, computed and summed. The third consists on representing a multi-dimensional chatter milling process by a system of three retarded differential equations (RDEs). After that, we prove that the characteristic function associated is a quasi-polynomial whose degree is superior to one. Then, looking back on theorems defining stability of RDEs on the basis of the characteristic quasi-polynomial root finder, some criteria are defined to determine the stability type (stable, asymptotically stable, unstable) of milling process. Hence, milling system behavior is determined in frequency domain. Afterwards, a case study of chatter milling process is borrowed from bibliography to apply the developed method of behavior prediction and Matlab software is used in computing phase. Finally, the outputs of this method are compared to milling simulation outcomes computed in time domain.

$\Omega$	Angular velocity
N	rotation frequency
$z$	Teeth number
$f_z$	Feed rate
$h$	Uncut chip thickness
$\theta_{k,L}$	Instantaneous engagement angle
$\kappa_k$	Axial immersion angle
$db_k$	Elemental uncut chip width
$dS_k$	Elemental cutting edge length
$Kc_r, Kc_t, Kc_a$	Radial, tangential and axial cutting coefficients
$Ke_r, Ke_t, Ke_a$	Radial, tangential and axial de sharing coefficients

## II. MILLING KINEMATICS

Machining process is assumed to be a series of workpiece/tool interactions where material is removed as chip. Particularly, in milling, tool is rotating around his axis with angular velocity  $\Omega$  and moving, simultaneously, at feed rate  $f_z$  to remove chip thanks to its  $z$  teeth indexed by  $L$  discretized into elemental cutting edges. Indeed, milling tool is discretized into  $nk$  disks of defined thickness  $db_k$  referenced by  $k$  where each disk is containing  $z$  elemental cutting edges indexed by  $(k, L)$ .



**Fig. 1 : Discretization of milling tool**

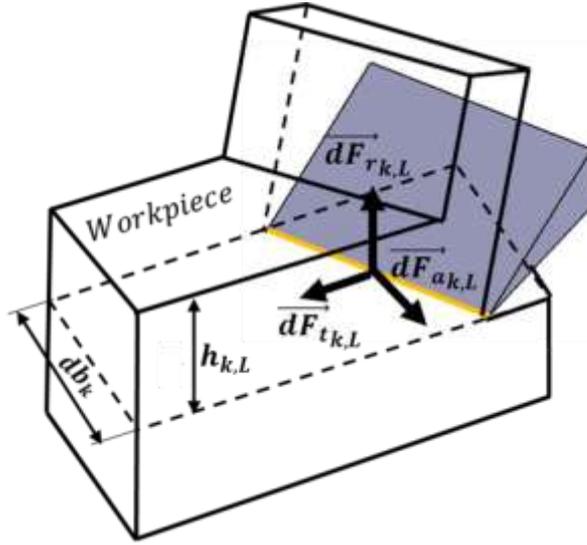
In the middle plane of the  $k^{th}$  elemental disk, we define a  $k^{th}$  med-plane perpendicular to the tool axis where each elemental cutting edge of the  $(k, L)^{th}$  edges ( $1 \leq L \leq z$ ) is located geometrically by an elemental angle engagement  $\theta_{k,L}^0$  measured from  $\vec{Y}$  axis of a reference frame tied to the spindle  $(O, \vec{X}, \vec{Y}, \vec{Z})$  and  $\kappa_k$ .  $\kappa_k$  represents the axial immersion of the  $k^{th}$  disk med-plane.

Milling kinematics induce to material removal and a new surface forming in each workpiece/tool interaction called local swept surface (**Error! Reference source not found.**). To shift from an interaction to the next, milling tool is rotating with an angle  $\Omega * t$  where  $t$  is the time factor. Then, at the time  $t$ , the instantaneous angular position  $\theta_{k,L}$  of the  $(k, L)^{th}$  elemental cutting edge is deducted as:

$$\theta_{k,L} = \theta_{k,L}^0 + \Omega * t \quad (1)$$

In the angular position  $\theta_{k,L}(t)$ , the  $(k, L)^{th}$  elemental cutting edge, whose length is noted  $dS_k$ , is applying a differential force  $\overline{dFtar}_{k,L}$  to remove the infinitesimal chip whose thickness and width are, respectively  $h_{k,L}$  and  $db_k$ . The  $(k, L)^{th}$  differential force  $\overline{dFtar}_{k,L}$  is applied on cutting edge middle and expressed as (2) based on linear model of milling force components [20] in a reference system attached to the elemental tooth.

$$\overline{dFtar}_{k,L} = \left\{ \begin{array}{l} \overline{dFt}_{k,L} = Kc_t * h_{k,L} * db_k + Ke_t * dS_k \\ \overline{dFa}_{k,L} = Kc_a * h_{k,L} * db_k + Ke_a * dS_k \\ \overline{dFr}_{k,L} = Kc_r * h_{k,L} * db_k + Ke_r * dS_k \end{array} \right\} \quad (2)$$



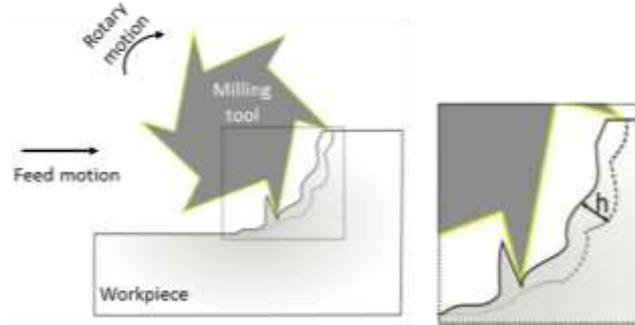
**Fig. 2 : Elemental milling force components**

Computation of linear model milling force depends not only on elemental cutter and chip geometry but also on tangential, radial and axial ploughing and shearing coefficients [21] [22] which are representing force applied respectively per unit of area or length.

### III. CHATTER MODELLING

In milling, discontinuity of edges engagement on material creates so-called cutting delay  $r$ . This phenomenon is called chatter and parameter  $r$  is defining duration between two successive teeth passage. In case of symmetric milling tool, this factor depends on teeth passing frequency  $N$  and number  $z$ .

$$r = \frac{60}{z * N}$$



**Fig. 3: Chip removal in chatter milling**

Figure 1 shows the impact of chatter on instantaneous chip thickness. Indeed, total chip thickness is the sum of dynamic thickness  $h_d$  induced by chatter and quasi-static thickness  $h_{st}$  removed by rigid removal system [23] (3).

$$h = h_{st} + h_d \quad (3)$$

In the context of a discretized milling tool into elemental cutting edges, we note  $h_{k,L}$  the instantaneous elemental chip thickness [23] express thicknesses quasi-static and dynamic elementary, respectively  $h_{k,L_{st}}$  and  $h_{k,L_d}$  as suite:

$$h_{k,L_{st}} = \sin \kappa_k * \sin \theta_{k,L} * f_z$$

$$h_{k,L} = h_{k,L_{st}} + \underbrace{\sin \kappa_k * \sin \theta_{k,L} * [\Delta x(t) + \Delta y(t) * \cot \theta_{k,L} - \Delta z(t) * \cot \kappa_k]}_{h_{k,L_d}(t)}$$

Elemental dynamic chip thickness  $h_{k,L_d}(t)$  is due to the delay in passage of the (k, L) th edge in matter and elemental cutting edge position at time t defined by  $\kappa_k$  and  $\theta_{k,L}(t)$ . Resulting chatter phenomenon has an impact on the three-dimensional geometry of the instant chip expressed by the instant terms  $(\Delta X(t), \Delta Y(t), \Delta Z(t))$  :

$$\bar{\Delta}(t) = \begin{cases} \Delta X(t) = X(t) - X(t - r) \\ \Delta Y(t) = Y(t) - Y(t - r) \\ \Delta Z(t) = Z(t) - Z(t - r) \end{cases}$$

Chatter is a dynamic phenomenon represented by distance between two vibratory displacements, respectively  $X(t), Y(t)$  and  $Z(t)$ , spaced out in time by r delay in the direction of respective reference axes X, Y and Z of spindle tool repair  $(\Delta X(t), \Delta Y(t), \Delta Z(t))$ .

We must remind that this section aim is to determine instantaneous global milling force  $\overline{F_{XYZ}}(t)$  expression. For this purpose, differential milling forces are sorted, so that associated to non-engaged elementary cutting edges (zero) are eliminated thanks to the window function  $g(\theta_{k,L})$  defined as:

$$g(\theta_{k,L}) = \begin{cases} 1 & \text{if } (k,L)^{th} \text{ tooth is engaged} \\ 0 & \text{if not} \end{cases}$$

Then, all non-zero differential milling forces are projected on a unique reference frame (milling tool reference frame, for example) thanks to  $\overline{T}_{k,L}(t)$  in order to be summed finally.

$$\overline{F_{XYZ}}(t) = \sum_{k=1}^{nk} \sum_{L=1}^z \overline{dF_{XYZ}}_{k,L}(t) = \sum_{k=1}^{nk} \sum_{L=1}^z \overline{T}_{k,L}(t) * g(\theta_{k,L}) * \overline{dF tar}_{k,L}(t) \quad (4)$$

Where:

$$\overline{T}_{k,L}(t) = \begin{pmatrix} \sin(\kappa_k) * \sin(\theta_{k,L}) & -\cos(\theta_{k,L}) & \sin(\theta_{k,L}) * \cos(\kappa_k) \\ \sin(\kappa_k) * \cos(\theta_{k,L}) & \sin(\theta_{k,L}) & -\cos(\theta_{k,L}) * \cos(\kappa_k) \\ -\cos(\theta_{k,L}) & 0 & \sin(\kappa_k) \end{pmatrix} \quad (5)$$

Finally, the sum obtained by formulas (2)-(5) is expressed as in (6):

$$\overline{F_{XYZ}}(t) = \begin{pmatrix} a_X \\ a_Y \\ a_Z \end{pmatrix} + \begin{bmatrix} \alpha_{XX} & \alpha_{XY} & \alpha_{XZ} \\ \alpha_{YX} & \alpha_{YY} & \alpha_{YZ} \\ \alpha_{ZX} & \alpha_{ZY} & \alpha_{ZZ} \end{bmatrix} \begin{pmatrix} \Delta X(t) \\ \Delta Y(t) \\ \Delta Z(t) \end{pmatrix} \quad (6)$$

In order to simplify, we set:

$$\bar{a}(t) = \begin{pmatrix} a_X \\ a_Y \\ a_Z \end{pmatrix}; \bar{\alpha}(t) = \begin{bmatrix} \alpha_{XX} & \alpha_{XY} & \alpha_{XZ} \\ \alpha_{YX} & \alpha_{YY} & \alpha_{YZ} \\ \alpha_{ZX} & \alpha_{ZY} & \alpha_{ZZ} \end{bmatrix}; \bar{\Delta}(t) = \begin{pmatrix} \Delta X(t) \\ \Delta Y(t) \\ \Delta Z(t) \end{pmatrix}$$

Where :

- $a_X = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * \left[ f_Z * \left( -Kc_r * \sin \theta_{k,L}^2 * \sin \kappa_k^2 - Kc_t \frac{\sin 2\theta_{k,L}}{2} * \sin \kappa_k - Kc_a * \sin \theta_{k,L}^2 * \sin \kappa_k \right) * db_k \right] + (-Ke_r * \sin \theta_{k,L} * \sin \kappa_k - Ke_t * \cos \theta_{k,L} - Ke_a * \sin \theta_{k,L} * \cos \kappa_k) * dS_k$
- $a_Y = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * \left[ f_Z * \left( -Kc_r * \frac{\sin 2\theta_{k,L}}{2} * \sin \kappa_k^2 + Kc_t * \sin \theta_{k,L}^2 * \sin \kappa_k - Kc_a * \frac{\sin 2\theta_{k,L}}{2} * \frac{\sin 2\kappa_k}{2} \right) * db_k \right] + (-Ke_r * \cos \theta_{k,L} * \sin \kappa_k + Ke_t * \sin \theta_{k,L} - Ke_a * \cos \theta_{k,L} * \cos \kappa_k) * dS_k$
- $a_Z = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * \left[ f_Z * \left( -Kc_r * \frac{\sin 2\kappa_k}{2} * \sin \theta_{k,L} - Kc_a * \sin \kappa_k^2 * \sin \theta_{k,L} \right) * db_k \right] + (-Ke_r * \cos \kappa_k - Ke_a * \sin \kappa_k) * dS_k$
- $\alpha_{XX} = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * db_k * \left( -Kc_r * \sin \theta_{k,L}^2 * \sin \kappa_k^2 - Kc_t \frac{\sin 2\theta_{k,L}}{2} * \sin \kappa_k - Kc_a * \sin \theta_{k,L}^2 * \sin \kappa_k \right)$
- $\alpha_{XY} = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * db_k * \left( -Kc_r * \frac{\sin 2\theta_{k,L}}{2} * \sin \kappa_k^2 - Kc_t * \cos \theta_{k,L}^2 * \sin \kappa_k - Kc_a * \frac{\sin 2\theta_{k,L}}{2} * \sin \kappa_k \right)$

- $\alpha_{XZ} = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * db_k * \left( Kc_r * \sin \theta_{k,L}^2 * \frac{\sin 2\kappa_k}{2} + Kc_t * \frac{\sin 2\theta_{k,L}}{2} * \cos \kappa_k + Kc_a * \sin \theta_{k,L}^2 * \cos \kappa_k \right)$
- $\alpha_{YX} = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * db_k * \left( -Kc_r * \frac{\sin 2\theta_{k,L}}{2} * \sin \kappa_k^2 + Kc_t * \sin \theta_{k,L}^2 * \sin \kappa_k - Kc_a * \frac{\sin 2\theta_{k,L}}{2} * \frac{\sin 2\kappa_k}{2} \right)$
- $\alpha_{YY} = \sum_{k=1}^{nk} \sum_{L=1}^z g(\theta_{k,L}) * db_k * \left( -Kc_r * \cos \theta_{k,L}^2 * \sin \kappa_k^2 + Kc_t * \frac{\sin 2\theta_{k,L}}{2} * \sin \kappa_k - Kc_a * \cos \theta_{k,L}^2 * \frac{\sin 2\kappa_k}{2} \right)$

Instantaneous milling force  $\overline{F}_{XYZ}(t)$  includes instantaneous cutting force  $\overline{a}(t)$  due to rigid body motion of the cutting tool and inducing to quasi-static chip thickness removal, besides of instantaneous chatter force  $\overline{\alpha}(t) * \overline{\Delta}(t)$  which depends on chatter vibration displacements and delay, implicitly. Chatter is a vibration type which is due to system flexibility. Then, instantaneous chatter milling force  $\overline{\alpha}(t) * \overline{\Delta}(t)$  is time response of dynamics milling system. So, on the basis of Newton's second law applied to the flexible milling system, a system of retarded differential equations is developed [24]:

$$\overline{M} \begin{bmatrix} \ddot{X}(t) \\ \ddot{Y}(t) \\ \ddot{Z}(t) \end{bmatrix} + \overline{C} \begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{Z}(t) \end{bmatrix} + \overline{K} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} = \overline{\alpha}(t) * \overline{\Delta}(t) \quad (7)$$

Where:

$$\overline{M} = \begin{bmatrix} M_x & 0 & 0 \\ 0 & M_y & 0 \\ 0 & 0 & M_z \end{bmatrix}; \quad \overline{C} = \begin{bmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_z \end{bmatrix}; \quad \overline{K} = \begin{bmatrix} K_x & 0 & 0 \\ 0 & K_y & 0 \\ 0 & 0 & K_z \end{bmatrix}$$

There, vibratory milling dynamics subsystem is built from transfer functions of decoupled system on a modal basis. Indeed, chatter milling dynamics is represented by motion equations of three oscillators in which each one is vibrating along one of spindle reference frame directions.

As we can see, factors of directional dynamic coefficients matrix  $\overline{\alpha}(t)$  are periodic at rotation frequency as milling tool rotates and engages the same elementary cutting edges in each interaction workpiece/tool. Then, based on immersion conditions and number of elemental cutting edges engaged, time varying dynamic coefficients can be expanded by their Fourier series components. It is straightforward to approximate the matrix directional coefficients by the constant term in the Fourier series. Then the expression (7) becomes:

$$\overline{M} \begin{bmatrix} \ddot{X}(t) \\ \ddot{Y}(t) \\ \ddot{Z}(t) \end{bmatrix} + \overline{C} \begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \\ \dot{Z}(t) \end{bmatrix} + \overline{K} \begin{bmatrix} X(t) \\ Y(t) \\ Z(t) \end{bmatrix} = \frac{\overline{\alpha}^0}{2} * \overline{\Delta}(t) \quad (8)$$

Where:

$$\overline{\alpha}^0 = \frac{1}{T} \int_0^T \overline{\alpha}(t) dt$$

$$T = \frac{1}{N}$$

The constant matrix directional coefficients  $\overline{\alpha}^0$  is time invariant but engagement dependent. So, each factor  $\alpha_{ij}^0$  can be expanded as:

$$\alpha_{ij}^0 = \sum_{k=1}^{nk} \sum_{L=1}^z \frac{1}{T} \int_0^T \alpha_{ij} dt \quad i, j \in \{x, y, z\} \quad (9)$$

#### IV. BEHAVIOR PREDICTION

Hale and Lunel [25] have defined five types of stability. On determination of milling process stability, a criterion is mandatory. As we have seen, chatter milling dynamics is modeled by system retarded differential equations where coefficients of vibratory displacements and its derivatives are classified into categories of constant and space-varying parameters. On the assumption of constant machining conditions and system parameters dynamics,

the only coefficients related to kinematic collision between tool and workpiece are matrix  $\overline{\alpha^0}$  coefficients. This is why stability analysis is mandatory in each interaction workpiece/tool. Of course, the literature suggests several methods to study dynamic systems stability involving delay [19], [26]–[28]]. We note, moreover, approaches based on the analysis of characteristic values associated to differential equations system modelling delayed process. Hence, Pontryagin method is used to determine asymptotic stability in Lyapunov sense of chatter milling process from characteristic function zeros of differential equations system.

### 1. Stability: types and criteria

Dynamic system stability can be determined from its related characteristic function zeros based on the following theorem [29], [30] :

**Theorem 1.** If all roots have negative real parts, then the trivial solution of (8) is asymptotically stable.

**Theorem 2.** If at least one root has a positive real part, then the trivial solution of (8) is unstable.

**Theorem 3.** If there are simple purely imaginary roots and the remaining roots have negative real parts, then the trivial solution of (8) is stable.

**Theorem 4.** If there is a multiple root among the purely imaginary roots, then the trivial solution of (8) is unstable. From these theorems, characterization of chatter process stability depends on computing and evaluating all characteristic function roots. Remember also that chatter characteristic function is an exponential polynomial because of Laplace transform of delayed functions. Therefore, stability system evaluation is time consuming. So, Quasi-Polynomial Mapping Based Rootfinder (QPmR) is proposed to decrease the amount of computing power and rendering time. In a similar way to that of Routh-Hurwitz criterion applied on ordinary differential equations systems, QPmR is a function for computation and analysis of spectrum of characteristic quasi-polynomials of both retarded and neutral time delay systems [31]–[33].

### 2. Stability computing

A frequency approach is proposed to evaluate stability of system based on analyzing the characteristic function of differential equations system modeling chatter system. Therefore, a new delayed differential equations system is redefined from the following dimensionless parameters:

$$\tau = \frac{t}{r}; r = \frac{1}{v}; \quad \lambda_x = \frac{\sqrt{K_x/M_x}}{2\pi v}; \lambda_y = \frac{\sqrt{K_y/M_y}}{2\pi v}; \quad \zeta_x = \frac{C_x}{2\sqrt{K_x M_x}}; \zeta_y = \frac{C_y}{2\sqrt{K_y M_y}}; \\ \lambda_z = \frac{\sqrt{K_z/M_z}}{2\pi v}; \quad \zeta_z = \frac{C_z}{2\sqrt{K_z M_z}}$$

With  $\tau$  dimensionless time parameter,  $\lambda_x, \lambda_y, \lambda_z$  dimensionless frequencies and  $\zeta_x, \zeta_y, \zeta_z$  damping ratios along the tree axes x, y and z, respectively, of Galilean reference frame.

Thanks to Laplace transform, RDEs system (8) is expressed in frequency domain (10) where  $s$  is the complex number and  $Id$  the identity matrix.

$$Id s^2 + 4\pi \begin{bmatrix} \lambda_x \zeta_x & 0 & 0 \\ 0 & \lambda_y \zeta_y & 0 \\ 0 & 0 & \lambda_z \zeta_z \end{bmatrix} s + 4\pi^2 \begin{bmatrix} \lambda_x^2 & 0 & 0 \\ 0 & \lambda_y^2 & 0 \\ 0 & 0 & \lambda_z^2 \end{bmatrix} + 4\pi^2 \begin{bmatrix} \frac{\lambda_x^2}{K_x} \alpha_{xx}^0 & \frac{\lambda_y^2}{K_y} \alpha_{xy}^0 & \frac{\lambda_z^2}{K_z} \alpha_{xz}^0 \\ \frac{\lambda_x^2}{K_x} \alpha_{yx}^0 & \frac{\lambda_y^2}{K_y} \alpha_{yy}^0 & \frac{\lambda_z^2}{K_z} \alpha_{yz}^0 \\ \frac{\lambda_x^2}{K_x} \alpha_{zx}^0 & \frac{\lambda_y^2}{K_y} \alpha_{zy}^0 & \frac{\lambda_z^2}{K_z} \alpha_{zz}^0 \end{bmatrix} (e^{-s} - 1) = 0 \quad (10)$$

Characteristic function  $P(s)$  of delayed vibratory milling system (8) is defined as:

$$P(s) = \begin{vmatrix} s^2 + 4\pi\lambda_x\zeta_x s + 4\pi^2\lambda_x^2 + \frac{4\pi^2\lambda_x^2\alpha_{xx}^0}{K_x}(e^{-s}-1) & \frac{4\pi^2\lambda_x^2\alpha_{xy}^0}{K_x}(e^{-s}-1) & \frac{4\pi^2\lambda_x^2\alpha_{xz}^0}{K_x}(e^{-s}-1) \\ \frac{4\pi^2\lambda_y^2\alpha_{yx}^0}{K_y}(e^{-s}-1) & s^2 + 4\pi\lambda_y\zeta_y s + 4\pi^2\lambda_y^2 + \frac{4\pi^2\lambda_y^2\alpha_{yy}^0}{K_y}(e^{-s}-1) & \frac{4\pi^2\lambda_y^2\alpha_{yz}^0}{K_y}(e^{-s}-1) \\ \frac{4\pi^2\lambda_z^2\alpha_{zx}^0}{K_z}(e^{-s}-1) & \frac{4\pi^2\lambda_z^2\alpha_{zy}^0}{K_z}(e^{-s}-1) & s^2 + 4\pi\lambda_z\zeta_z s + 4\pi^2\lambda_z^2 + \frac{4\pi^2\lambda_z^2\alpha_{zz}^0}{K_z}(e^{-s}-1) \end{vmatrix}$$

Delayed function is converted to exponential function through Laplace transform. Then, characteristic function  $P(s)$  is an exponential polynomial where  $s^6 e^3$  is the principal term.

Coefficients of the characteristic quasi-polynomial  $P(s)$  are formulas which call parameters related to structural

dynamics of milling system besides the coefficients matrix  $\overline{\alpha}^0$  of vibratory displacements. So, characteristic function coefficients determination are based on calculating  $\overline{\alpha}^0$  components  $\left\{ \alpha_{ij}^0 = \sum_{k=1}^{n_k} \sum_{L=1}^z \frac{1}{T} \int_0^T \alpha_{ij} dt \mid i, j \in \{x, y, z\} \right\}$ . Then, stability of RDEs system (8) in the Lyapunov sense can be evaluated on the basis of characteristic function roots. In order to achieve this, we use Matlab software to develop an algorithm (**Error! Reference source not found.**) where we use the function QPmR [33], [34] to compute characteristic quasi-polynomials roots of RDEs systems. Finally, stability milling process is determined by applying the above theorems.

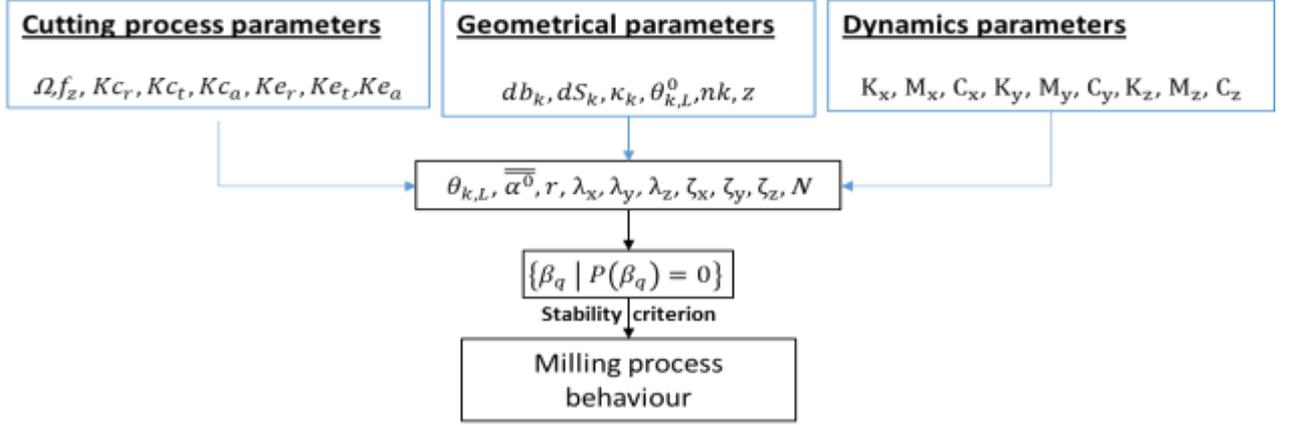


Fig. 4: Milling process behavior computation

## V. APPLICATION

In the aim of calculating  $\overline{\alpha}^0$  matrix elements, we need data related to cutting process parameters and tool geometry. Also, cutting system dynamics will allow us to get a closer look at its stability. For this, we exploit the experimental conditions of Li et al. [24] where a helical milling tool having a helical angle of  $30^\circ$ , a diameter of 10 mm and two flutes is grooving on an AL7075-T6 aluminum alloy part with constant feed  $f_z = 0.05 \text{ mm/dent}$ . Cutting law parameters and dynamics parameters associated to milling system are defined as:

Cutting law conditions	Radial		Tangentiel		Axial	
	$K_{rc} [\text{N/mm}^2]$	$K_{re} [\text{N/mm}]$	$K_{tc} [\text{N/mm}^2]$	$K_{te} [\text{N/mm}]$	$K_{ac} [\text{N/mm}^2]$	$K_{ae} [\text{N/mm}]$
	168.0	30.8	796.0	27.7	222	1.5
Dynamics parameters	Natural frequency [Hz]		$\omega_x = 807$		$\omega_y = 777.8$	
	Damping ratio		$\zeta_x = 0.047$		$\zeta_y = 0.052$	
	Stifness [N/m]		$K_x = 1.23 * 10^7$		$K_y = 0.76 * 10^7$	

A Two-DOF structural dynamics of milling system is considered. These two degrees of freedom are the displacements  $X(t)$  and  $Y(t)$  in the two orthogonal directions, respectively  $\vec{X}$  and  $\vec{Y}$  axes. Finally chatter vibration is described by two equations of motion placed in the following matrix form:

$$\begin{bmatrix} \ddot{X}(t) \\ \ddot{Y}(t) \end{bmatrix} + 4\pi \begin{bmatrix} \zeta_x \omega_x & 0 \\ 0 & \zeta_y \omega_y \end{bmatrix} \begin{bmatrix} \dot{X}(t) \\ \dot{Y}(t) \end{bmatrix} + 4\pi^2 \begin{bmatrix} \omega_x^2 & 0 \\ 0 & \omega_y^2 \end{bmatrix} \begin{bmatrix} X(t) \\ Y(t) \end{bmatrix} = 2\pi^2 \begin{bmatrix} \frac{\omega_x^2 \alpha_{xx}^0}{K_x} & \frac{\omega_x^2 \alpha_{xy}^0}{K_y} \\ \frac{\omega_x^2 \alpha_{yx}^0}{K_x} & \frac{\omega_x^2 \alpha_{yy}^0}{K_y} \end{bmatrix} \bar{\Delta}(t)$$

Under these conditions, we define three configurations for the torque (rotation frequency, cutting depth). For each configuration, we examine roots of the associated characteristic function using a computer program written on Matlab script.

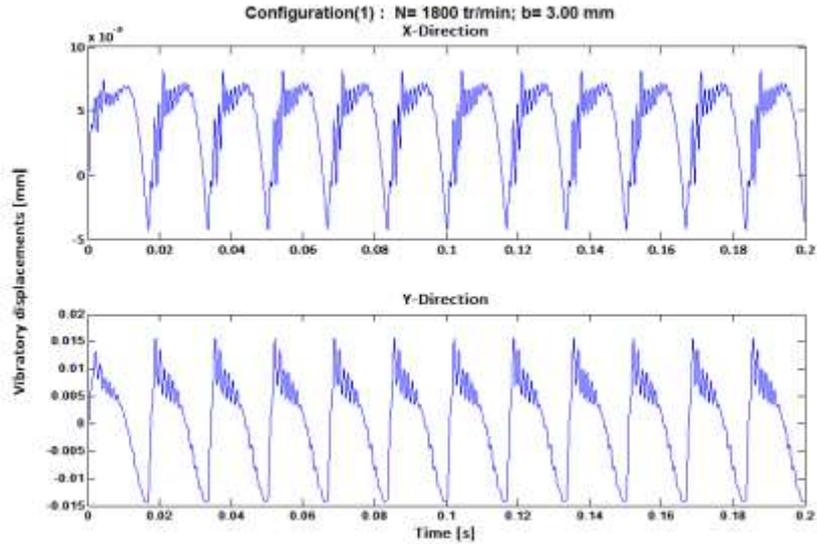
**Table 1: Milling process configurations**

Configurations	Rotation frequency [ <i>tr/min</i> ]	Cutting depth [ <i>mm</i> ] ( $db_k * nk$ )
(1)	1800	$db_k = 0.15 \text{ mm} ; nk = 20$
(2)	5000	$db_k = 0.05 \text{ mm} ; nk = 33$
(3)	10000	$db_k = 0.1 \text{ mm} ; nk = 25$

## VI. RESULTS AND DISCUSSION

For each of the three configurations mentioned in **Error! Reference source not found.**, we inject tool parameters, cutting law coefficients and process parameters associated with in the input of the script. After that, we evaluate the Lyapunov stability of the RDEs system (8). Based on roots analysis of quasi-polynomial characteristic function, the QPmR function returns an asymptotic stability for configuration (1) and milling process instability for configurations (2) and (3).

In order to evaluate QPmR function outputs, we use dde23 Matlab function to solve the system of DDEs with constant delays representing the two degree-of-freedom structural dynamics of milling system. Then, we obtain the signals of the regenerative milling system displacements with respect to the three configurations (1), (2), (3) on the axes  $\vec{X}$  and  $\vec{Y}$ .



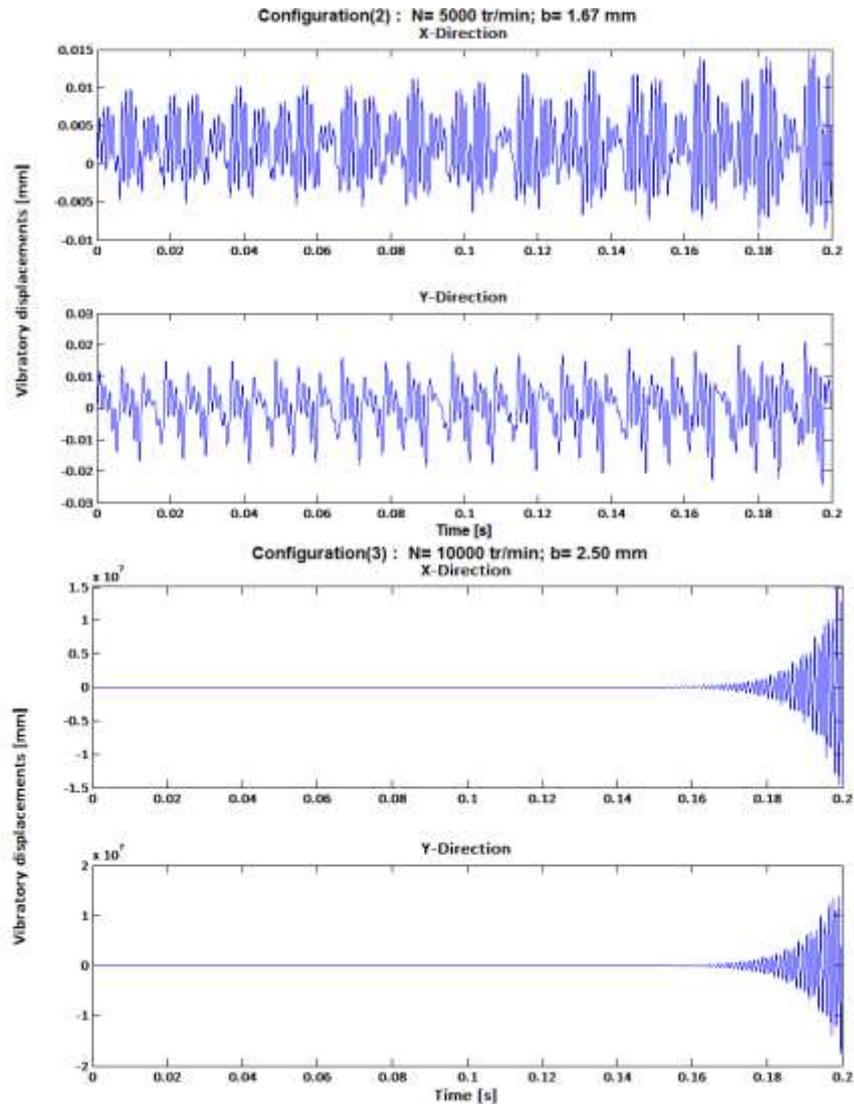


Fig. 5: Vibratory displacements of the configurations (1), (2), (3)

**Error! Reference source not found.** proves the nature of milling system stability evaluated through the frequency approach. So, output signals of the configuration (1) are periodic at tooth passing and bounded. At the output of the configuration (2), we remark that the envelopes of the signals are linear. Nevertheless, at the simulation of the configuration (3), we obtain signals with exponential envelopes. In conclusion, the numerical results (extracts from the simulation of the regenerative milling) are qualitatively consistent with the analytical prediction (evaluation of the characteristic roots).

## VII. CONCLUSION

Stability lobes find its limits in predicting stability of multi-dimensional chatter cutting process. Then, a new approach is proposed to determine chatter milling process stability in sense of Lyapunov by looking back to theorems of RDEs system stability in frequency domain. For a case study of milling process, three configurations of machining conditions are integrated in the inputs of the QPmR Matlab function to determine milling stability for each configuration. Impact of chatter on stability milling system is visualized in time domain by resolving the RDEs system thanks to the dde23 Matlab function.

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#### CITE AN ARTICLE

**Mechkouri , M. H., Chaaba, A., Moraru, G., & Veron, P. (2017). PREDICTION OF MULTI-DIMENSIONAL MILLING BEHAVIOR .INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY, 6(10), 329-339.**