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# Multi-scale shape optimisation of lattice structures: an evolutionary-based approach.

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## Abstract

This study deals with the problem of the least-weight design of a lattice structure subject to constraints of different nature. To face this problem, a general multi-scale optimisation procedure is proposed. This approach aims at optimising simultaneously both global and local geometric parameters defining the shape of the representative volume element (RVE) of the lattice at the mesoscopic scale. The optimisation procedure involves design requirements defined at different scales: geometric and manufacturing constraints are involved at the mesoscopic scale, whilst thermodynamic constraints on the positive definiteness of the stiffness tensor of the lattice (modelled as an equivalent homogeneous anisotropic medium) intervene at the macroscopic scale. Finally, since lattice structures usually undergo compressive loads, a requirement on the first local buckling load is considered too. The proposed approach is based on (a) the Non-Uniform Rational Basis Splines (NURBS) curves theory to describe the shape of the struts composing the lattice, (b) the strain energy homogenisation technique of periodic media to perform the scale transition and (c) a special genetic algorithm to perform optimisation calculations. The optimised solutions provided by the presented method are characterised by a weight saving of about 39% with slightly enhanced mechanical properties when compared to conventional octahedral lattice structures.

*Keywords:* Lattice structures, homogenisation, NURBS curves, genetic algorithms, finite element method.

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## 1. Introduction

Lattice structures (or lattices) represent a new generation of products which are widely used in different engineering fields (aerospace, automotive, naval and biomedical industry) due to their high strength-to-mass ratio, superior thermal insulation and high impact properties.

Generally speaking, lattice structures are made up of a periodical pattern of a unit cell, also called *representative volume element* (RVE). Therefore, the lattice equivalent (or effective) properties at the upper scale can be easily predicted by applying a suitable homogenisation procedure on the RVE. Apart the topology characterising the RVE, the main feature of a lattice structure is the relative density which is defined starting from the density of the bulk material composing the lattice [1].

The spreading of new manufacturing technologies, like additive layer manufacturing (ALM) processes, allows for producing parts with complex geometry, which are difficult to obtain through traditional processes. ALM is particularly interesting in the production of porous structures like lattices [2]. Nevertheless, ALM technology is not free from process requirements/limitations that must be integrated into the formulation of the design problem.

Two different design approaches are available in the literature: on the one hand, those based on analytical models for describing the lattice behaviour at the macroscopic scale [1, 3–13]; on the other hand, those procedures based on general numerical homogenisation schemes applied to a detailed finite element (FE) model of the lattice RVE [12, 14–17].

Deshpande *et al.* [3] proved that the mechanical response of the lattice depends on the strain field generated into the cell wall or struts. In particular, lattices composed of struts can be classified as bending dominated or stretching dominated, if either bending or axial efforts prevails. The Maxwell’s stability criterion is an useful tool to obtain preliminary information about the nature of the lattice [18]. A micromechanical model of the octet-truss RVE was proposed by Dong [4]: this work aims at exploring the influence on the spherical nodes among the struts on the overall mechanical properties of the lattice by means of the so-called body diagram method.

Challapalli *et al.* [19] proposed a method to find the constitutive elastic behaviour of an octet-truss orthotropic structure. By means of analytical equations, they found the mechanical response of the structure transforming the global stresses and strains fields into the local ones evaluated in the local frame of each struts composing the RVE. The results proved that the elastic behaviour of the sample depends upon the printing orientation.

Ling *et al.* [20] proved that the mechanical behaviour of octet-truss lattices depends on both relative density and the intrinsic material properties, by means of experimental characterisation tests (quasi-static and dynamic compressive loading tests). In particular, they focused on the specific energy absorption capability of the octet-truss configuration.

Refai *et al.* [21] developed a method making use of an explicit description of the lattice representative volume element (RVE) to investigate and compare the influence of the lattice RVE topology on the macroscopic elastic response. A large numerical campaign has been conducted on 17 geometries. The effect of the RVE local features, i.e. the presence of a fillet radius and/or of a spherical node between struts composing the RVE as well as the surface roughness, on the effective elastic properties is investigated. Results show

that both the relative density and the geometrical features of the RVE strongly affect the equivalent macroscopic elastic behaviour of the lattice.

Other authors put much emphasis on general cellular structures, e.g. honeycomb panels, perforated plates, square cell structures. Recently, Malek and Gibson [5] derived analytical equations, to determine the elastic properties of a honeycomb made of hexagonal cells. The proposed model is able to predict the full 3D set of elastic properties by integrating the effect of nodes between inclined and vertical members in hexagonal unit cells.

Different lattice geometries are also studied in literature. For example, and without any ambition of exhaustiveness, Dallago and Luchin [1] attempted to model a simple 2D square pattern through simplifying formulae and by making use of the Euler-Bernoulli beam theory to predict the macroscopic elastic response. Finite element analyses on the same cell proved that the joints between the cell walls have a relevant effect on the mechanical behaviour of the structure which cannot be captured by the simple Euler-Bernoulli beam model.

Concerning the design methods related to lattice structures, there is a relatively small amount of works dedicated to this topic in the literature. Generally speaking, design approaches devoted to lattice structures can be formally grouped into two different classes: strategies based on analytical models of the lattice RVE and methods based on the development of suitable FE models of the RVE. Concerning the first class, in [15, 16] the so-called size matching and scaling (SMS) method is proposed to design the mesoscale RVE of the lattice structure avoiding time-consuming optimisation, by using a combination of a solid-body finite element analysis and a library of pre-defined RVE lattice configurations to generate the lattice topology. Through the SMS method the designer can easily generate the FE model of the whole component, modelled as an equivalent homogeneous material, by using a suitable mapped mesh whose properties are taken from a pre-defined library of lattice topologies.

Chu *et al.* [14] described a similar procedure, where different geometrical configurations of lattice RVE are used to fill regions of a part through a two step process. Firstly the local strain field resulting from an analysis is investigated, then an appropriate RVE topology is determined in order to withstand the local stress/strain field. The RVE geometrical parameters are computed by using a suitable strategy based on element strains resulting from the FE analysis.

As it can be inferred from the previous works, design methods based on an analytical model of the lattice RVE share two main weaknesses. On the one hand, the analytical model is based on a set of simplifying hypotheses and is limited only to a specific geometry satisfying such hypotheses. On the other hand, this kind of models is not able to correctly predict the macroscopic elastic behaviour of the lattice which is strongly affected by the lattice local features, e.g. fillet radii, spherical nodes among adjacent struts, etc. Therefore, design strategies making use of an analytical model of the lattice RVE are doubtless very efficient when the design problem involves only global requirements (e.g. mass, overall stiffness). Conversely, they are inadequate when local phenomena must be taken into account, e.g. local buckling load of the lattice RVE, fatigue strength (which is influenced by the lattice local feature [21]), etc.

Unlike the previous methodologies, the design strategies making use of a FE model of the lattice RVE are undoubtedly really general, yet time consuming. In particular, these

strategies allow for evaluating the effect of the geometrical details of the lattice RVE on the problem at hand, though they require the use of a suitable numerical homogenisation procedure. A proper homogenisation procedure is of paramount importance because of two reasons. On the one hand, it allows for defining an equivalent homogeneous anisotropic medium which replaces the true lattice RVE topology within the numerical model of the component at the macroscopic scale (thus leading to a considerable simplification of the FE model of the structure). On the other hand, numerical homogenisation schemes are really general and allow for integrating the full set of lattice parameters as design variables into the problem at hand. Examples of this design strategy dealing with the multi-scale optimisation of sandwich panels with a honeycomb core are given in [17, 22, 23].

As far as lattice structures are concerned, in literature only a limited number of works using FE models of the lattice RVE in the framework of a design approach could be found. Some examples dealing with the optimisation of the lattice structure are given in [24–26].

In [24] Azman proposed a method for evaluating the mechanical response of lattices. The main goal of this work is to provide design criteria for helping designers in choosing the best configuration and pattern to be used in a given application. The equivalent lattice structure Young’s and shear moduli are found by means of FE analysis performed on different lattice patterns (e.g. octet-truss, cubic, hexa-truss, open-cell) at different relative density values.

A parametric optimisation of the RVE geometrical variables is proposed by Bai *et al.* [25]. In this study the strut cross-section geometrical parameters were taken as design variables: the goal is the mass minimisation of the lattice subject to a constraint on the equivalent elastic moduli. The solution search was carried out by means of the firefly random-based optimisation algorithm, coupled to a FE model of the lattice in order to evaluate the mechanical response of each different configuration.

Chen *et al.* [26] proposed a design method to optimise the cross-sectional area of the generic strut forming the lattice RVE. The main goal of this approach was the minimisation of the relative density subject to a constraint on the strain energy of the lattice RVE. Different benchmarks were considered in order to demonstrate the effectiveness of the method.

The design approach proposed in this work is a general multi-scale optimisation strategy for lattice structures. The main goal is the shape optimisation of the lattice RVE by considering both global and local design requirements intervening at different scales. The design method aims at integrating all the geometrical variables defining the shape of the lattice RVE in the problem formulation without introducing simplifying hypotheses on the macroscopic behaviour of the structure. The design strategy relies on:

- the use of the Non-Uniform Rational B-Spline (NURBS) curves to describe the shape of the neutral axis of the struts composing the lattice RVE;
- the strain energy homogenisation technique of periodic media [27] to perform the scale transition;
- a special genetic algorithm able to deal with optimisation problems defined over a domain of variable dimension, i.e. the code ERASMUS (EvolutionaRy Algorithm for optimiSation of ModUlar Systems) [28, 29], to perform the solution search.

The effectiveness of the proposed strategy is proven on a meaningful benchmark: the least-weight design of a lattice by considering requirements on the equivalent elastic behaviour, on the first buckling load of the lattice RVE as well as a manufacturing constraint on the

ratio of the local curvature radius to the local thickness of the generic strut composing the RVE.

The paper is structured as follows. In Section 2 the design problem is briefly introduced. Section 3 briefly recalls the fundamentals of the NURBS curves theory and of the considered homogenisation technique. Section 4 gives the details of the mathematical formulation of the optimisation problem and of the numerical strategy as well. In Section 5 the FE models of the lattice RVE invoked during the optimisation process are presented, whilst the numerical results are discussed in Section 6. Finally, Section 7 ends the paper with some concluding remarks and perspectives.

## 2. Problem description

Selective laser melting (SLM) is an additive process that allows manufacturing components layer by layer enabling complex shapes. In this study, Ti6Al4V alloy is considered due to its excellent mechanical and corrosion properties, even at high temperature. The behaviour of the bulk material composing the lattice RVE is supposed to be isotropic and purely elastic: the Ti6Al4V properties are reported in Table 1.

The multi-scale optimisation strategy presented in this study is applied to a lattice octahedral configuration. The RVE of the octahedron (and its main geometrical parameters) is illustrated in Fig. 1.

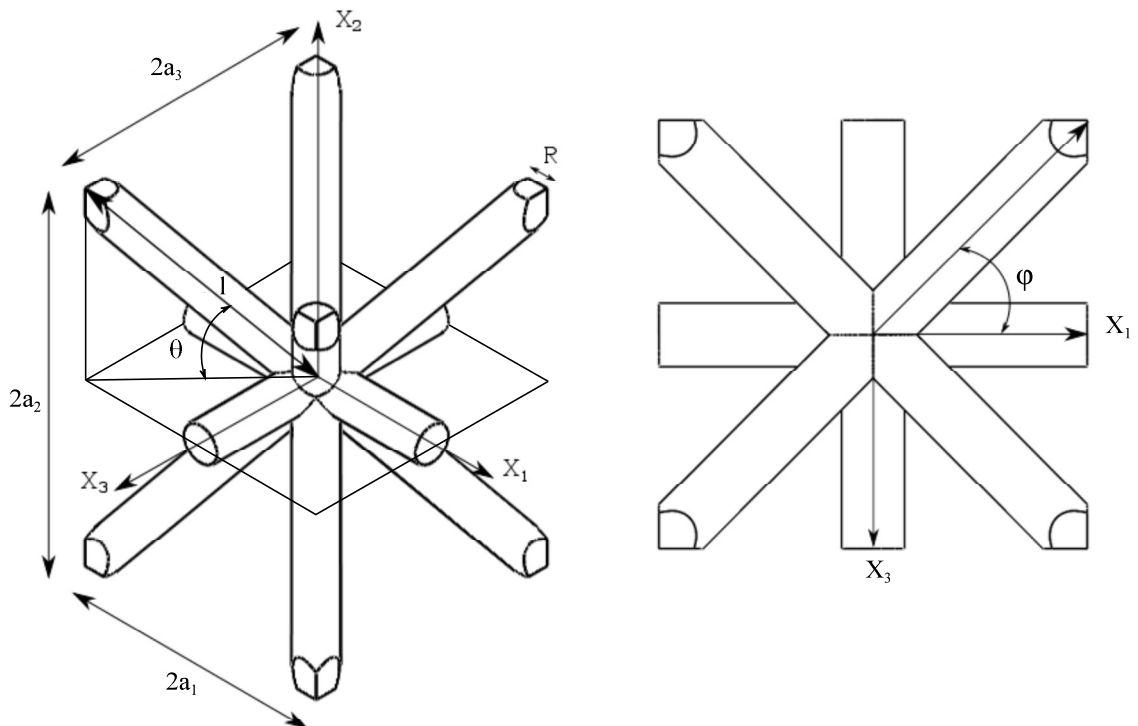


Figure 1: Geometric parameters for the reference configuration of the lattice octahedral RVE.

The RVE overall volume is defined as

$$V_{\text{RVE}} = 8a_1a_2a_3, \quad (1)$$

where

$$a_1 = l \cos \theta \cos \varphi, \quad a_2 = l \sin \theta, \quad a_3 = l \cos \theta \sin \varphi. \quad (2)$$

In Eq. (2)  $l$  is the length of the generic strut composing the RVE, while  $\theta$  and  $\varphi$  are the characteristic angles of the octahedron defined in Fig. 1; moreover, each strut has a circular cross-section of radius  $R$ .

The proposed multi-scale optimisation strategy aims to determine the optimum shape of the struts composing the octahedron in order to minimise the lattice relative density and to satisfy a given set of design requirements of different nature.

- *Geometrical constraints.* The RVE geometry must be characterised by three planes of orthogonal symmetry. Moreover no overlap between the struts composing the octahedron is allowed.
- *Manufacturing constraint.* The ratio of the local curvature radius to the cross-section diameter of each strut must be lower than or equal to an admissible value characterising the SLM technology.
- *Thermodynamic constraint.* The equivalent elastic tensor of the lattice at the macroscopic scale (i.e. evaluated through a suitable homogenisation method) must be positive definite.
- *Structural constraint.* Since the octahedral configuration is stretch-dominated [21] and the struts composing the RVE are really slender, the most critical failure mechanism is due to buckling rather than yielding (regardless of the shape of the lattice branches). Therefore, since the lattice must withstand compressive loads, a requirement on the *first buckling load* must be integrated into the problem formulation. In particular, the first buckling load of the lattice must be greater than or equal to a given *reference value* (which is computed on a reference configuration of the lattice RVE).

The value of the geometric parameters for the reference configuration are given in Table 1, while the related performances in terms of elastic properties, relative density and first buckling load are listed in Table 2.

Geometrical parameters (reference values)			
$l$ [mm]	$R$ [mm]	$\theta$ [deg]	$\varphi$ [deg]
2	0.075	45	45
Material properties			
Titanium	$E$ [GPa]	$\nu$	$\rho$ $\left[ \frac{\text{kg}}{\text{mm}^3} \right]$
	110	0.3	$4.4 \times 10^{-6}$

Table 1: Geometrical and material parameters of the lattice octahedral RVE.

### 3. Theoretical framework

#### 3.1. The NURBS curves theory

This section focuses on the fundamentals of B-Spline and NURBS curves. B-Spline and NURBS entities constitute a further development of the well-known Bézier's curves and surfaces [30]. In this work the B-spline curves are used to describe the neutral axis of

Reference configuration performances	
$\rho_{eq}^{(ref)} \left[ \frac{\text{kg}}{\text{mm}^3} \right]$	$1.6 \times 10^{-7}$
$\sigma_c^{(ref)} [\text{MPa}]$	3.28
$E_1^{(ref)} [\text{MPa}]$	622.42
$E_2^{(ref)} [\text{MPa}]$	198.29
$E_3^{(ref)} [\text{MPa}]$	622.42
$G_{23}^{(ref)} [\text{MPa}]$	453.55
$G_{13}^{(ref)} [\text{MPa}]$	451.95
$G_{12}^{(ref)} [\text{MPa}]$	453.55
$\nu_{23}^{(ref)}$	0.3016
$\nu_{13}^{(ref)}$	0.03146
$\nu_{12}^{(ref)}$	0.3016

Table 2: Reference RVE configuration performances: relative density, first buclinkg load and elastic properties.

each strut. Among the valuable bibliographic references on the topic, it has been decided to adopt the notation of [30].

The parametric explicit form of a NURBS curve is:

$$\mathbf{C}(u_1) = \sum_{i_1=0}^{n_1} R_{i_1,p_1}(u_1) \mathbf{P}_{i_1}, \quad (3)$$

where  $\mathbf{C}(u_1) = \{x_1(u_1), x_2(u_1), x_3(u_1)\}$  are the Cartesian coordinates of a point belonging to the curve, whilst  $R_{i_1,p_1}(u_1)$  is the generic rational basis function having the form

$$R_{i_1,p_1}(u_1) = \frac{N_{i_1,p_1}(u_1)w_{i_1}}{\sum_{j=0}^{n_1} N_{j,p_1}(u_1)w_j}. \quad (4)$$

In Eqs. (3) and (4),  $u_1$  is a dimensionless parameter defined in the range  $[0, 1]$ ,  $N_{i_1,p_1}(u_1)$  are the basis functions, recursively defined by means of Bernstein polynomials [30],  $p_1$  is the maximum degree,  $w_{i_1}$  are the weights and  $\mathbf{P}_{i_1} = \{X_{i_1}^{(1)}, X_{i_1}^{(2)}, X_{i_1}^{(3)}\}$  are the Cartesian coordinates of the control points. The set of the  $(n_1 + 1)$  control points (CPs) form the so-called *control polygon*. The blending functions  $N_{i_1,p_1}(u_1)$  are defined as

$$N_{i_1,0}(u_1) = \begin{cases} 1, & \text{if } U_{i_1}^{(1)} \leq u_1 < U_{i_1+1}^{(1)}, \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

$$N_{i_1,q}(u_1) = \frac{u_1 - U_{i_1}^{(1)}}{U_{i_1+q}^{(1)} - U_{i_1}^{(1)}} N_{i_1,q-1}(u_1) + \frac{U_{i_1+q+1}^{(1)} - u_1}{U_{i_1+q+1}^{(1)} - U_{i_1+1}^{(1)}} N_{i_1+1,q-1}(u_1), \quad q = 1, \dots, p_1, \quad (6)$$

where  $U_{i_1}^{(1)}$  is the  $i_1$ -th component of the following non-periodic, non-uniform *knot vector*:

$$\mathbf{U}^{(1)} = \underbrace{\{0, \dots, 0\}}_{p_1+1}, U_{p_1+1}^{(1)}, \dots, U_{m_1-p_1-1}^{(1)}, \underbrace{\{1, \dots, 1\}}_{p_1+1}. \quad (7)$$

It is noteworthy that the size of the knot vector is  $m_1 + 1$ , with

$$m_1 = n_1 + p_1 + 1. \quad (8)$$

The knot vector is a non-decreasing sequence of real numbers that can be interpreted as a discrete collection of values of the dimensionless parameter  $u_1$ , splitting the curve in arcs. The components of  $\mathbf{U}^{(1)}$  are called *knots* and each knot can have a multiplicity  $\lambda$ .

Among the properties characterising blending functions, one of the most important is the *partition of unit property*, i.e.

$$\sum_{i_1=0}^{n_1} N_{i_1, p_1}(u_1) = 1, \quad \forall u_1 \in [0, 1]. \quad (9)$$

This property allows for defining the simpler B-Spline curve starting from the more general definition of NURBS curve of Eqs. (3)-(4). Indeed, B-Spline curves constitute a subset of NURBS curves because the weights  $w_{i_1}$  take the same value. Considering the partition of unit property, Eq. (4) simplifies into  $R_{i_1, p_1}(u_1) = N_{i_1, p_1}(u_1)$  and the formula of a B-Spline curve reads:

$$\mathbf{C}(u_1) = \sum_{i_1=0}^{n_1} N_{i_1, p_1}(u_1) \mathbf{P}_{i_1}. \quad (10)$$

### 3.2. The finite element-based homogenisation method

At the mesoscopic scale, the RVE of the lattice structure can be interpreted, from a mechanical point of view, as an heterogeneous medium composed of two phases, i.e. the bulk material and the void. Conversely, at the macroscopic scale it can be modelled as an equivalent homogeneous anisotropic continuum whose mechanical response is described by a set of *effective* (or equivalent) material properties.

The set of effective elastic properties of the RVE can be determined through a suitable numerical homogenisation technique. Indeed, although the bulk material constituting the lattice is isotropic, its macroscopic behaviour (i.e. after homogenisation) can be anisotropic because the effective elastic properties depend upon the RVE topology which strongly affects the stiffness tensor of the homogeneous material at the upper scale by introducing some preferential orientations (according to the symmetries of the lattice RVE).

The macroscopic elastic behaviour of the RVE is determined using the strain energy homogenisation technique of periodic media [27]. This technique makes use of the repetitive unit of the periodic structure to evaluate the resulting properties at the macroscopic scale. The basic feature of the strain energy homogenisation technique is the assumption that the RVE of the periodic structure and the corresponding volume of the homogeneous solid undergo the same deformation, having, hence, the same total strain energy. This homogenisation scheme has proven to be an efficient numerical procedure able to determine the equivalent properties of different heterogeneous materials characterised by complex RVE topologies. The strain energy homogenisation technique of periodic media based on volume averaged stresses has already been utilised in other works, see [17, 22, 23, 31, 32] for further details.

In order to evaluate the elastic response of the lattice RVE at the macroscopic scale, two main hypotheses have been considered:

- linear, elastic behaviour for the bulk material of the RVE;

- the buckling of the cell struts/walls is disregarded.

In order to compute the RVE effective elastic properties, suitable boundary conditions (BCs) must be considered and an appropriate post-processing procedure must be carried out on the complex stress field resulting from the application of these BCs.

In order to evaluate the components of the stiffness tensor  $\mathbf{C}$  of the lattice structure at the macroscopic scale, the RVE is submitted to an average strain field  $\varepsilon_{ij}^0$ , with  $i, j = 1, 2, 3$  (tensor notation). The six independent components of the average strain tensor are applied by considering a set of periodic boundary conditions (PBCs) on the RVE [27]:

$$\begin{cases} u_i(a_1, x_2, x_3) - u_i(-a_1, x_2, x_3) = 2a_1\varepsilon_{i1}^0, & (-a_2 \leq x_2 \leq a_2, -a_3 \leq x_3 \leq a_3), \\ u_i(x_1, a_2, x_3) - u_i(x_1, -a_2, x_3) = 2a_2\varepsilon_{i2}^0, & (-a_1 \leq x_1 \leq a_1, -a_3 \leq x_3 \leq a_3), \\ u_i(x_1, x_2, a_3) - u_i(x_1, x_2, -a_3) = 2a_3\varepsilon_{i3}^0, & (-a_1 \leq x_1 \leq a_1, -a_2 \leq x_2 \leq a_2), \end{cases} \quad (11)$$

where  $i = 1, 2, 3$ .

These PBCs result in a complex stress field inside the RVE. The applied average strains always meet the following condition:

$$\bar{\varepsilon}_{ij} = \frac{1}{V_{\text{RVE}}} \int_{V_{\text{EFF}}} \varepsilon_{ij} dV = \varepsilon_{ij}^0, \quad i, j = 1, 2, 3. \quad (12)$$

In the previous equation  $V_{\text{EFF}}$  is the real volume of the lattice RVE which is different from the overall volume of the RVE, i.e.  $V_{\text{RVE}}$ , due to the presence of voids. The constitutive law for the equivalent homogeneous material (at the macroscopic scale) can be obtained as (Voigt's notation):

$$\bar{\sigma}_\alpha = C_{\alpha\beta} \bar{\varepsilon}_\beta, \quad \alpha, \beta = 1, \dots, 6. \quad (13)$$

In the previous equation Einstein's summation convention on repeated indexes is tacitly assumed. The components of the stiffness tensor  $C_{\alpha\beta}$  are then determined by solving six static analyses on the RVE and by imposing the previous PBCs, where only one component at time of the strain  $\varepsilon_\beta^0$  is different from zero for each one of the six problems. For each static analysis, the volume-averaged value of each component of the stress field  $\bar{\sigma}_\alpha$  can be easily computed and the stiffness matrix of the equivalent homogeneous material can be calculated one column at time:

$$C_{\alpha\beta} = \frac{1}{V_{\text{RVE}}\varepsilon_\beta^0} \int_{V_{\text{EFF}}} \sigma_\alpha(x_1, x_2, x_3) dV, \quad \varepsilon_\gamma^0 = 0, \quad \gamma = 1, \dots, 6, \quad \gamma \neq \beta. \quad (14)$$

The engineering moduli of the equivalent homogeneous anisotropic continuum can be calculated starting from the components of the compliance tensor  $\mathbf{S}$ :

$$\mathbf{S} = \mathbf{C}^{-1}. \quad (15)$$

#### 4. Formulation of the design problem

The design strategy proposed in this work aims to find the optimum shape of the elements composing the lattice RVE in order to minimise its relative density and to satisfy

the set of design requirements described in Section 2.

It is noteworthy that the design requirements intervene at different scales. Geometric and manufacturing constraints are involved at the mesoscopic scale, i.e. the scale of the RVE, whilst thermodynamic constraints concern the macroscopic scale wherein the lattice is modelled as an equivalent homogeneous anisotropic medium. Finally, the structural constraint on the first local buckling load takes place at the mesoscopic / macroscopic scale transition because it is imposed on an array of  $N_{\text{RVE}}$  lattice RVEs which are modelled explicitly.

#### 4.1. Design variables

The geometric design variables describing the topology of the octahedral lattice RVE are of two types: *local* and *global*. Local variables describe the shape of the struts constituting the RVE, while global ones uniquely define the RVE overall size. The global variables are the parameters  $R$ ,  $L$  and  $H$  defined in Fig. 2(a). It is useful to introduce the corresponding dimensionless counterpart as:

$$\bar{R} = \frac{R}{R_{\text{ref}}}, \quad \bar{L} = \frac{L}{2 \cos \theta_{\text{ref}} \cos \varphi_{\text{ref}} l_{\text{ref}}}, \quad \bar{H} = \frac{H}{2 \sin \theta_{\text{ref}} l_{\text{ref}}}, \quad (16)$$

where  $R_{\text{ref}}$ ,  $l_{\text{ref}}$ ,  $\theta_{\text{ref}}$  and  $\varphi_{\text{ref}}$  are the reference values given in Table 1.

As far as local variables are concerned, the shape of the neutral axis of the oblique strut is represented through a B-Spline curve, see Eq.(10) with  $p = 3$ ,  $n + 1 = 8$  and the following non-periodic and uniform knot vector is used:

$$U = \left\{ 0, 0, 0, 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1, 1, 1, 1 \right\}. \quad (17)$$

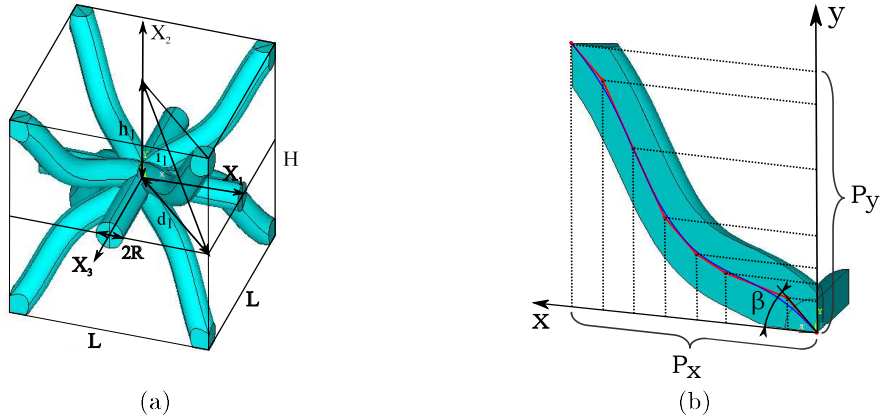


Figure 2: Global and local geometric design variables.(a) Global variables and (b) local variables (B-Spline control polygon is in red, while B-spline curve is in blue).

The CPs coordinates are defined in the oblique  $x - y$  plane (see Fig. (b)) as:

$$\begin{aligned} x_i &= \bar{x}_i d_1, \quad \text{with } i = 0, \dots, 7, \\ y_i &= \bar{y}_i h_1. \end{aligned} \quad (18)$$

Where  $d_1 = \frac{L}{\sqrt{2}}$  and  $h_1 = \frac{H}{2}$ , while  $\bar{x}_i$  and  $\bar{y}_i$  are the dimensionless CPs coordinates varying between zero and one. First and last CPs of the curve have a fixed position, whilst the  $C_1$  condition at the intersection of struts at the origin (see Fig.2(b)), required for a correct volume generation, is preserved by setting the  $y$ -coordinate of the second CP as:

$$y_1 = x_1 \tan \left( \arccos \left( \frac{d_1}{i_1} \right) \right), \text{ with } i_1 = \sqrt{d_1^2 + \frac{H^2}{2}}. \quad (19)$$

The set of global and local dimensionless design variables can be grouped into the following vector:

$$\boldsymbol{\xi} = \{\bar{R}, \bar{H}, \bar{L}, \bar{x}_1, \dots, \bar{x}_{n-1}, \bar{y}_2, \dots, \bar{y}_{n-1}\}. \quad (20)$$

The dimensionless geometric design variables together with their nature and bounds are summarised in Table 3.

Variable	Type	Lower bound	Upper bound	Discretisation step
$\bar{R}$	Discrete	1	3	0.02
$\bar{H}$	Discrete	0.5	2	0.025
$\bar{L}$	Discrete	0.5	2	0.025
$\bar{x}_i$	Discrete	0.01	0.99	0.01
$\bar{y}_i$	Discrete	0.01	0.99	0.01

Table 3: Design space.

#### 4.2. Objective and constraint functions

The goal of the optimisation process presented in this study, is the least-weight design of the lattice subject to a given set of requirements. Accordingly, the relative density of the lattice has been chosen as an objective function, i.e.

$$f(\boldsymbol{\xi}) = \rho_{eq}(\boldsymbol{\xi}) = \rho \frac{V_{\text{EFF}}(\boldsymbol{\xi})}{V_{\text{RVE}}(\boldsymbol{\xi})}. \quad (21)$$

The geometric design variables involved into the optimisation problem are not only limited by the lower and upper bounds defined in Table 3, rather they have to meet also the design requirements imposed to the problem at hand. Firstly, the shape of the lattice must satisfy the non-self-intersecting condition: this constraint equation cannot be written in a closed form and can only be checked numerically (this check is automatically performed by the FE code used to build the model of the RVE). In particular, the intersection between two non-adjacent segments of the control polygon is not allowed in order to avoid self-intersection of the volume extruded along the B-Spline curve. This task is achieved by sorting in ascending order the CPs  $x$  coordinate. Moreover, the lattice RVE is characterised by three planes of orthogonal symmetry.

Secondly, the manufacturing constraints related to the SLM technology must be considered. Among the different technological constraints inherent the SLM manufacturing process, in this work the attention is put on the admissible ratio between the minimum radius of curvature and the diameter of the oblique strut of the lattice RVE:

$$g_1(\boldsymbol{\xi}) = 2RSF - \min(r(u))(\boldsymbol{\xi}) \leq 0, \quad (22)$$

where  $\text{SF} = 1.5$  is a safety factor and  $\min(r(u))(\boldsymbol{\xi})$  is the smallest radius of curvature evaluated by means of the following formulae:

$$\kappa(u) = \frac{\|\mathbf{C}'(u) \times \mathbf{C}''(u)\|}{\|\mathbf{C}'(u)\|^3}, \quad r(u) = \frac{1}{\kappa(u)}. \quad (23)$$

As far as the thermodynamic requirement is concerned, a set of constraints must be considered to ensure the positive definiteness of the stiffness matrix of the lattice (at the macroscopic scale). These constraints can be written as follows (see [33] for more details):

$$\begin{aligned} g_2(\boldsymbol{\xi}) &= -E_1 < 0, \\ g_3(\boldsymbol{\xi}) &= -E_2 < 0, \\ g_4(\boldsymbol{\xi}) &= -E_3 < 0, \\ g_5(\boldsymbol{\xi}) &= -G_{12} < 0, \\ g_6(\boldsymbol{\xi}) &= -G_{13} < 0, \\ g_7(\boldsymbol{\xi}) &= -G_{23} < 0, \\ g_8(\boldsymbol{\xi}) &= |\nu_{12}| - \sqrt{\frac{E_1}{E_2}} < 0, \\ g_9(\boldsymbol{\xi}) &= |\nu_{13}| - \sqrt{\frac{E_1}{E_3}} < 0, \\ g_{10}(\boldsymbol{\xi}) &= |\nu_{23}| - \sqrt{\frac{E_2}{E_3}} < 0, \\ g_{11}(\boldsymbol{\xi}) &= 2\nu_{12}\nu_{13}\nu_{23}\frac{E_3}{E_1} + (\nu_{12})^2\frac{E_2}{E_1} + (\nu_{23})^2\frac{E_3}{E_2} + (\nu_{13})^2\frac{E_3}{E_1} - 1 < 0. \end{aligned} \quad (24)$$

$E_1, E_2, E_3, G_{12}, G_{13}, G_{23}, \nu_{12}, \nu_{13}$  and  $\nu_{23}$  are the effective elastic properties (engineering moduli) of the homogeneous transversely isotropic solid replacing the lattice at the macroscopic scale which are determined via the numerical homogenisation method discussed in Section 3.2. It is noteworthy that the set of constraints of Eq. (24) are implicitly imposed on the geometric design variables (both global and local ones) of the RVE.

Finally, since lattice structures are usually employed in applications involving compressive loads, a constraint on the first local buckling stress is imposed in order to prevent undesired localised instability phenomena of the RVE struts:

$$g_{12}(\boldsymbol{\xi}) = \sigma_c^{(\text{ref})} - \sigma_c \leq 0, \quad (25)$$

where  $\sigma_c^{(\text{ref})}$  is the first buckling load of the reference configuration of the RVE presented in Section 2 whose value is given in Table 2.

Therefore, the lattice least-weight design problem can be stated as a classical constrained non-linear programming problem (CNLPP) as follows:

$$\begin{aligned} &\min_{\boldsymbol{\xi}} f(\boldsymbol{\xi}), \\ &\text{subject to :} \\ &g_j(\boldsymbol{\xi}), \quad j = 1, \dots, 12. \end{aligned} \quad (26)$$

### 4.3. The numerical strategy

The CNLPP of Eq.(26) is a non-convex problem in terms of both global and local geometrical variables. Its non-convexity is due, on one side, to the nature of the objective function and, on the other, to the optimisation constraints.

Problem (26) is characterised by 14 design variables, see Eq. (20), and 12 optimisation constraints according to Eqs. (22)-(24). The GA ERASMUS (EvolutionaRy Algorithm for optimiSation of ModUlar Systems)[28, 29] has been used as optimisation tool for the solution search of problem (26) and has been coupled to the FE models of the lattice (which are built into the ANSYS® environment) to compute the first buckling load of the structure and the equivalent elastic properties at the macroscopic scale as well. The GA ERASMUS was already successfully applied to solve different kinds of real-world engineering problems, see [17, 34–38] for example.

As shown in Fig. 3, for each individual at each generation seven FE analysis are performed: six static analyses for the evaluation of the lattice macroscopic elastic properties and, subsequently, an eigenvalue buckling analysis on an array of the lattice composed of a given number of RVEs (see Section 5 for more details on the FE models involved into the optimisation process).

The FE models of the RVE make use of both global and local geometrical parameters (given by ERASMUS and elaborated by ANSYS® which generates the B-Spline curve representing the oblique strut of the RVE) in order to perform the numerical homogenisation of the lattice and also to calculate the local buckling load of the current configuration.

The results of the analyses are elaborated by the GA to execute the genetic operations which are repeated until the user-defined convergence criterion is met.

The generic individual represents a potential solution for the optimisation problem. The genotype of the individual for the problem (26) is characterised by one chromosome composed of 14 genes, each one related to a component of the design variables vector of Eq.(20).

## 5. The finite element models of the lattice

The FE models of the lattice have been created within the commercial FE code ANSYS®. These models are generated through *ad-hoc* ANSYS Automatic Parametric Design Language (APDL) scripts in which the mesh has been properly related to the relevant geometric variables of the RVE to take into account for different topologies. At the mesoscopic scale, the lattice RVE is modelled by means of 20-nodes solid elements (SOLID186), with three degrees of freedom (DOFs) per node. A FE model made of solid elements is needed to provide a realistic representation of the RVE geometry and to accurately estimate the 3D stress field generated inside the RVE and which strongly influences its macroscopic elastic behaviour. The RVE model is shown in Fig. 4(a). Since the 3D FE model of the lattice RVE is used for homogenisation purposes, six static analyses are needed in order to determine the equivalent stiffness tensor (and hence the effective elastic moduli) at the macroscopic scale according to Eq. (14). To this purpose, the PBCs of Eq. (11) have been applied through constraint equations (CEs) among homologous nodes belonging to the opposite faces of the RVE.

A convergence study in terms of the average element size (not reported here for the sake of brevity) has been conducted in order to verify its influence on the effective elastic properties of the different RVE topologies. The mesh size has been chosen equal to 0.075 mm for a cell size  $L$  equal to 2 mm.

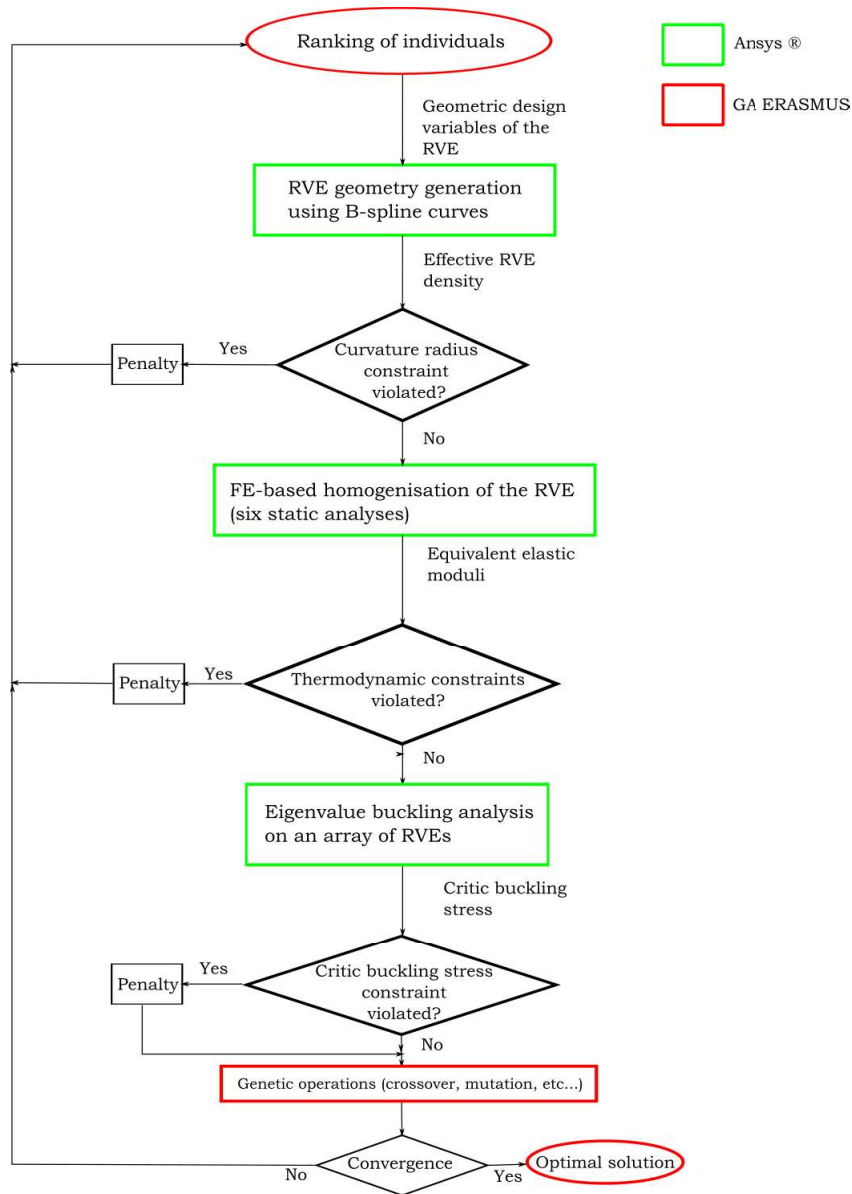


Figure 3: Optimisation procedure diagram.

As far as the eigenvalue buckling analysis is concerned, a simplified model made of beam elements and composed of a  $2 \times 2 \times 2$  RVEs has been built in order to achieve a compromise between accuracy of results and low computational effort. In this model, BEAM188 elements (with three nodes and six DOFs per node) based on the Timoshenko's kinematic model are used to represent the struts of the RVE with a circular cross-section of radius  $R$ . To model the rigid ends of the specimen constituted of  $2 \times 2 \times 2$  RVEs, multi-point constraints MPC184 elements are defined between central master nodes (A and B in Fig. 4(b)) and those belonging to the bottom and top faces (slave nodes), respectively.

Then, the following BCs are imposed on nodes A and B:

$$\begin{aligned}
 \text{node A : } & u_i = 0, \beta_i = 0, \\
 \text{node B : } & u_x = u_z = 0, \beta_i = 0, F_y = -1N, \\
 & i = x, y, z.
 \end{aligned} \tag{27}$$

Finally, the critical buckling stress of the RVEs array is computed by dividing the first buckling load to the area of the rigid end, i.e.  $A_{\text{rigid}} = 4L^2$  according to Figs. 2 and 4(b).

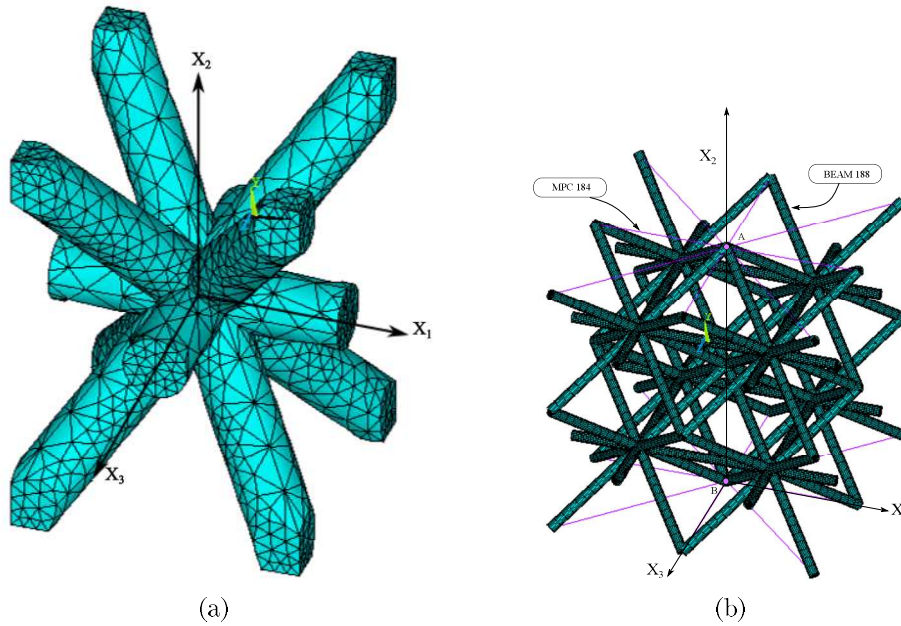


Figure 4: Numerical models involved in the optimisation process: (a) 3D model of the RVE, (b) Simplified model of the array of RVEs made of beam elements.

## 6. Numerical results

The genetic parameters tuning the behaviour of the GA used to perform the solution search for problem (26) are listed in Table 4. Moreover, the automatic dynamic penalisation (ADP) method has been considered as a constraint-handling technique, see [28]. For more details on the optimisation tool and the meaning of the values of the different parameters tuning the GA, the reader is addressed to [29].

The overall optimisation process has required approximately ten days (30 s for all the FE analyses executed for the generic individual) using two cores of a machine with an Intel Xeon E5-2697v2 processor (2.70 – 3.50 GHz). The value of both local and global geometric design variables for the optimised configuration are listed in Table 5.

The optimised lattice is characterised by an equivalent density  $\rho_{eq} = 9.74 \cdot 10^{-8} \frac{\text{kg}}{\text{mm}^3}$ . This value has been obtained after only 49 iterations and represents a remarkable reduction (39.12%), when compared to the relative density of the reference configuration given in Table 1. Furthermore, the optimised RVE satisfies the full set of design requirements, especially that on the buckling stress. In fact, the critic buckling stress provided by the optimised RVEs is  $\sigma_c = 3.363 \text{ MPa}$ , which is about 2.41% greater than the reference value given in Table 1.

Genetic parameters	
$N_{pop}$	2
$N_{ind}$	100
$N_{gen}$	150
$p_{cross}$	0.85
$p_{mut}$	0.01
Isolation time	15
Selection	Roulette-wheel
Elitism	Active

Table 4: Genetic parameters of the GA ERASMUS.

Design variable	Value
$\bar{R}$	1.13
$\bar{H}$	1.825
$\bar{L}$	1.3
$\bar{\mathbf{x}}$	(0.0, 0.189, 0.409, 0.619, 0.909, 0.979, 0.979, 1.0)
$\bar{\mathbf{y}}$	(0.0, 0.059, 0.129, 0.189, 0.369, 0.579, 0.729, 1.0)

Table 5: Global and local dimensionless geometric design variables for the optimised configuration of the lattice.

The trend of the objective function along the generations for the best individual is illustrated in Fig. 5. As it can be inferred from this figure, the best individual (and, thus, all the individuals within the two populations) is infeasible since at least one constraint is not satisfied. However, thanks to the ADP method, the GA is able to find a feasible solution after only 25 iterations and to converge towards a pseudo-optimal solution after 49 generations.

Fig. 6 shows the shape of the optimised configuration and the deformed shape of the RVEs array related to the first buckling mode as well. The trend of both Young's and shear moduli from generation 20 to generation 49 is illustrated in Fig 8, whilst the corresponding values for both the reference and the optimised configurations are provided in Table 6. As it can be inferred from these results, the optimised RVE is globally less stiff than the reference counterpart. However, the local buckling behaviour is enhanced thanks to a different geometric configuration of the oblique strut that, in the optimised configuration, is aligned with the direction of the applied load, i.e. along the  $y$ -axis (see Fig. 6(b)).

## 7. Conclusions

The design strategy presented in this paper is a numerical optimisation procedure characterised by several features that make it an innovative, effective and general method for the multi-scale design of lattice structures. In the present work, this strategy has been applied to the multi-scale optimisation of an octahedral lattice. The design process is not submitted to restrictions: both global and local geometric parameters describing the lattice RVE shape at the mesoscopic scale are considered as optimisation variables. This allows searching for a true global minimum, hard to be obtained otherwise.

The proposed optimisation method relies, on the one hand, on the NURBS curves theory to describe the shape of the neutral axis of each strut composing the lattice RVE

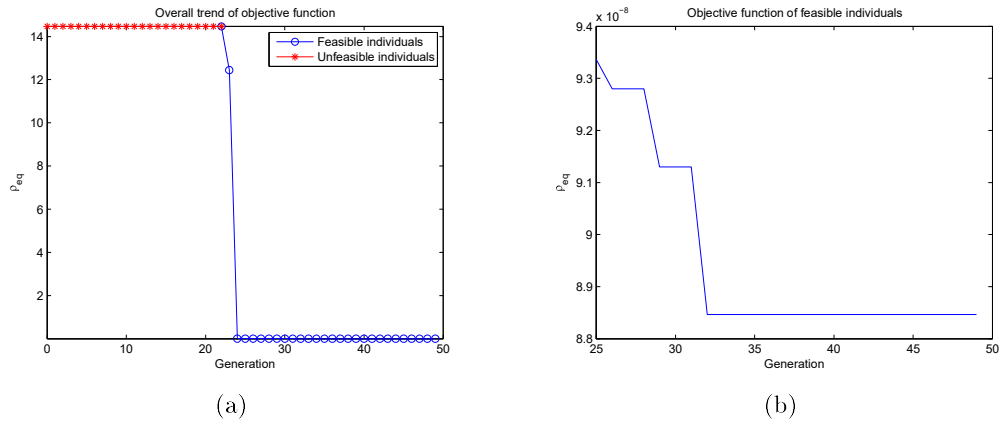


Figure 5: Best individual's objective function vs. generations. (a) Overall trend with infeasible solutions during the first 25 generations and (b) feasible individuals starting from iteration 25.

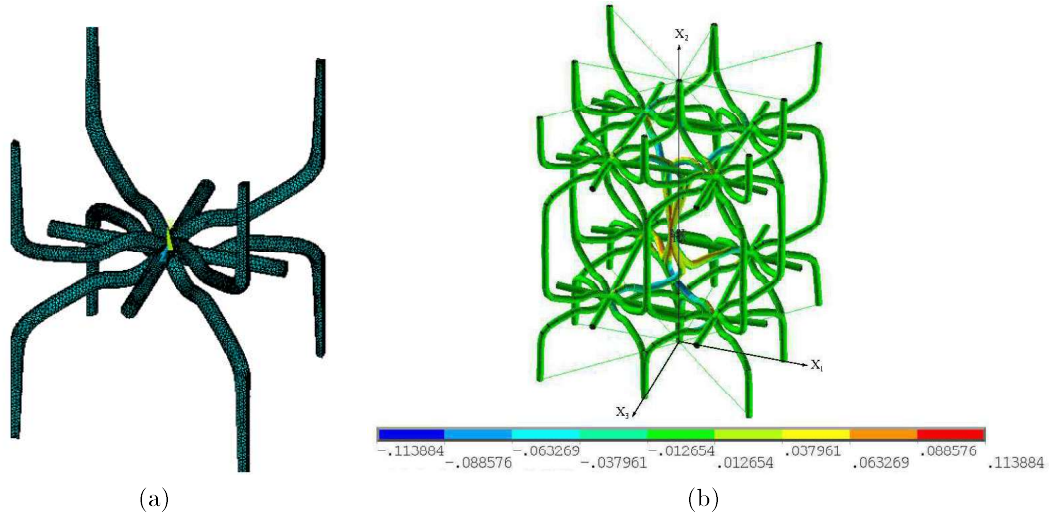


Figure 6: Optimised configuration of the lattice RVE: (a) the optimum shape of the RVE and (b) the deformed shape related to the first buckling load for the array of the lattice RVEs.

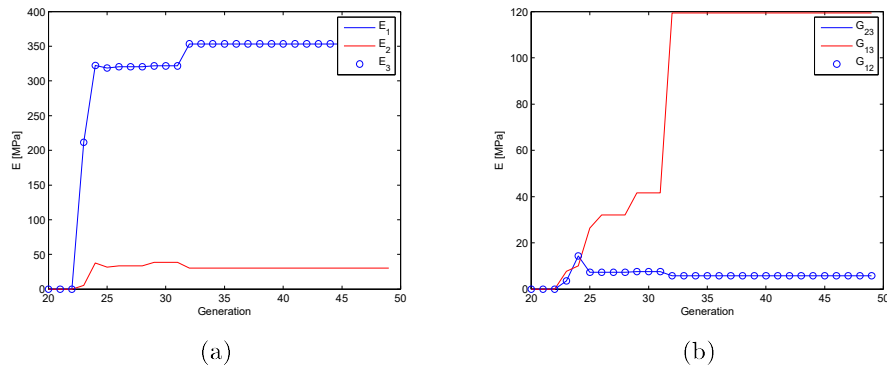


Figure 7: (a) Young's moduli and (b) Shear moduli vs. generations.

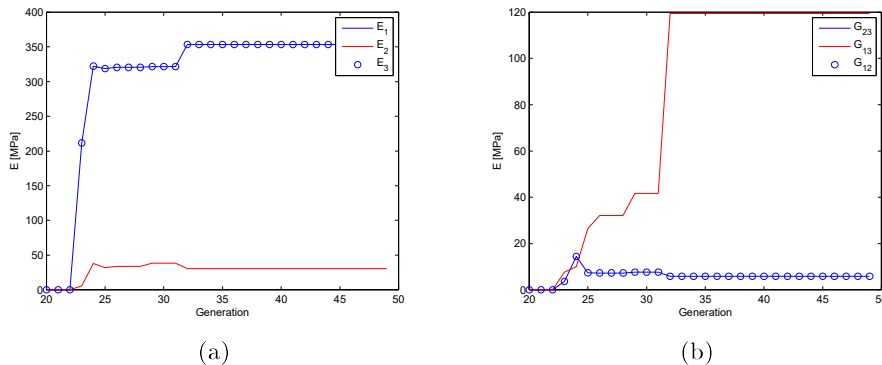


Figure 8: (a) Young's moduli and (b) Shear moduli vs. generations.

Elastic moduli	Reference solution	Optimised solution
$E_1$ [MPa]	622.42	369.86
$E_2$ [MPa]	198.29	12.2
$E_3$ [MPa]	622.42	369.86
$G_{23}$ [MPa]	453.55	2.674
$G_{13}$ [MPa]	451.95	65.52
$G_{12}$ [MPa]	453.55	2.674
$\nu_{23}$	0.3016	0.028
$\nu_{13}$	0.03146	0.15
$\nu_{12}$	0.3016	0.028

Table 6: Elastic moduli for the reference and the optimised configurations of the lattice RVE.

and, on the other hand, on a general numerical homogenisation technique.

The multi-scale design problem of the lattice structure involves several design requirements defined at two characteristic scales: the mesoscopic scale of the RVE and the macroscopic one where the lattice is modelled as an equivalent homogeneous anisotropic medium. The effectiveness of the optimisation method is tested on a meaningful case study: the least-weight design of the lattice subject to constraints of different nature. Such constraints include: manufacturing and geometric requirements on the lattice shape (mesoscopic scale), thermodynamic constraints on the stiffness tensor of the lattice (macroscopic scale), a requirement on the local buckling strength of the lattice (transition from mesoscopic scale to macroscopic one).

The utilisation of an evolutionary strategy, together with the fact that the problem is stated in the most general sense, allows finding some non-conventional configurations that are more efficient than the standard ones. In fact, the considered example proves that, when standard lattice topologies are abandoned and all the parameters characterising the lattice shape are included within the design process, a significant weight saving can be obtained: up to 39% (with respect to a reference octahedral configuration) with equivalent mechanical properties in terms of the critical buckling stress.

Generally speaking, the proposed approach can be considered interactive because the result of the optimisation process is not uniquely an optimum configuration of the product (recall that the optimisation problem discussed in this work is strongly non-convex), rather a family of optimal (or/and pseudo-optimal) solutions on which the designer (or the

engineer) can intervene by defining further design requirements (aesthetics, different functionalities, product customization, etc.) in order to sort the best configurations (according to the aforementioned criteria). Moreover, the proposed multi-scale optimisation strategy is problem-independent and the interface with the user (designer, engineer, student, etc.) takes place at three different levels: (i) when the user defines the mathematical formulation of the design problem (starting from the technical specification of the considered application); (ii) when the user sets the parameters tuning the behaviour of the optimisation tool; (iii) when the user implements the numerical models (which are used to evaluate the physical responses involved into the problem formulation) to be interfaced with the optimisation tool.

As far as perspectives of this study are concerned, research is ongoing in order to deal with the more general case of multi-scale topology optimisation of lattice structures. The idea is to develop a multi-scale topology optimisation based on a CAD-compatible descriptor of the topology at each iteration as currently done in other research works developed at I2M laboratory in Bordeaux [39, 40]. Moreover, other useful design criteria should be introduced in the problem formulation: lattice fatigue strength, process-related constraints as the presence of support material, part distortion, overhang angle, etc.

A further interesting perspective of this work could be the implementation of a suitable surrogate model to approximate the physical response of the product at each pertinent scale. On the one hand, such a surrogate model could be directly interfaced with the optimization algorithm in order to find the optimal solutions in a faster way. On the other hand, this meta-model could be used a posteriori to analyse each optimal configuration of the product in a unique and common environment shared by multiple users having different backgrounds. In this way, the optimal solutions can be quickly analysed (i.e. in quasi-real time fashion), and the best configuration can naturally arise as a result of a collective choice by smartly exploiting the skills of the different users. Research is ongoing on all the previous points.

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