# Contents

# Preface

The objective of this book is to present a consistent methodology for the study and numerical computation of the thermomechanical response of materials and structures, with a particular focus on composites. We recognize that there are plenty of studies in the literature that successfully treat, both theoretically and computationally, the thermomechanical response of various categories of dissipative materials or composites and that are not included in this manuscript. Our aim though is not to present an extensive review of these studies, but to propose to the scientific community a general framework for studying the vast majority of dissipative materials and composites under fully coupled thermomechanical loading conditions. We cover many aspects of the modeling process, so that the reader is able to find how to: (i) identify the conservation laws and thermodynamic principles that need to be respected by any solid material, (ii) construct proper constitutive laws for various types of dissipative processes, both rate-independent and rate-dependent, by utilizing an appropriate thermodynamic framework, (iii) design robust numerical algorithms that permit accuracy and efficiency in the calculations of very complicated constitutive laws, (iv) extend all the previous points to the study of composites, utilizing rigorous homogenization theories for materials and structures with both periodic and random microstructure. For the last point, the book explores the concepts of periodic homogenization, namely the asymptotic expansion homogenization method, as well as various micromechanics theories based on the Eshelby approach. We believe that our book, with the topics it covers, will be useful to both young and advanced researchers that want to obtain a general guide to properly studying the thermomechanical response of dissipative materials and composites, and to identifying robust and accurate computational schemes.

Chapter 1 is devoted to a quick presentation of tensor calculus, both in Cartesian and curvilinear coordinate systems, as well as the various symbols denoting tensor operations that are utilized throughout the book. While the indicial notation with the Einstein summation rule is very helpful in many situations, the large number of indices, mainly introduced in Chapter 3 on computational methods and Chapters 5 and 6 on homogenization theories, requires a more elegant representation of tensors. Thus, the tensorial notation with bold fonts for vectors and higher order tensors is chosen in the book. The first chapter also discusses the Voigt notation, which is particularly useful for representing second and fourth order tensors as vectors and matrices respectively, simplifying the computational procedures. In addition, a special notation for isotropic fourth order tensors is included in section 1.1.3.

Chapter 2 is a short summary of the continuum mechanics theory and the identification of constitutive laws based on thermodynamic principles. The first four sections discuss the general principles of continuum mechanics (kinetics, kinematics, conservation laws, thermodynamics) and their reduction when small deformation procedures are considered. The fifth section of the chapter focuses on the description of constitutive laws for dissipative materials using a proper thermodynamic framework. The presented framework is quite general in order to include many types of mechanisms: viscoelasticity, plasticity, viscoplasticity and continuum damage constitutive laws are discussed in this section. Even though not mentioned explicitly, the framework is also capable of describing phase transformation mechanisms, such as martensitic transformation occurring, for example, in shape memory alloys. The chapter closes with the presentation of a thermomechanical parameters identification strategy through appropriate experimental protocol for an elastoplastic material.

Chapter 3 presents rigorous integration methods that make it possible to identify and computationally simulate the response of materials and structures under quasi-static loading conditions when nonlinear mechanisms appear. The chapter introduces the general iterative scheme that can be applied for solving a boundary value problem using the finite element method (though without entering into details on finite element computations) and focuses mainly on the numerical implementation of the constitutive law for a homogeneous material. The presented methodologies are based on the well known return mapping algorithm scheme. Analytical description on the methods is given for the case of rate independent plasticity. Numerical applications in plasticity and viscoelasticity are also included in the chapter. The numerical procedures discussed here, even though applied to homogeneous materials, can be considered as the basis for the design of numerical algorithms applied to composites.

Chapter 4 introduces the notion of homogenization, which is presented in two different frameworks: one that focuses on homogenization from an engineering point of view, and one that describes the principles of mathematical homogenization. Both frameworks eventually lead to the same conclusions, even though they start from a different theoretical background. The engineering oriented homogenization accounts for composites with periodic or random microstructure and its concepts are mainly utilized in Chapter 6. On the other hand, the mathematical homogenization is the basis of the asymptotic expansion homogenization method that is utilized in Chapter 5 to describe the homogenization of composites with periodic microstructure.

Chapter 5 focuses on composite materials and structures with periodic microstructure. The asymptotic expansion homogenization method is employed for identifying the microscopic and macroscopic principles that the actual composite obeys (kinetics, kinematics, conservation laws, thermodynamics). For quasi-static processes and composites with dissipative material constituents a numerical scheme is proposed for solving the fully coupled thermomechanical problem iteratively and simultaneously in the two scales (microscopic and macroscopic). An illustrative example of multilayered composite material is discussed and various numerical applications with elastic, plastic and viscoplastic responses are presented.

Chapter 6 discusses homogenization techniques for composite materials and structures with random microstructure. The described methodologies belong to the mean field or Eshelby based theories. After presenting the classical Eshelby problems the chapter introduces three homogenization techniques: the Mori-Tanaka, the self consistent and the Ponte-Castañeda and Willis method. For quasi-static processes and composites with dissipative material constituents an iterative numerical scheme is proposed for solving the fully coupled thermomechanical homogenization problem by using the concentration tensors calculated with an Eshelby-based approach. An illustrative example of a composite material consisting of a matrix phase and spherical particles is presented and various numerical applications with elastic and plastic responses are examined.

Appendices 1 and 2 are devoted to large deformation processes and their connection with the homogenization theories. Appendix 1 discusses the average theorems in the reference configuration of a deformable body, i.e. the stress is measured through the Piola stress tensor and the deformation is identified through the deformation gradient tensor. Appendix 2 presents the extension of the asymptotic expansion homogenization method for composites with periodic microstructure, using the same stress and deformation measurements (though without entering into details of proper microscopic and macroscopic thermodynamic potentials). While these two appendices do not provide the global picture of a general homogenization framework in large deformation processes, they can be considered as important contributions towards such a target.

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