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# **Dimensioning a product in preliminary design through different exploration techniques**

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**Abstract:** Once a design concept has been chosen and parameterised, the embodiment design stage consists of choosing materials and dimensions to ensure a 'good matching' with the expected performances. In this context of preliminary design stages, several approaches exist, which correspond to

slightly different complexities and issues and must consequently be used at different moments. We consider in this paper three families of approaches:

- 1 exploring design (parametric) dimensioning under uncertainty (through constraint programming techniques, representations of feasible design points or Pareto frontiers)
- 2 robust design and multidisciplinary optimisation
- 3 design for reliability.

We advocate and state in this paper that these approaches must be used in that order of increasing complexity. Indeed, applying an approach allows one to quickly figure out inadequacies with performance specifications or initial allowable bounds of design parameters and then to backtrack or to refine the design issue before proceeding to the next stage or approach. We illustrate that phenomenon by successively applying the three approaches on a dimensioning issue of a two-member truss structure. We clearly show that the successive optimal designs obtained are notably different, but that the optimal point obtained in a given approach is used to explore its surroundings within the next approach.

**Keywords:** design exploration; robust design; design for reliability; constraint programming; Pareto solutions; embodiment design; parametric design; design optimisation.

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## **Introduction and overview of embodiment design methods**

The *exploration of design parametric dimensioning under uncertainty* can be made using:

- fuzzy sets theory applied to design engineering (Antonsson and Otto, 1995)
- Monte Carlo simulations (*i.e.*, generating feasible design points) followed by an exploration within the feasible design space (see, for example, Stump *et al.*, 2004)
- constraint programming techniques (Yannou and Harmel, 2005).

We have already experimented that *constraint programming techniques* may be convenient (as soon as they are well tuned) for quickly achieving an encompassing approximation of the design space (see Yannou and Hamdi, 2004). In addition, a further Monte Carlo sampling within this approximate space has been shown (see Yannou *et al.*, 2005) to be very efficient for straightforwardly obtaining a good ratio of feasible design points (*i.e.*, respecting the different design constraints and specifications). An *exploration of a set of feasible design points* is a valuable alternative and may even be preferred to the use of a global objective function (or a global preference aggregation function) because in preliminary design, the weighting of the different objectives may be very subjective. This is why we believe that exploration lets the designer be more opportunistic and even lets him/her acquire a better comprehension of the potential of his/her parameterised design. Lastly, techniques for generating and *exploring the Pareto optimal solutions* are evoked. On the example of the truss structure, it is shown that the design issue is already well constrained and that a tradeoff is useful to somewhat loosen some specifications in order to keep a sufficient degree of freedom to tackle robust design and design for reliability.

The *multidisciplinary design* uses multiobjective optimisation algorithms to find the best design that fulfils the numerous performances to be reached. These performances are evaluated using numerous disciplinary theories. It is often difficult to take the different theories into account in the unique formalism of an optimisation approach. We propose to use a *metamodel* (or *surrogate model*) (Papalambros, 2002) that enables one to formalise

multidisciplinary knowledge and to evaluate several performances using just one kind of mathematical formulation. To illustrate the elaboration of a metamodel, the *design of experiments method* and an *identification method* (least mean square method) are used to identify predefined mathematical functions that link design parameters to performance.

Once these functions are validated, they represent with a unique form several physical phenomena and can be used to evaluate, approximately but very quickly, the behaviour of the design that can be evaluated. This evaluation can be made according to different criteria such as robustness or reliability.

As defined by Taguchi, a *robust design* is a design of a product so that its functionality varies minimally despite disturbing factor influences, which can be associated with environmental factors, usage factors or technical factors such as design parameters. The aim is not to find the best performing design according to the set of performances to be reached, but to guarantee a higher level of performance whatever the perturbations on the definition, process, usage or environment parameters are. The use of desirability functions to formalise the optimisation goal and the signal/noise ratio in the design of experiments enables us to find the most robust solution on the truss structure.

The *design for reliability* aims to find the optimal solution that fulfils a given reliability condition. The fluctuation of loads, the variability of material properties and the uncertainties regarding the analysis models contribute to make the performance of the optimal design different from the expected one. In this sense, the optimisation process has a large effect on the structural safety and/or reliability. However, the safety factor approach cannot ensure the required safety level, as it does not explicitly consider the probability of failure regarding some performance criteria. In other words, the optimal design resulting from deterministic optimisation procedures does not necessarily ensure the required reliability level. The design for reliability allows us to consider the safety margin evolution, leading to the settlement of the best compromise between the life cycle cost and the required reliability. This task is further complicated due to the inherent nondeterministic nature of the input information. For this reason, many analysis methods have been developed to deal with the statistical nature of data. The process efficiency is mandatory to deal with realistic engineering problems (Kharmanda *et al.*, 2002); the metamodels can thus be very helpful in achieving the reliability-based optimal design with a reasonable computation effort. The solution obtained on the reliability basis is rather robust as the uncertain parameters are penalised during the design process, compared to a greater commitment of the well-controlled parameters. Practically, the design problem is formulated as a minimisation of the cost function under some prescribed reliability targets (Aoues and Chateaufneuf, 2008).

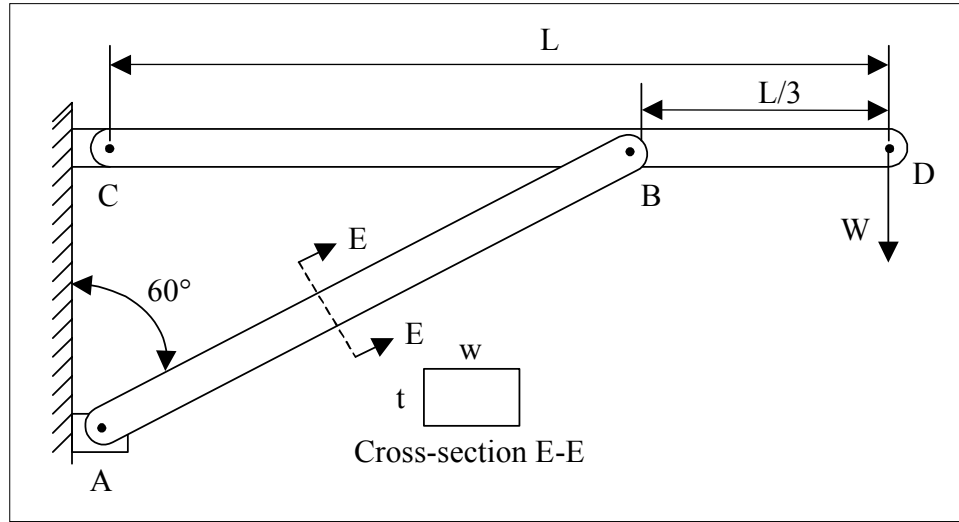
For the robust design and design for reliability approaches, the truss structure example has been made more complex in adding two performances: fundamental frequency and section area.

This paper can be considered a brief survey of the main approaches that can be followed during the embodiment design stage of a product. Its purpose, through a practical example of the parametric dimensioning of a truss structure, is to concurrently apply these approaches so as to figure out that it is worthy to apply them successively in an ever-refining embodiment design process. The paper is structured into five more sections: a presentation of the truss structure dimensioning issue, one section for each of the three approaches and a section of concluding remarks.

## The truss structure dimensioning design issue: a first modelling

Our case study consists of dimensioning the two members of the truss structure shown in Figure 1. This problem was originally proposed by Wood and Antonsson (1989) to compute imprecise performance parameters from imprecise design parameters via fuzzy sets theory. This example has also been used by Scott and Antonsson (2000) in a different parameterised form to select an optimal Pareto solution that could not be selected via a linear aggregation function using importance weights. For this example, we use the exact parameterisation and initial design variable ranges of the truss structure described by Wood and Antonsson (1989), but we have chosen the more complex design constraints and performance parameters used by Scott and Antonsson (2000).

**Figure 1** The parameterisation of the truss structure



The requirement is to design a mechanical structure supporting an overhanging vertical load at a distance  $L$  from the wall with a minimal mass. One possible configuration (see Figure 1) consists in a two-member pin-jointed bracket with a horizontal member ( $CD$ ) and a compression member ( $AB$ ) attached to the wall at an angle of  $60^\circ$ . The common pin is located at two thirds of  $L$  from the wall. Both members have rectangular cross-sections:  $w_{AB} \times t$  for ( $AB$ ) and  $w_{CD} \times t$  for ( $CD$ ),  $w$  standing for width and  $t$  for thickness. Additional design decisions have been made: the material of both members is steel, and we impose  $w_{CD} = w_{AB} - 0.025$ . The designer has to make decisions for the values of the following *design parameters*:  $t$ ,  $w_{AB}$  and  $L$ . Moreover, the specification of the overhanging load  $W$  is imprecise at the beginning of the design process, varying from 15–20 kN; consequently,  $W$  is treated as a fourth design variable.

The two *mechanical constraints* to satisfy are:

- 1 the maximum bending stress,  $\sigma_b$ , in member ( $CD$ ) must be less than or equal to the allowable bending limit,  $\sigma_r$  (here 225 MPa for steel)
- 2 the compression force  $F_{AB}$  in member ( $AB$ ) must be less than or equal to the buckling limit  $F_b$ .

The maximum bending stress,  $\sigma_b$ , is located at point B and is given by the following formulas involving  $W_{CD}$ , the weight of member (CD):

$$\sigma_b = \frac{2L \left( W + \frac{W_{CD}}{6} \right)}{w_{CD} t^2} \text{ with } \begin{cases} W_{CD} = \rho g w_{CD} t L \\ W_{CD} = w_{AB} - 0.025. \end{cases} \quad (1)$$

The compression force in member (AB) is given by the following formulas involving  $W_{AB}$ , the weight of member (AB):

$$F_{AB} = \sqrt{\left\{ \frac{9}{2\sqrt{3}} \left( W + \frac{W_{CD}}{2} + \frac{W_{AB}}{3} \right) \right\}^2 + \left\{ \frac{3}{2} \left( W + \frac{W_{CD}}{2} \right) \right\}^2} \quad (2)$$

with  $W_{AB} = \rho g w_{AB} t L_{AB}$  and  $L_{AB} = \frac{4\sqrt{3}}{9} L$ .

The buckling limit in member (AB) is given as:

$$F_b = \frac{\pi^2 E I_{AB}}{L_{AB}^2} = \frac{9\pi^2 E w_{AB} t^3}{64 L^2}. \quad (3)$$

The *performance variables* are the mass  $M$  of the structure (to be minimised) and the safety factor  $s$ , i.e., the amount of overdimensioning beyond the satisfaction of the two mechanical constraints. The mass  $M$  is given by:

$$M = W_{AB} + W_{CD}. \quad (4)$$

The safety factor of the truss structure  $s$  is the minimum between the safety factor below the allowable bending limit,  $\sigma_b$ , namely,  $s_\sigma$ , and the safety factor below the buckling limit,  $F_b$ , namely,  $s_F$ , which is expressed as:

$$s_\sigma = \frac{\sigma_r}{\sigma_b}, \quad s_F = \frac{F_b}{F_{AB}}, \quad s = \min(s_\sigma, s_F). \quad (5)$$

The two mechanical constraints may be merely expressed by:  $s_\sigma \geq 1$ ,  $s_F \geq 1$  or simply by the single constraint:

$$s \geq 1. \quad (6)$$

### 3 Exploration of design parametric dimensioning under uncertainty

*Design space exploration* during embodiment design is an active research field. It consists in exploring the relationships between the choices of design parameter values and the performance variable values. This exploration provides the designer with a deep understanding of the potential of the given design concept that is studied, in comparison with a direct optimisation of an objective function (function of the performance values).

Often, a design space exploration is performed within a ‘design under uncertainty’ process, which is a process of dimensioning the product in progressively and consciously narrowing the domains (of allowable values) of design parameters while respecting the specifications on performance variables and constraints (like the aforementioned mechanical constraints). Three families of techniques are used to support the uncertainty reduction forward (from parameter domains to performance domains) and backward (from performances to parameter domains):

- 1 *fuzzy sets theory* in design engineering (see Antonsson and Otto (1995), not detailed here)
- 2 *probabilistic techniques*: most of these approaches consist in generating a number of feasible design points (complying with the constraints) and apply graphical postprocessing to visualise correlations between variables, the *Pareto frontier* or a *preference structure* among the design solutions (see Stump *et al.*, 2004). The generation of feasible design points is often a statistical (Monte Carlo) generation of potential candidates sampled within initial variable domains, followed by the checking of constraints, which may become inefficient if the design problem is highly constrained since a majority of candidates that are generated do not belong to the (small) feasible solution space.
- 3 *Constraint Programming* (CP) techniques. They are not only well adapted to a *numerical* exploration of dimensional values for both performance variables and design parameters. They are also adapted to *topological* explorations of design concepts or architectures under constraints and, in addition, to the consideration (and exploration) of different *configurations of system functioning constraints* (such as external load conditions), depending on the considered life cycle stages (like flight phases, in the case of Scaravetti *et al.* (2006b)). Scaravetti *et al.* (2006a) studied the way that CP techniques modelled under the Constraint Satisfaction Problem (CSP) formalism (which is generally the case for CP techniques) must be used within an industrial deployment of a design project: which constrained variables, which constraints, evaluation of several alternatives, choice within component catalogues).

With *Constraint Programming* (CP) over reals, *performance variables* and *design parameters* are modelled as intervals of allowable values. These *constrained variables* may be equated to uniform distributions of values in probabilistic modelling. CP techniques consist of sophisticated evolutions of interval analysis or *interval arithmetics* (see Moore, 1979) applied to a set of analytical constraints. Starting from a set of *initial domains* for the constrained variables and from a set of mathematical constraints linking the variables, different CP *consistency* or *filtering techniques* (such as *Hull*, *Box*, *weak-3B* or *3B*; see, for instance, Benhamou *et al.* (1999) and Yannou and Harmel (2005)) try to contract as much as their consistency degree allows the variable domains so as to eliminate infeasible values. This domain contraction stage is called the *filtering* stage. One tries to result in the most tightened Cartesian product of intervals, ensuring at any moment that any feasible solution is kept inside. This last important property refers to the *completeness* property and guarantees that the contraction process results in an outer design space approximation.



In the second stage, the mechanism of *domain splitting* (bisection for instance) is recursively applied in parallel with the *filtering* mechanism. A *search tree* is built until a stopping criterion (*e.g.*, width of the domains, number of solutions) is reached. This *branch-and-prune* algorithm allows pruning out large parts of the design space whenever a domain is found to be empty. At the end of the process, the design space is approximated by a number of elementary and disjointed Cartesian products of small intervals, denoted as *boxes*. The resulting *hull of boxes* provides the designer with valuable information about the potential values remaining for any design variable at this stage. Finally, a graphical representation of this *collection of n-dimensional boxes* (*n* being the number of constrained design variables) is easy and convenient for obtaining good pictures of the resulting design space (by 2D or 3D projections on pairs or triplets of design variables).

Figure 2 illustrates the four outer approximations of the design space that we can consider in a CP computation process, namely:

- 1 the initial domains
- 2 the filtered domains after the uncertainty reduction propagation has been made for the first time
- 3 the hull of boxes, *i.e.*, the projection on variable domains of the collection of boxes that have not been considered inconsistent after the domain splitting process (with no guarantee of any actual solution inside)
- 4 the collection of boxes itself.

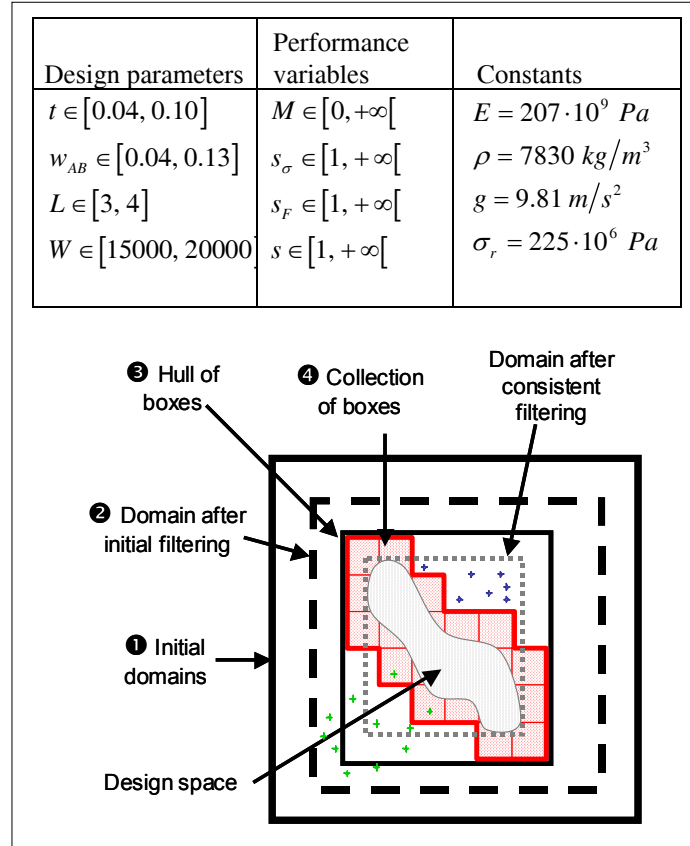
It is obvious that these four outer approximations of the design space are ordered in an increasing rank of refinement.

It has already been shown in Yannou and Hamdi (2004) that the graphical representations of the *collections of boxes* could be meaningful for the designer(s) to perform relevant analyses of variable correlations and tendencies and to make good decisions in a multistage ‘design under uncertainty’ process. This is a first important utility of CP techniques in preliminary parametric dimensioning. Two stages of such a dimensioning process are illustrated in Table 1 with two cases:

- 1 The case of a ‘not-so-constrained’ design problem, which means that the initial domain is not large compared to the effective solution space. This is the situation of Case 1 of the specification constraints on the truss structure (the safety factor is just constrained to be greater than or equal to 1).
- 2 The case of a ‘highly constrained’ design problem, which means that the initial domains are much larger than the actual solution space. This is Case 2 of the specification constraints on the truss structure.

Note that in both cases, the increasingly constrained problem is performed throughout the constraints on performances. It means that, here, the designer(s) can really design in a functional manner, starting from the need and propagating consequences towards the solution (the means). This interesting facility is here permitted by the back-propagation properties of the CP filtering mechanisms.

**Figure 2** Initial interval domains of design parameters and performance variables for the truss-structure problem (left); the four successive outer approximations of the design space available after a Constraint Programming (CP) computation (right) (see online version for colours)

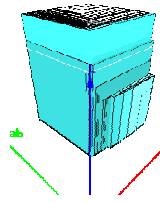
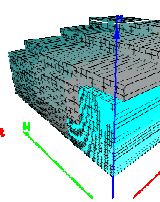
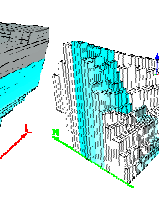
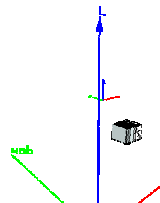
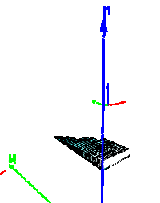
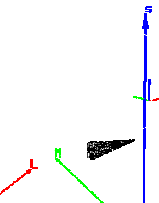


Another interesting case of domain reductions by CP techniques may be seen on the design exploration of an aircraft air-conditioning system by Scaravetti *et al.* (2006b). The design problem consists in optimising the internal structures of the heat exchangers while satisfying the functioning constraints imposed by the system environment. The design variables are geometric and structural variables (length, surface type and pass number in the exchangers) but also thermodynamic variables (pressure, mass flow rate, temperature). The performance variables are linked to efficiencies, mass and drag induced.

After having sufficiently explored the design space of our truss structure and after several domain (and uncertainty) reductions, the designer(s) has converged towards a small design space of interest. It is now time to use *probabilistic techniques* to result in a cloud of *feasible design points* that one could apprehend one by one. We can perform a brute *Monte Carlo simulation* in sampling 100 000 design points by random trials within the initial domains of the design parameters (provided in leftmost column of table in Figure 2). After the checking of mechanical constraints, we obtained (see also Yannou *et al.* (2005) 15 000 feasible design points in the not-so-constrained case (Case 1) versus

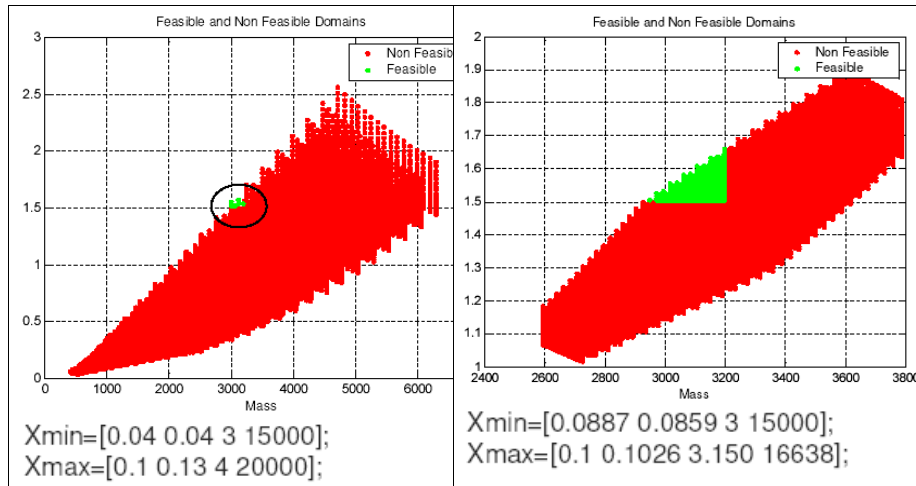
only four in the highly constrained case (Case 2). These feasible and unfeasible design points have been represented in Figure 3 (left) to figure out the very low ratio of feasible design points. It is then problematic when the designer wants to finely explore the design space since the design space is not dense enough to display a continuous variation of performances. A second utility of CP computation techniques has been proposed by Yannou *et al.* (2005): the ratio of feasible design solutions is much more efficient when sampling inside the collection of boxes (obtained from a primary CP computation). In Case 2, the number of feasible design points is 9500 instead of four (Yannou *et al.*, 2005), a great gain in efficiency.

**Table 1** CP computation of the truss structure – considering two series of specification constraints (see online version for colours)

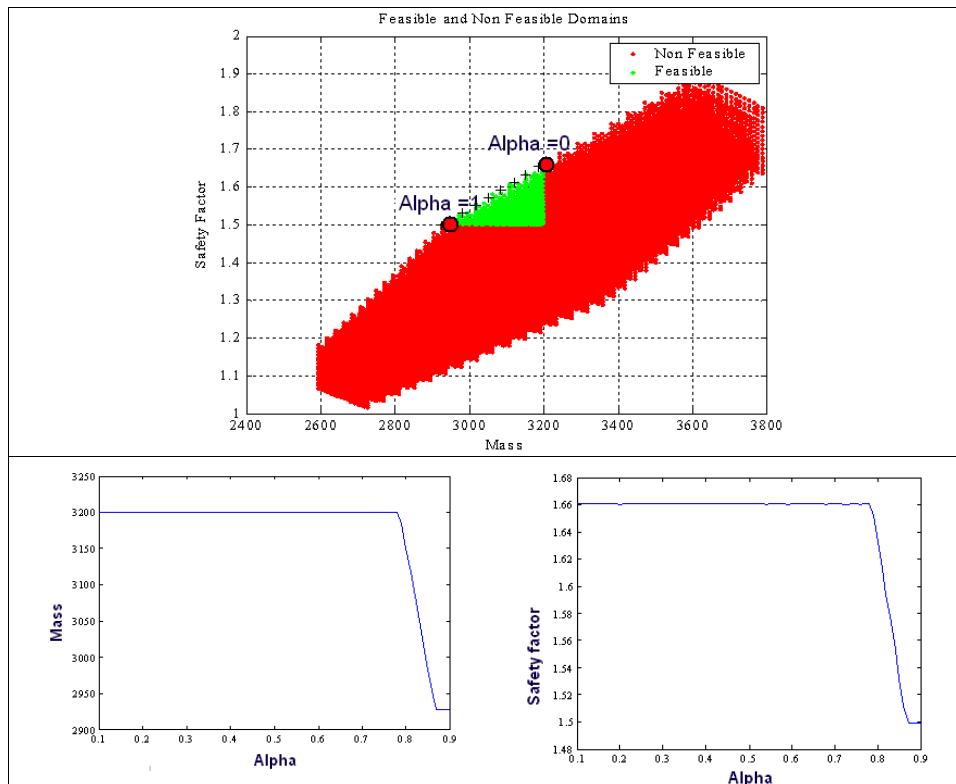
Specification constraints	$\{t, w_{AB}, L\}$ projection	$\{L, W, M\}$ projection	$\{W, M, s\}$ projection	Hull of boxes
Case 1 $s \geq 1$				$t \in [0.0621, 0.1]$ $w_{AB} \in [0.0654, 0.13]$ $L \in [3, 4]$ $W \in [15000, 20000]$  $M \in [2077.9, 6300.9]$ $s \in [1, 2.567]$
Case 2 $s \geq 1.5$ $M \leq 320$				$t \in [0.0887, 0.1]$ $w_{AB} \in [0.0859, 0.1026]$ $L \in [3, 3.150]$ $W \in [15000, 16638]$  $M \in [2926.5, 3200]$ $s \in [1.5, 1.664]$

Only a subset of the feasible design points may be eligible as the best or preferred solution: this is the set of *Pareto optimal solutions*. This set of Pareto solutions is represented as a curve in the performance space: this is the *Pareto frontier*. By definition, a Pareto frontier (see Messac *et al.*, 2003) is the locus of nondominated solutions, a dominated solution being a solution (design point) for which it exists at least another solution outranking it for all the performances. Highlighting the subset of Pareto solutions among the feasible design points is then very meaningful for designers. In Stump *et al.* (2004) and in Scaravetti *et al.* (2006b), the authors have developed graphical interactive tools in informing the designer on the performance values of a clicked optimal Pareto solution (the click is made in a *performance plane or space*) and in highlighting the corresponding design parameter values (made in a *design parameter plane or space*).

**Figure 3** Representation of feasible and unfeasible design points in the performance space for Case 2 with a sampling of 100 000 design points within two different sets of domains  $X = [t, w, L, W]$  (see online version for colours)



**Figure 4** An optimisation of an objective function (function of  $s$  and  $M$ ), varying the  $\alpha$  parameter of linear combination, describes the whole Pareto frontier (see online version for colours)



At this stage, it would be useful to benefit from a procedure to choose one of the Pareto optimal solutions as the preferred solution. Here, *optimisation techniques* are often used. An objective function must then be built as a function of the performances ( $M$  and  $s$ ). A traditional form of this objective function is given by the following formula:

$$\alpha \frac{M - M_{\min}}{M_{\max} - M_{\min}} + (1 - \alpha) \frac{s_{\max} - s}{s_{\max} - s_{\min}} \quad \text{with } \alpha \in [0,1]. \quad (7)$$

The weighting factor  $\alpha$  must be identified by the designers. It is well known that, in case of a convex Pareto frontier, varying the  $\alpha$  results in running all along the Pareto frontier (see Figure 4). But, in case of a concave Pareto frontier, some portions are not covered by an optimisation process and, consequently, some Pareto solutions practically become noneligible. This nontrivial issue may be solved by using more sophisticated forms of the objective functions (see, for instance, Scott and Antonsson, 2000).

#### 4 Robust design

The aim of this section is to use a robust design approach as described by Taguchi (Fowlkes and Creveling, 1995) on the truss example. The first section has presented how to explore the design space and to identify the space of decisions among the values of the different design parameters in order to reach fixed specifications or performances. Then, knowing the ranges of parameters that enable us to reach the specifications, several criteria can be formalised to focus not only on a convenient design, but also on the best design. The criteria that we propose to use is the robustness, *i.e.*, we search for the design solution that guarantees the level of performances whatever the variabilities on the design parameters are. That is to say, the design parameters are chosen in order to have the performance levels at least sensitive as possible to the variabilities of design parameters, process variables and environmental factors. These uncontrolled variabilities (for instance, due to tolerances) should affect at least as possible the levels of performances. The problem is not to suppress or control the variabilities but to minimise their effects on performances.

Two different levels of design parameter values are used. The first level (concerned with the first section of this paper) of the scale is dealing with the variation of design parameters (different values of each design parameters that should be chosen to define the design). The second level is concerned with the variabilities around a design parameter value. These variabilities are small compared to the possible variation of the design parameter values but it is very important to be able to take these variabilities into account for decision-making in the whole design process (Crossland *et al.*, 2003; Ullman and D'ambrosio, 1995).

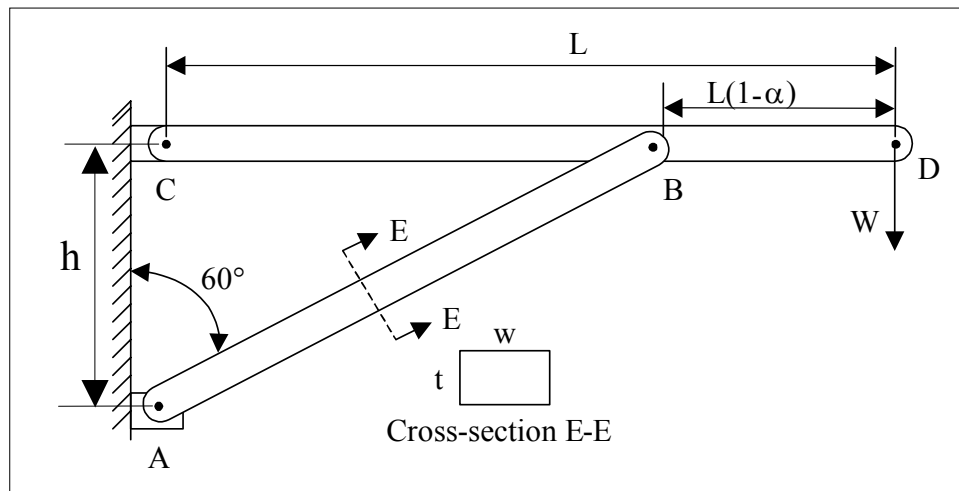
In this section, we will take some design parameter values, identified for the truss example in the first part of the paper as convenient values, to reach the specifications in terms of performances. Around these values, some variabilities are considered to take into account some uncertainties on design parameters, process variables and environmental factors. Then, a robust design approach is used to propose the most robust design in the design space under uncertainties.

#### 4.1 Specificities of the truss example for robust design

Managing the quality of the truss design requires ensuring attainable performance levels in a constrained and uncertain context. Even if a common practice is to find the first convenient design that enables one to reach the functional specifications, numerous research works are conducted to improve the design methodologies in order to be able to find *the best* design instead of *a good* design. Suh (2001), with his Axiomatic Design, is one of the first authors to propose an approach that enables one to guide design activities based on two axioms that evaluate the design all along the design process. Quality Function Deployment is especially appropriated to evaluate the quality of the product all along the design process. However, we will use the vocabulary introduced by the informational spaces proposed by Suh in the following description of the robust design approach. Designing is then a transformation between the variables through these different spaces. Even if the role of history and knowledge is not highlighted by this model, it will be used to formalise the truss problem in order to use a robustness criterion for its design evaluation (El-Haik, 2005).

On the truss structure, the performances or functional requirements initially considered are  $M$ ,  $s_\sigma$ ,  $s_F$ ,  $s$  and the design parameters are  $t$ ,  $w_{AB}$ ,  $L$ ,  $W$  as described in Figure 5. Other design parameters are added in the problem formulation in order to have more sensitive performances. Two more design parameters that we consider in this section are  $h$ , the distance between the top and the bottom of the truss, and  $\alpha$ , which parameterises the position of the joint linking the two members of the truss along the horizontal member.

**Figure 5** The new parameters of the truss structure

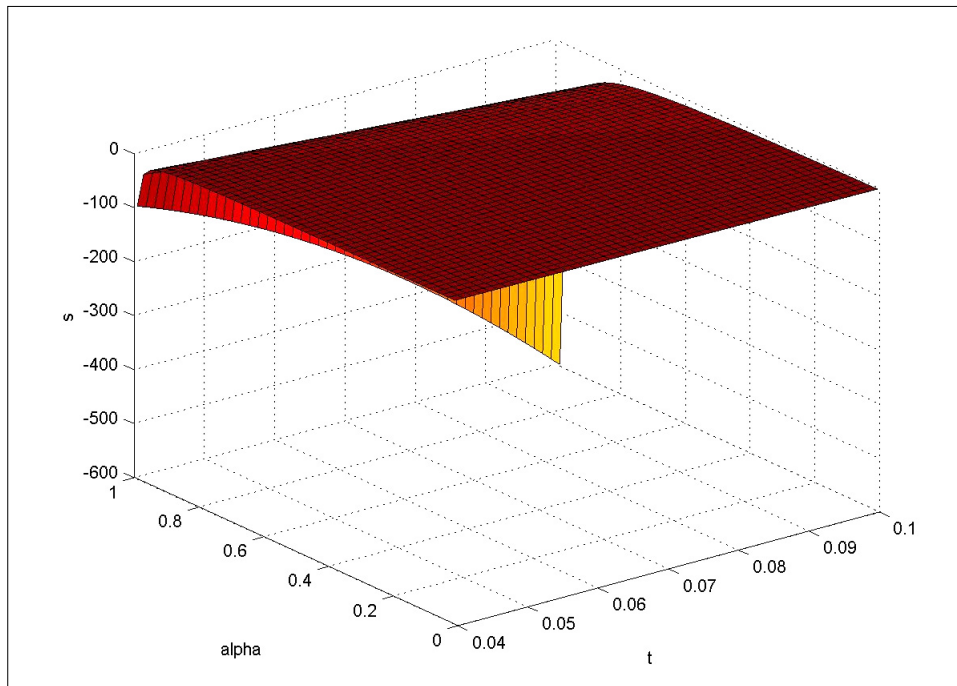


#### 4.2 The Taguchi robust design approach

Axiomatic Design provides two axioms (the *independence* and *minimal information* axioms) that enable the evaluation of the quality of the design (Suh, 2001). If these axioms present a great interest for new innovative designs on ‘uncoupled’ systems, this design context is unusual when complex products are improved from one design to

another and the constraints imply that the axioms cannot be respected in practice. For instance, on the previous truss design formulation, the two axioms are not respected by the formulation used in engineering design. Thus, under specific conditions, a multiobjective optimisation formulation can be formulated in order to find the solution that provides the best level of performances. However, the design solution obtained is often sensitive to variabilities on design parameters due to the process (uncertainties managed as tolerances), the environment (uncertainties not controlled but imposed by the environment such as temperature, humidity, *etc.*) and also due to desired variabilities such as users' preferences and contexts of use of the product. For instance, the best solution for the truss structure previously defined is very sensitive to several variabilities such as illustrated in Figure 6, which is not much desirable for the customer in terms of service quality. In this figure, the performance  $s$  is provided with respect to the  $h$  and  $t$  design parameters.

**Figure 6** An example of a response surface which provides  $s$ , one of the performances {FRs} with respect to some sensitive design parameters of {DPs},  $t$  and  $\alpha$  (see online version for colours)



In this case, the robust design approach enables us to find a good solution that is, in addition, robust to design parameters' variabilities. Robust design was introduced by Taguchi and is defined in Fowlkes and Creveling (1995). The usual design approach aims at finding a design solution that reaches functional specifications and at minimising the uncertainties on design parameters. If some design parameters can be defined accurately, some tolerances always exist and perturbing factors can always be considered. Then, the robust design approach proposes to take into account the variabilities in the performance evaluations instead of avoiding it. Taguchi's proposition is based on the Design of

Experiment (DoE) method to evaluate the sensitivity of the performances taking uncertainties into account. These uncertainties or variabilities are introduced as noise factors in the Taguchi table of the DoE as a small variability around each value of the design parameters. The impact of these variabilities around the design parameters' values can then be analysed and the most robust solution can be obtained by maximising the signal-to-noise ratio.

The DoE considered on the truss example is provided in Table 2. The mechanical laws are a bit more sophisticated than those formulated by Equations (1) to (6) and are not provided here for reasons of brevity. These laws are used to evaluate the truss performances and fulfil the performance evaluation (the last two columns of Table 2). For each line of the DoE, a set of design parameter values are considered and the level of variability around the fixed value is also provided.

**Table 2** The first lines of the table of experiments used to identify a linear model with interactions

$t$	$w_{AB}$	$L$	$W$	$h$	Noise on $\alpha$	$M$	$s$
-1	-1	-1	-1	-1	-1	142	141
-1	-1	-1	-1	-1	1	503,3	504,3
-1	-1	-1	-1	1	-1	507	506
-1	-1	-1	-1	1	1	657,1	658,1
-1	-1	-1	1	-1	-1	142	141
-1	-1	-1	1	-1	1	503,3	504,3
-1	-1	-1	1	1	-1	507	506
-1	-1	-1	1	1	1	657,1	658,1
-1	-1	1	-1	-1	-1	189,3	188,3
-1	-1	1	-1	-1	1	671	672
-1	-1	1	-1	1	-1	553,1	552,1
-1	-1	1	-1	1	1	794,9	795,9
-1	-1	1	1	-1	-1	189,3	188,3
-1	-1	1	1	-1	1	671	672
-1	-1	1	1	1	-1	553,1	552,1
-1	-1	1	1	1	1	794,9	795,9
-1	1	-1	-1	-1	-1	979,8	978,8
-1	1	-1	-1	-1	1	2154	2155
-1	1	-1	-1	1	-1	2166	2165
-1	1	-1	-1	1	1	2654	2655
-1	1	-1	1	-1	-1	979,8	978,8
-1	1	-1	1	-1	1	2154	2155
-1	1	-1	1	1	-1	2166	2165
-1	1	-1	1	1	1	2654	2655
-1	1	1	-1	-1	-1	1306	1305
-1	1	1	-1	-1	1	2872	2873

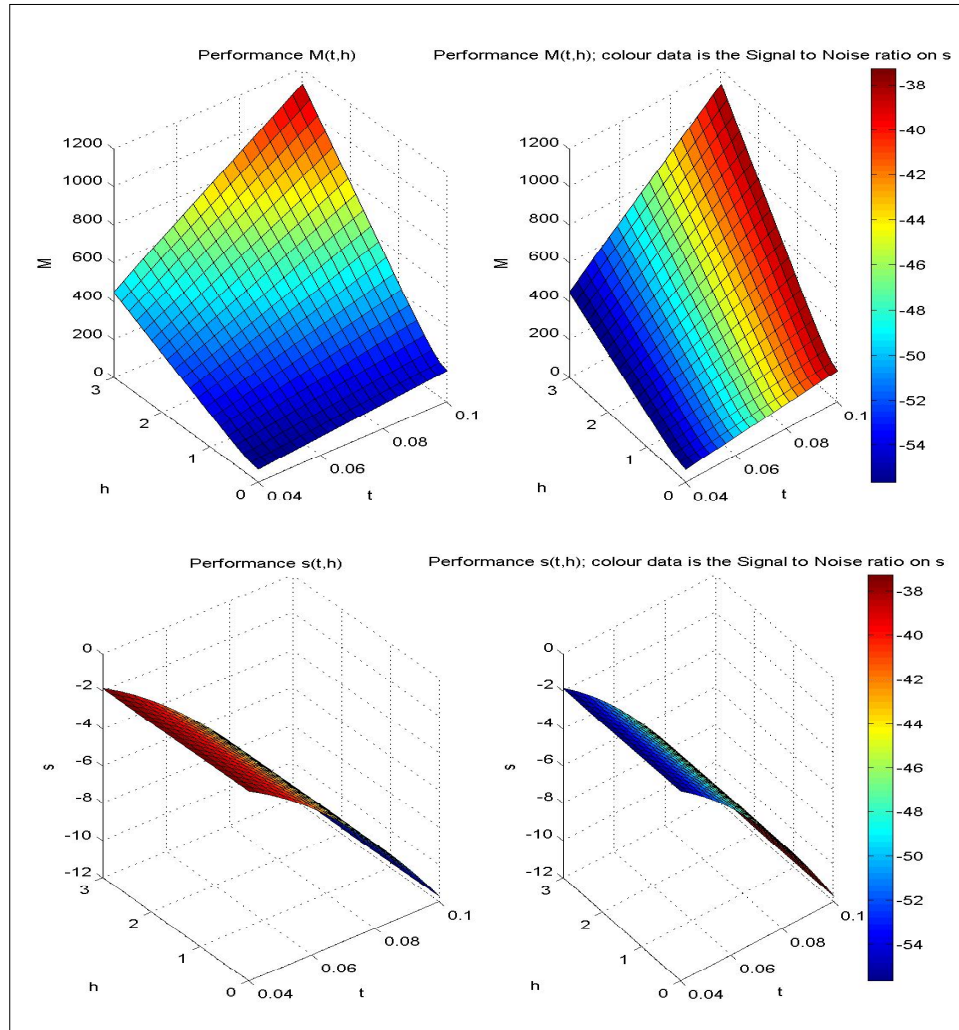


**Table 2** The first lines of the table of experiments used to identify a linear model with interactions (continued)

$t$	$w_{AB}$	$L$	$W$	$h$	Noise on $\alpha$	$M$	$s$
-1	1	1	-1	1	-1	2489	2488
-1	1	1	-1	1	1	3275	3276
-1	1	1	1	-1	-1	1306	1305
-1	1	1	1	-1	1	2872	2873
-1	1	1	1	1	-1	2489	2488
-1	1	1	1	1	1	3275	3276
1	-1	-1	-1	-1	-1	354,9	353,9
1	-1	-1	-1	-1	1	1258	1259
1	-1	-1	-1	1	-1	1267	1266
1	-1	-1	-1	1	1	1643	1644
1	-1	-1	1	-1	-1	354,9	353,9
1	-1	-1	1	-1	1	1258	1259
1	-1	-1	1	1	-1	1267	1266
1	-1	-1	1	1	1	1643	1644
1	-1	1	-1	-1	-1	473,2	472,2
1	-1	1	-1	-1	1	1678	1679
1	-1	1	-1	1	-1	1383	1382
1	-1	1	-1	1	1	1987	1988
1	-1	1	1	-1	-1	473,2	472,2
1	-1	1	1	-1	1	1678	1679
1	-1	1	1	1	-1	1383	1382
1	-1	1	1	1	1	1987	1988
1	1	-1	-1	-1	-1	2450	2449
1	1	-1	-1	-1	1	5385	5386
1	1	-1	-1	1	-1	5415	5414
1	1	-1	-1	1	1	6635	6636
1	1	-1	1	-1	-1	2450	2449
1	1	-1	1	-1	1	5385	5386
1	1	-1	1	1	-1	5415	5414
1	1	-1	1	1	1	6635	6636
1	1	1	-1	-1	-1	3266	3265
1	1	1	-1	-1	1	7180	7181
1	1	1	-1	1	-1	6222	6221
1	1	1	-1	1	1	8187	8188
1	1	1	1	-1	-1	3266	3265
1	1	1	1	-1	1	7180	7181
1	1	1	1	1	-1	6222	6221
1	1	1	1	1	1	8187	8188

Then the response surfaces (fitted metamodel) are built for each performance and the data of the robustness level is added on the response surfaces as a colour level. For instance, Figure 7 provides the representation of a performance (here  $M$  and  $s$ ) with respect to two design parameters ( $h$  and  $t$ ) and taking into account the level of the signal-to-noise due to the variabilities on  $\alpha$  (the darker the colour on the surface response, the higher the signal-to-noise ratio).

**Figure 7** Examples of a response surface, on which robustness data are added (right part of figure) (see online version for colours)



Then, the most robust solution is found (given in Table 3) by maximising the signal to noise ratio for the performances under consideration. The choice of the performance(s) to guarantee has to be done. In Table 2, we can find the most robust design solution considering the robustness of the  $s$  performance, taking the variabilities on the  $\alpha$  design parameter into account.

**Table 3** The best solution with the robustness criteria on  $s$ , with variabilities on  $\alpha$ 

<i>Design approach</i>	<i>t</i>	<i>w<sub>AB</sub></i>	<i>L</i>	<i>W</i>	<i>h</i>	<i>α</i>	<i>M</i>	<i>s</i>
Robust design	0.1	0.13	3.42	17 115	1.5	0.5		

If the most robust design solution is searched, then combination functions can be used, such as the desirability (or usability) functions to determine a global robustness. The most robust solution can be:

- the one that provides the less sensitive global desirability, taking design parameter variabilities into account
- the solution that maximises the global desirability level (or minimises a global loss function calculated on the whole responses), with a desirability level (or a loss level) associated with each signal to noise ratio provided by the impact of the variabilities on each performance.

#### 4.3 *Results and interests of robust design for uncertainty management during the embodiment design*

In order to make a decision under uncertainties in an engineering design context, and in particular in the embodiment design stage (defined in Pahl and Beitz, 1996), the robustness analysis can be used. The results provided, using a response surface with a colour grid to evaluate the signal to noise ratio, enables the designer to choose not only the best design solution in the design space but also the less sensitive one to the design parameters' variabilities due to uncertainties. The robustness can be studied performance by performance or considering a set of performances to guarantee.

### **Design for reliability**

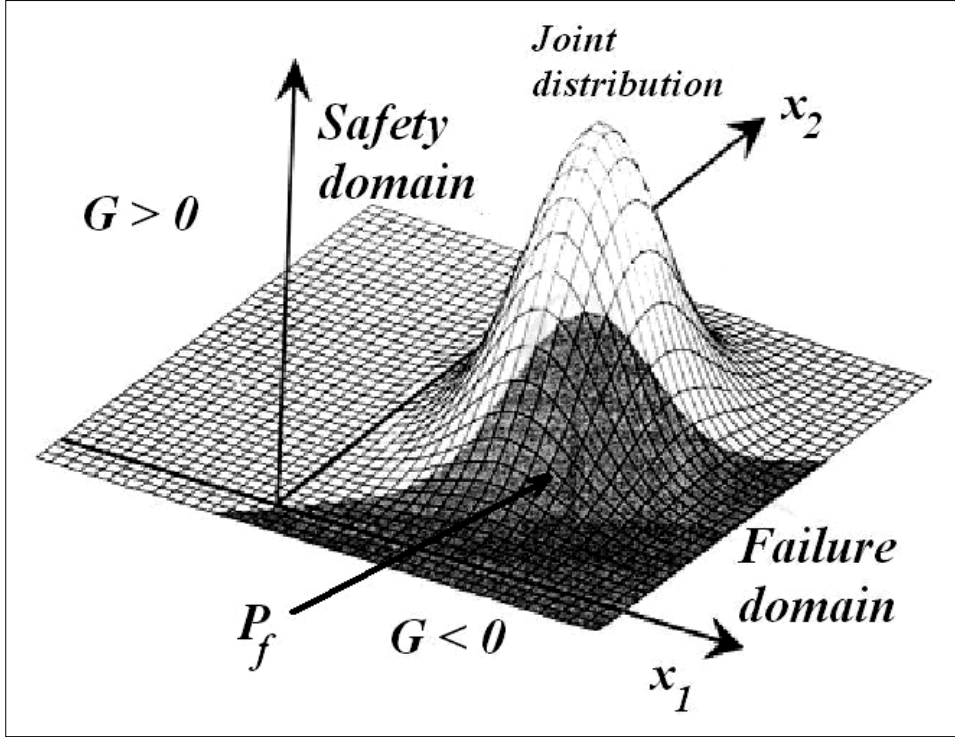
The design of structures requires the verification of a certain number of rules resulting from the knowledge of physics and the mechanical experience of the designers and constructors. These rules come from the necessity to limit the loading effects such as stresses and displacements. Each rule represents an elementary event and the occurrence of several events leads to a failure scenario. In addition to the deterministic variables  $d_k$  to be used in the system control and optimisation, the uncertainties are modelled by stochastic variables affecting the failure scenario. The knowledge of these variables is not, at best, more than statistical information and we admit a representation in the form of random variables (denoted  $X_i$  whose realisations are  $x_i$ ). For a given design rule, the basic random variables are defined by their probability distribution with some expected parameters.

#### 5.1 *Reliability analysis*

Safety is defined as the state where the structure is able to fulfil all the functional requirements – mechanical and serviceability – for which it is designed. To evaluate the failure probability with respect to a chosen failure scenario, a performance function

$G(x_i, d_k)$  is defined by the condition of good operation of the structure. The limit between the state of failure  $G(x_i, d_k) \leq 0$  and the state of safety  $G(x_i, d_k) > 0$  is known as the limit state surface  $G(x_i, d_k) = 0$  (see Figure 8).

**Figure 8** Parameter joint distribution and failure probability



In the First Order Reliability Method, the reliability level is defined by an invariant reliability index  $\beta$ , which is evaluated by solving the constrained optimisation problem:

$$\beta = \min V(T_i(x_j)) = \sqrt{\sum_i (T_i(x_j))^2} \quad (8)$$

under the constraint :  $G(x_i, d_k) \leq 0$

where  $V(\cdot)$  is the distance between the median point and the failure subspace in the normalised space and  $T_i(\cdot)$  is an appropriate probabilistic transformation; the solution to this problem is called the *design point*. At the first-order approximation, the failure probability  $P_f$  is given as a function of the reliability index:

$$P_f = \Pr [G(x_i, d_k) \leq 0] \approx \Phi(-\beta) \quad (9)$$

where  $\Pr(\cdot)$  is the probability operator and  $\Phi(\cdot)$  is the standard Gaussian cumulated function.

## 5.2 Reliability-based design optimisation

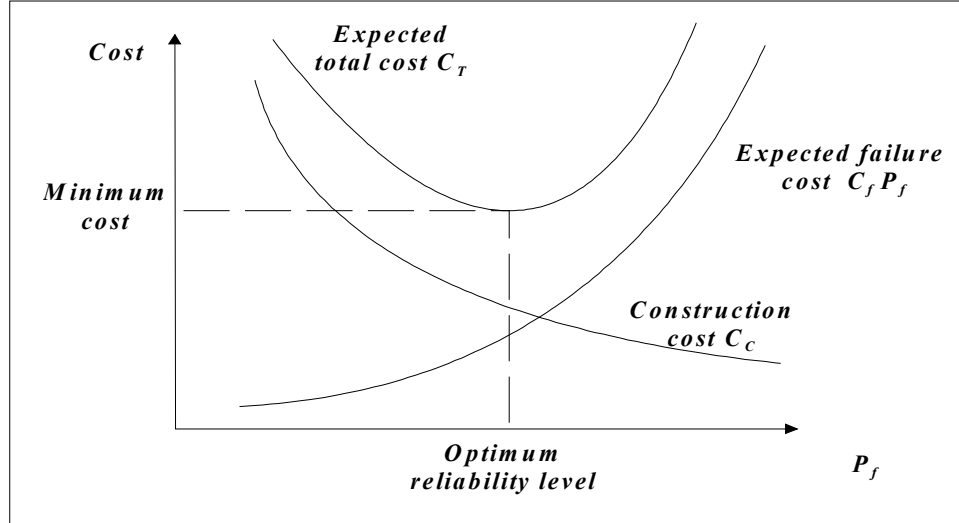
The Reliability-Based Design Optimisation (RBDO) aims at searching for the best compromise between cost reduction and reliability assurance, by taking the system uncertainties into account; therefore, the RBDO leads to an economical and safe design. It offers a good alternative to the safety factor approach, which is based on deterministic considerations and cannot take into account the reduction of safety margins during the optimisation procedure. In RBDO models, there are two kinds of variables:

- 1 the design variable  $d_k$ , which is the deterministic variable to be defined in order to optimise the design. They represent the control parameters of the mechanical system (*e.g.*, dimensions, materials, loads) and of the probabilistic model (*e.g.*, means and standard deviations of the random variables)
- 2 the random variable  $x_i$ , which represents the structural uncertainties, identified by probabilistic distributions. These variables can be related to geometrical dimensions, material characteristics and applied external loading.

Basically, the RBDO aims at minimising the total expected cost  $C_T$  (see Figure 9), which is given in terms of initial manufacturing and construction costs  $C_c$  and direct failure cost  $C_f$ :

$$C_T = C_c(d_k) + C_f P_f(x_i, d_k). \quad (10)$$

**Figure 9** Expected total cost in terms of failure probability



Due to difficulties in the failure cost estimation  $C_f$  (especially when dealing with human lives), the direct use of the above equation is not that easy. A practical formulation consists in minimising the initial cost under the constraint of satisfying a target safety level  $\beta$ :

$$\begin{cases} \min_{d_k} C_c(D_k) \\ \text{subject to : } \beta(d_k, x_i) \geq \beta_t \\ d_k^L \leq d_k \leq d_k^U \end{cases} \quad (11)$$

where  $d_k^L$  and  $d_k^U$  are the lower and upper bounds of the  $k$ -th design variable respectively.

This formulation represents two embedded optimisation problems. The outer one concerns the search for optimal design variables to minimise the cost and the inner one concerns the evaluation of the reliability index in the space of random variables. The coupling between the optimisation and reliability problems is a complex task and leads to a very high calculation cost. The major difficulty lies in the evaluation of the structural reliability, which is carried out by a particular optimisation procedure. In the random variable space, the reliability analysis implies a large number of mechanical calls, where in the design variable space, the search procedure modifies the structural configuration and hence requires the reevaluation of the reliability level at each iteration. For this reason, the solution of these two problems (optimisation and reliability) requires very important computational resources, which seriously reduces the applicability of this approach.

### 5.3 Application to truss structure

The truss structure illustrated in Figure 1 is now optimised by considering uncertainties. The structural limit states are written as:

$$\begin{aligned} G_1 &= f_Y - \sigma_b(t, w_{AB}, L, W) \\ G_2 &= F_b(t, w_{AB}, L) - F_{AB}(t, w_{AB}, L, W), \end{aligned} \quad (12)$$

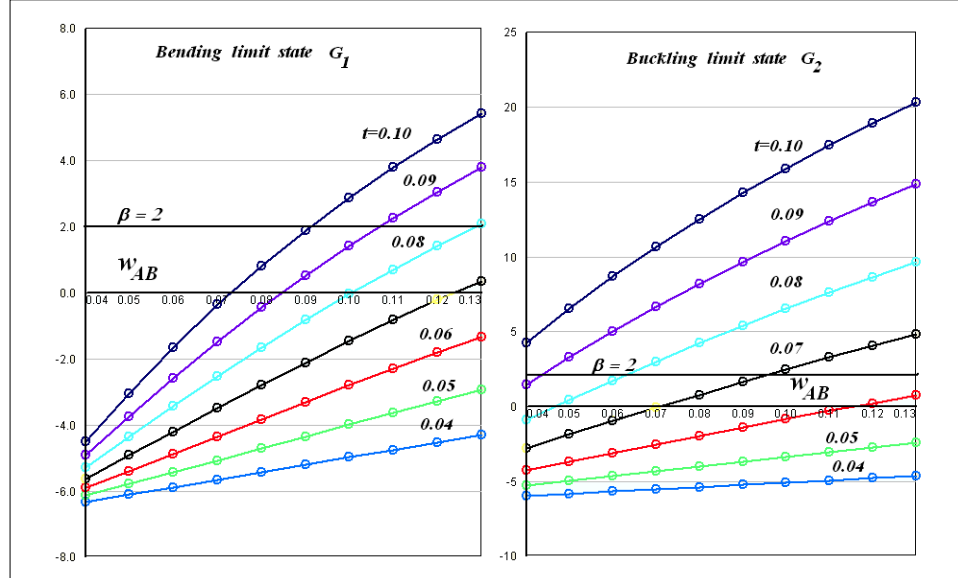
where  $f_Y$  is the yield stress,  $F_b$  is the buckling load of member  $AB$ ,  $\sigma_b$  is the bending stress at point  $B$  and  $F_{AB}$  is the normal force in member  $AB$  (detailed expressions are given in Section 2 of the paper). For a target reliability  $\beta_t = 2$  (corresponding to a failure probability of 1%), the reliability-based optimisation problem is written as:

$$\begin{aligned} \min_{t, w_{AB}} M &= W_{AB} + W_{CD} = \rho g t L \left( \frac{4\sqrt{3}}{9} w_{AB} + (w_{AB} - 0.025) \right) \\ \text{subject to : } \beta_1 &\geq 2 \quad \text{and} \quad \beta_2 \geq 2 \end{aligned} \quad (13)$$

where  $\beta_1$  and  $\beta_2$  are the reliability indexes related to  $G_1$  and  $G_2$  respectively. In this example, the uncertainties are related to the applied load  $W$ , the material strength  $f_Y$  and the truss length  $L$ , where the coefficients of variation are 15%, 8% and 2% respectively.

Figure 10 shows the reliability index evolution for the two limit states, in terms of the design variables  $w_{AB}$  and  $t$ . For the required safety level  $\beta \geq 2$ , the safe design space is reduced to points above this level. The search for the minimum weight in this subspace leads to the optimal solution given by  $w_{AB} = 0.091$  m and  $t = 0.10$  m, corresponding to a global mass of 4180 kg. At this solution, the bending limit state is observed as the most critical one. This result also indicates a safety factor of 1.38 for this limit state.

**Figure 10** Reliability index evolution in terms of truss dimensions (see online version for colours)



### Concluding remarks: three complementary approaches for embodiment design

The studied approaches of parametric design exploration have been shown to be complementary and interactive. The exploration of the design space allows us to define the region of potentially interesting feasible solutions, which can be efficiently used for further investigations. The use of metamodels thus becomes precise as the search region is shrunk. These metamodels can then be used for multidisciplinary design considering several performance objectives and constraints. As the design solution is almost described, it becomes necessary to carry out a design for reliability in order to take parameter and functioning fluctuations into account, as well as uncertainties. Finally, the whole process allows us to reach a cost-based robust and reliable design. The application to a simple truss illustrates the advantages and difficulties in the different stages of the proposed process. The three considered successive design stages of design exploration, robust design and design for reliability are more and more sophisticated since they need more and more modelling information to provide a result. This is the reason why they must be successively used within a design process. Indeed, applying one of the three approaches allows us to quickly figure out inadequacies with performance specifications or initial allowable bounds of design parameters and then to backtrack or to refine the design issue before proceeding to the next stage or approach. First, the design space is more and more precisely defined like in a Toyota-like set-based approach of design under uncertainty (see Ward *et al.*, 1994; Finch *et al.*, 1997). Second, we clearly show that the successive optimal designs obtained by the three categories of methods are notably different, but that the optimal point obtained in a given approach is used to explore its surroundings in the next approach. This paper is just an illustration of this progressive and increasingly complex preliminary parametric design process.

Table 4 summarises that the best considered designs are significantly different in the three design stages. The comparison of the different methods is not that easy, as each one is based on specific assumptions. For example, the reliable design indicates that the safety factor should be reduced to 1.38 instead of 1.506; this leads to enlarging the design exploration space. However, if the designer requires higher reliability levels, the safety factor is automatically reduced at the optimal design.

**Table 4** The best solutions found during the stages of design exploration, robust design and reliable design

<i>Design approach</i>	<i>t</i>	<i>w<sub>AB</sub></i>	<i>L</i>	<i>W</i>	<i>h</i>	<i>α</i>	<i>M</i>	<i>s</i>
Opportunistic design exploration (given in Yannou and Hamdi (2004))	0.0995	0.0927	3.01	16 570			3190	
Robust design	0.10	0.13	3.42	17 115	1.5	0.5		
Reliable design	0.10	0.091	4.00	15 385			4180	1.38

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