# An Approximation Approach for an Integrated Part Quality Inspection and Preventive Maintenance Planning in a Nonlinear Deteriorating Serial Multi-stage Manufacturing System

**M. Rezaei-Malek*,**, A. Siadat*, J.-Y. Dantan*,**
**R. Tavakkoli-Moghaddam*,****

**LCFC, Arts et Métier ParisTech, 57078 Metz, France*
*(Tel: +33-646-71-6545; e-mail: mohammad.rezaeimalek@ensam.eu)*
***School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

**Abstract:** This paper develops a mixed-integer non-linear programming model for an integrated planning of part quality inspection and preventive maintenance activities. This model considers a serial multi-stage manufacturing system while its production stages are deteriorating nonlinearly. The deterioration is assumed a non-linear process and an approximation approach based on the separable programming is developed to cope with the non-linearity. The model determines the optimum times and locations for performing inspection and preventive maintenance activities while minimizing total manufacturing cost. The theoretical achievement of the presented approach is investigated through a numerical example.

## 1. INTRODUCTION

Part Quality Inspection Planning (PQIP) is a challenging problem for Serial Multi-stage Manufacturing Systems (SMMSs), specially deteriorating ones. The PQIP problem contains the questions like when, where and how a part quality inspection activity should be performed along a manufacturing process. In addition, after detection of a non-conforming item, the determination of proper defect management strategies (i.e. scrap, rework and replacement/repair) is needed to prevent the defect propagation throughout the system and defect delivery to end-consumer.

Preventive Maintenance (PM) is employed as an advantageous tool for maintaining the system in a good production state for producing high quality products. Since the performance of the PM activities affects production parameters such as Defective Production Rate (DPR)–which is the main input information for the PQIP problem, there is a strong dependency of the PQIP on the PM plan. The PM plan itself depends on the system status and its deterioration trend. This dependency connection has been also mentioned in the new paradigm “Production Quality” proposed by Colledani et al. (2014) as a potential research direction to be considered for increasing the efficiency of manufacturing systems. As a first attempt, Rezaei-Malek et al. (2018a) developed a mathematical model for the integrated planning of part quality inspection and PM activities for a linear-deteriorating SMMS while minimizing total manufacturing cost. However, they pointed out the superficiality of the linear deteriorating assumption because almost all the real manufacturing systems degrade in a nonlinear way. This paper assumes that the deterioration process of the manufacturing stages in a SMMS is non-linear and presents a separable programming approach to approximate the optimum integrated plan.

## 2. LITERATURE REVIEW

The PQIP problem has been investigated by the researchers since 1964. Mandroli et al. (2006) reviewed the PQIP problem literature based on the considered manufacturing system and modelling characteristics. However, they did not do any analysis about the integration of this problem with the other related issues such as production and maintenance planning. These kinds of connections are well elaborated by Colledani et al. (2014) and Rezaei-Malek et al. (2018b).

Gunter and Swanson (1985) explored the effects of the inspection location decisions on the production planning. For a given required number of final conforming products, they determined the required production rate of each manufacturing stage. They assumed that each inspection station can detect and scrap all the nonconforming items free of error. Park et al. (1988) determined the required quantity of raw materials to produce the demanded quantity within the planning horizon while considering incoming raw material reliability and machine processing reliability. For four different configurations of free-error inspection stations, Tayi and Ballou (1988) optimized lot size and determined optimal reprocessing batch size for the rejected items. Raghavachari and Tayi (1991) extended the modelling approach of Tayi and Ballou (1988) and developed a variant of the shortest path algorithm to deal with the problem of determining concurrently the optimal initial lot size, inspection configuration and reprocessing decisions. Narahari and Khan (1996) compared possible alternative ways of locating inspection stations to optimize cycle time and throughput for a re-entrant manufacturing system. Raz et al. (2000)

addressed the economic joint optimization of the inspection policy and the production batch size. The optimum intensity, sequence and timing of inspection effort with several possible inspection activities at each stage were investigated by Kogan and Raz (2002). Kakade et al. (2004) incorporated the throughput rate into the objective function instead of modelling it as a constraint. This allowed the optimal inspection decision to pose a new cycle time for the line. Penn and Raviv (2007) and Penn and Raviv (2008) considered unreliable serial production lines with known failure probabilities for each operation. They simultaneously determined the locations of quality control stations along the line and the production rate while maximizing the steady state expected net profit per time unit. Shiau et al. (2007) proposed an integrated process planning and inspection planning. They considered manufacturing capability, inspection capability, and tolerance specified by customer requirement and developed a mathematical model to select and assign the proper inspection and workstations while minimizing unit manufacturing cost. Manufacturing shop scheduling and the optimal allocation and sequencing of inspection stations was investigated by Sadegheih (2007) through genetic algorithms and simulated annealing techniques. Galante and Passannanti (2007) developed a genetic algorithm to address an interacted problem of operation scheduling and inspection strategy in the job-shop environment. Kim and Gershwin (2008) did an analysis on the effective production rate for three different versions of long flow lines that differ in the locations of the inspection stations and in the sets of machines that each inspection station monitors.

As can be seen, although the connection between production logistic planning and the PQIP problem has being investigated analytically in the literature, to the best of our knowledge, the link of the PQIP and maintenance planning problems has not been properly explored yet. For the first time, Rezaei-Malek et al. (2018a) proposed a mixed integer linear programming for the integrated planning of part quality inspection and PM activities. They assumed that each production stage degrades linearly and accordingly the probability which an item incurs a defect is increasing. However, most of the time, in real manufacturing systems, production stages degrade non-linearly. Regarding this non-linearity, this paper proposed a non-linear programming for the integrated problem and approximate an optimum solution through applying separable programming approach.

## 3. PROBLEM DESCRIPTION

Consider a SMMS which produces a single type product through $n$ identical serial manufacturing stages. Raw materials enter this discrete part manufacturing process and each manufacturing stage realizes a specific Quality Characteristic (QC) for the product. However, the manufacturing stages are not technologically capable to process all the items in perfect quality. To prevent this defects propagation and defects delivery to end-customer(s), there is possibility to perform an inspection after each production stage to detect non-conforming items. Each inspection is capable to detect the defect incurred in the preceding manufacturing stage. It should be mentioned that Error type I and II exist for the inspection process. Some of the rejected items are repairable and the rest should be scrapped while posing known expenses to the system.

Almost all the manufacturing stages are deteriorating in time. When the deterioration process starts, stage runs at a lower quality rate (Muchiri et al., 2013). To be exact, the probability that each item incurs a defect, $\varepsilon$, after each stage is increasing in time. Fig. 1 shows the behavior of this probability for a sample stage during its lifespan. At the end of lifespan, the manufacturing stage is down and a Corrective Maintenance (CM) is done to bring it back to the *as good as new* condition. Meanwhile, it is an opportunity to do a PM activity in a known cost before a complete failure, which depends on the extent of remained time to the complete failure. In this way, the defects propagation and accordingly material and energy wastage would be decreased. Therefore, there is a probability that an item incurs a defect after Stage $j$ in period $t$, $\varepsilon_{jt}$, which is a function of number of periods have been passed since the last PM activity on this stage, $g(\omega_{jt})$.

The SMMS needs a simultaneous plan for part quality inspection and PM activities to produce a certain number of conforming items during a planning horizon (consisting of equal time periods) while minimizing total cost including production, inspection, repair, scrap, and maintenance. Below, the problem assumptions are provided. Then, the required notations are defined. Finally, a non-linear mixed-integer programming model is developed for the afore-mentioned problem.

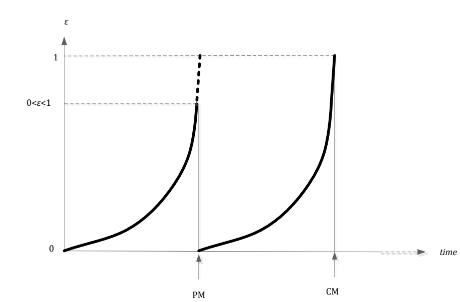


**Fig. 1.** Impact of the PM and CM on the value of $\varepsilon$

### *3.1 Assumptions*

- Probability of occurrence of Error type-I and Error type-II for each inspection activity are known and constant during the planning horizon,
- Repairable and scrap fraction of rejected items are certain for each inspection activity and are fixed during the planning horizon,
- Repair operation is done without any error and transforms the nonconforming items to the conforming ones,
- Manufacturing stages are identical in lifespan and deterioration behaviour,
- A PM activity brings a manufacturing stage back to the as good as new condition (i.e., $g(0) = 0$),

- Although the state of a manufacturing stage is stochastic, this paper assumes that, without any PM, a sudden failure does not happen before the end of the lifespan.

*3.2 Indices*

$j$ index of production stages ($j = 1, \ldots, n$),
$t,t'$ index of time periods ($t$, $t'= 1, \ldots, T$).

*3.3 Parameters*

$n$ number of production stages in the SMSPL,
$w_{t01}$ number of unit material that enters the SMSPL in the period $t$,
$\alpha_j$ probability of Type-I error at the inspection station $j$,
$\beta_j$ probability of Type-II error at the inspection station $j$,
$\varepsilon_{t0}$ non-conforming fraction of material that enters the SMSPL in the period $t$,
$I_j$ unit inspection cost at the inspection station $j$,
$R_{j1}$ unit replacement/repair cost of a conforming item rejected by the inspection station $j$,
$R_{j2}$ unit replacement/repair cost of a non-conforming item rejected by the inspection station $j$,
$p_j$ unit production cost in the stage $j$,
$p_0$ unit material cost entering the SMSPL,
$S_j$ scrap cost per unit at the stage $j$,
$C$ penalty cost of delivering a non-conforming item to customer,
$RE$ revenue generated by delivering an item to customer,
$f_{j1}$ fraction of the rejected items repaired at the inspection station $j$,
$f_{j2}$ fraction of the rejected items scraped at the inspection station $j$,
$g(.)$ $\varepsilon_{tj}$ behavior function,
$v(.)$ $mc_{tj}$ behavior function,
$K$ number of unit material entering the SMSPL in each period.

*3.4 Variables*

$d_{tj}$ 1; if inspection station associated to the stage $j$ is established in the period $t$, 0; otherwise,
$md_{tj}$ 1; if PM activity associated to the stage $j$ is performed in the beginning of the period $t$, 0; otherwise,
$mc_{tj}$ cost of performing a PM activity for the stage $j$ in the period $t$; $mc_{tj}=v(\omega_{tj})$,
$w_{tj1}$ expected conforming items entering the production stage $j$ in the period $t$,
$w_{tj2}$ expected non-conforming items entering the production stage $j$ in the period $t$,
$u_{tj1}$ $= \alpha_j(1-\varepsilon_{tj})w_{tj1}d_{tj}$; expected conforming items rejected at the $j^{\text{th}}$ inspection station in the period $t$,
$u_{tj2}$ $= (1-\beta_j)\ (\varepsilon_{tj}w_{tj1}+w_{tj2})\ d_{tj}$; expected non-conforming items rejected at the $j^{\text{th}}$ inspection station opportunity in the period $t$,
$\omega_{tj}$ number of periods have been passed since the last PM activity on the operation stage $j$ in the period $t$,
$\varepsilon_{tj}$ probability of a conforming item acquires a defect during processing in the stage $j$ in the period $t$; $\varepsilon_{tj}=g(\omega_{tj})$,
$FI$ fixed inspection cost
$IC$ total inspection cost of the SMSPL,
$RC$ total replacement/repair cost of the SMSPL,
$PC$ total production cost of the SMSPL,
$MC$ total PM cost of the SMSPL,
$SC$ total scrap cost of the SMSPL,
*Profit* what is left over at the very end after the company has paid for the penalty cost of non-conforming goods sold.

*3.5 Mathematical Model*

Based on the above-mentioned definitions, the mixed-integer non-linear mathematical formulation of the problem is provided as follows.

Model 1

$$\text{Min } z = RC + SC + PC + IC + MC - profit \quad (1)$$

s.t.

$$RC = \sum_{t=1}^{T}\sum_{j=1}^{n} f_{j1}R_{j1}u_{tj1} + \sum_{t=1}^{T}\sum_{j=1}^{n} f_{j1}R_{j2}u_{tj2} \quad (2)$$

$$SC = \sum_{t=1}^{T}\sum_{j=1}^{n} f_{j2}S_j u_{tj1} + \sum_{t=1}^{T}\sum_{j=1}^{n} f_{j2}S_j u_{tj2} \quad (3)$$

$$PC = \sum_{t=1}^{T}\sum_{j=1}^{n} p_j\ (w_{tj1} + w_{tj2}) \quad (4)$$

$$IC = \sum_{t=1}^{T}\sum_{j=1}^{n} I_j\ (w_{tj1} + w_{tj2})\, d_{tj} + FI\ d_{tj} \quad (5)$$

$$MC = \sum_{t=1}^{T}\sum_{j=1}^{n} mc_{tj}\, md_{tj} \quad (6)$$

$$profit = \sum_{t=1}^{T} RE\ w_{t,n+1,1} - C\ w_{t,n+1,2} \quad (7)$$

$$w_{t,1,1} = \left(1-\varepsilon_{t0}\right) w_{t01} \qquad \forall t \quad (8)$$

$$w_{t,1,2} = \varepsilon_{t0}\, w_{t01} \qquad \forall t \quad (9)$$

$$\begin{aligned} w_{t,j+1,1} = {} & (1-\varepsilon_{tj})w_{tj1} + (f_{j1}(\alpha_j(1-\varepsilon_{tj}) + (1-\beta_j)\varepsilon_{tj}) \\ & - \alpha_j(1-\varepsilon_{tj}))w_{tj1}d_{tj} \\ & + f_{j1}(1-\beta_j)w_{tj2}d_{tj} \qquad \forall t, j \neq n \end{aligned} \quad (10)$$

$$\begin{aligned} w_{t,n+1,1} = {} & (1-\varepsilon_{tn})w_{tn1} + (f_{n1}(\alpha_n(1-\varepsilon_{tn}) + (1-\beta_n)\varepsilon_{tn}) \\ & - \alpha_n(1-\varepsilon_{tn}))w_{tn1}d_{tn} \\ & + f_{n1}(1-\beta_n)w_{tn2}d_{tn} \qquad \forall t \end{aligned} \quad (11)$$

$$\begin{aligned} w_{t,j+1,2} = {} & w_{tj2} + \varepsilon_{tj}w_{tj1} - (1-\beta_j)\varepsilon_{tj}w_{tj1}d_{tj} \\ & - (1-\beta_j)w_{tj2}d_{tj} \qquad \forall t, j \end{aligned} \quad (12)$$

$$\omega_{tj} = \left(\omega_{t-1,j} + 1\right)\left(1 - md_{tj}\right) \qquad \forall j, t \neq 1 \quad (13)$$

$$\omega_{1,j} = 0 \qquad \forall j \quad (14)$$

$$md_{1,j} = 1 \qquad \forall j \quad (15)$$

$$\varepsilon_{tj} = g\ (\omega_{tj}) \qquad \forall t, j \quad (16)$$

$$mc_{tj} = v\ (\omega_{tj}) \qquad \forall t \quad (17)$$

$$\begin{aligned} & w_{tj1}, w_{tj2}, mc_{tj}, u_{tj1}, u_{tj2}, \omega_{tj}, \varepsilon_{tj}, RC, IC, \\ & \quad PC, MC, SC, profit \geq 0 \qquad \forall t, j \end{aligned} \quad (18)$$

$$md_{tj}, d_{tj} : \text{binary} \qquad \forall t, j \quad (19)$$

Objective function (1) minimizes the total cost including repair, scrap, production, inspection, preventive maintenance, and negative form of profit. Equations (2) – (7) calculate these different cost components. Equations (8) – (12) calculate the expected number of conforming and non-conforming items entering the different production stages in each period. Equation (13) calculates the number of passed time periods after the last performed PM activity. It is assumed that a PM activity is done for all the production stages in the first period (please see Equations (14) and (15)). Equation (16) calculates the value of $\varepsilon_{tj}$ which is a function of $\omega_{tj}$ (see Fig. 1). Equation (17) also shows that the maintenance cost of the machine $j$ in the period $t$ is a function of $\omega_{tj}$. Equation (18) is a non-negativity limitation, and Equation (19) indicates that opening an inspection station and doing a PM activity are binary decisions.

## 4. METHODOLOGY

Model 1 is an MINLP model. Because solving a linear program is much easier than a non-linear one, especially when the problem is a large-sized one, firstly we attempt to linearize Model 1.

There are some terms consisting of the products of binary and non-negative real variables. The linearization procedure is performed by adding new variables and constraints to Model 1 as shown in Table 1.

By doing the aforementioned linearization process, Model 1 is still an MINLP model because it includes the nonlinear function of $\omega_{tj}$ (i.e., $\varepsilon_{tj} = g(\omega_{tj})$) and the product of the variable $\varepsilon_{tj}$ and the other non-negative real variables (i.e., $w_{tj1}$, $wd_{tj1}$ and $wd_{tj2}$). We can linearize the model by using the piecewise linear approximation. However, before the usage of the piecewise linear approximation, the following transformation for each multiplication $\varepsilon_{tj}$ and the other non-negative real variables (i.e., $w_{tj1}$, $wd_{tj1}$ and $wd_{tj2}$). For instance, for the multiplication $\varepsilon_{tj}$ and $w_{tj1}$:

1. introduce two new variables $u_1$ and $u_2$ into the model;
2. relate $u_1$ and $u_2$ to $\varepsilon_{tj}$ and $w_{tj1}$ by the relations

$$u_1 = \frac{1}{2}\left(\varepsilon_{tj} + w_{tj1}\right) \text{ and } u_2 = \frac{1}{2}\left(\varepsilon_{tj} - w_{tj1}\right)$$

3. replace the term $\varepsilon_{tj}\, w_{tj1}$ in the model by $u_1^2 - u_2^2$

This transformation should be repeated for the other multiplications (please see Table 1). The model now contains non-linear functions $u_1^2$ and $u_2^2$ of single variables and is therefore separable. These non-linear terms can be dealt with by piecewise linear approximations. It is important to remember that $u_2$ might need to take negative values. When the possible ranges of values for $u_1$ and $u_2$ are considered it may be necessary to either translate $u_2$ by an appropriate amount or treat it as a 'free' variable. A 'free' variable in linear programming is one which is not restricted to non-negative. Now, the piecewise linear approximations of the non-linear separate functions can be done as be explained following for the $\varepsilon_{tj}$.

For $\varepsilon_{tj}$ , which has non-linear behavior like Fig. 1, the piecewise linear approximation should be applied. A piecewise linear approximation to this function (see Fig. 1) is depicted in Fig. 2. The curve in Fig. 1 is divided into $n$ different straight line portions.

Our purpose is to eliminate the non-linear term $\varepsilon_{tj}=g(\omega_{tj})$ from Model 1. This can be done by replacing it by the single linear term $\vartheta_{tj}$. Now, it is possible to relate $\vartheta_{tj}$ to $\varepsilon_{tj}$ by the following relationships.

$$\omega_{tj} - \tau_1 \times \lambda 1_{tj} + \tau_2 \times \lambda 2_{tj} + \tau_3 \times \lambda 3_{tj} + \ldots + \tau_{n-1} \times \lambda n-1_{tj} + \tau_n \times \lambda n_{tj} = 0 \;\forall\, t, j$$

$$\vartheta_{tj} - \tau'_1 \times \lambda 1_{tj} + \tau'_2 \times \lambda 2_{tj} + \tau'_3 \times \lambda 3_{tj} + \ldots + \tau'_{n-1} \times \lambda n-1_{tj} + \tau'_n \times \lambda n_{tj} = 0 \;\forall\, t, j$$

$$\lambda 1_{tj} + \lambda 2_{tj} + \lambda 3_{tj} + \ldots + \lambda n-1_{tj} + \lambda n_{tj} = 1 \quad \forall\, t, j$$

**Table 1. Separate form of the multiplication of two variables**

| | |
|---|---|
| $\varepsilon_{tj}\, w_{tj1}$: | $u_1^2 - u_2^2$ |
| $\varepsilon_{tj}\, wd_{tj1}$: | $u_3^2 - u_4^2$ |
| $\varepsilon_{tj}\, wd_{tj2}$: | $u_5^2 - u_6^2$ |

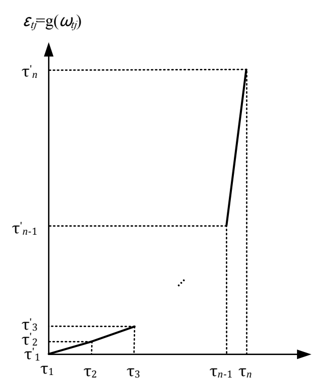


**Fig. 2.** Piecewise linear approximation of $\varepsilon$

**Table 1. Linearization process**

| | New variables | |
|---|---|---|
| | $wd_{tj1} = w_{tj1} \times d_{tj}$ | $wd_{tj2} = w_{tj2} \times d_{tj}$ |
| New constraints | $wd_{tj1} \le \mathrm{M} \times d_{tj}$ | $wd_{tj2} \le \mathrm{M} \times d_{tj}$ |
| | $wd_{tj1} \le w_{tj1}$ | $wd_{tj2} \le w_{tj2}$ |
| | $wd_{tj1} \ge w_{tj1} - (1 - d_{tj}) \times \mathrm{M}$ | $wd_{tj2} \ge w_{tj2} - (1 - d_{tj}) \times \mathrm{M}$ |
| | $wd_{tj1} \ge 0$ | $wd_{tj2} \ge 0$ |
| | New variables | |
| | $mcd_{tj} = mc_{tj} \times md_{tj}$ | $\omega md_{t-1,t,j} = \omega_{t-1,j} \times md_{tj}$ |
| New constraints | $mcd_{tj} \le \mathrm{M} \times md_{tj}$ | $\omega md_{t-1,t,j} \le \mathrm{M} \times md_{tj}$ |
| | $mcd_{tj} \le mc_{tj}$ | $\omega md_{t-1,t,j} \le \omega_{t-1,j}$ |
| | $mcd_{tj} \ge mc_{tj} - (1 - md_{tj}) \times \mathrm{M}$ | $\omega md_{t-1,t,j} \ge \omega_{t-1,j} - (1 - md_{tj}) \times \mathrm{M}$ |
| | $mcd_{tj} \ge 0$ | $\omega md_{t-1,t,j} \ge 0$ |

$\lambda i$ are new variables introduced into Model 1. They can be interpreted as 'weights' to be attached to the vertices of the curve in Fig. 2. However, it is necessary to add another stipulation regarding $\lambda i$.

At most two adjacent $\lambda i$ can be non-zero (21)

The stipulation (21) assures that corresponding values of $\vartheta_{tj}$ and $\varepsilon_{tj}$ lie on one of the straight line segments.

## 5. EXAMPLE

To show the presented model works well and provides reasonable and useful results, we solve a similar example to which was investigated by Rezaei-Malek et al. (2017). In addition, the results of the presented approach, which considers the non-linear deteriorating behavior (see Fig. 3), is compared to the developed model by Rezaei-Malek et al. (2018a), which considered the linear deteriorating stages (see Fig. 4). The data of that 3-stage system is used for all the 12 periods. The considered linear and non-linear deterioration processes of all the three stages are considered the same which are illustrated in Fig. 3. The maintenance cost is a function of $\omega_{tj}$ as we have $mc_{tj}$=50+10 $\omega_{tj}$. The number of unit material entering the SMSPL is 100 units per period and 5% of them are non-conforming, i.e. $\varepsilon_{t0}$ = 0.05. The revenue generated by delivering a conforming item to customer is 10 US$ and the penalty cost of a non-conforming item shipment is estimated 20 US$. The considered vertices for the piecewise linear approximation of the non-linear functions are shown in Table 3. The rest of required data is provided in Table 2. The model is coded in GAMS25.0.3 and solved with a zero absolute gap by the CPLEX 12 solver on the data of example, using a laptop with Core i5 CPU, 2.4 GHz and 8 GB of RAM.

Since the linear deterioration form leads to suboptimality, the most important achievement of this research is to approach to a real condition by considering non-linear deterioration process. Tables 4 and 5 shows the place and period that the PM and inspection activities are done considering linear and non-linear deterioration, respectively. As it can be concluded, in the nonlinear deterioration assumption, model decided to replace some of the Inspection activities by PMs and this resulted in a cost increase around 12%. However, the nonlinear model achieves 15% more conforming items.

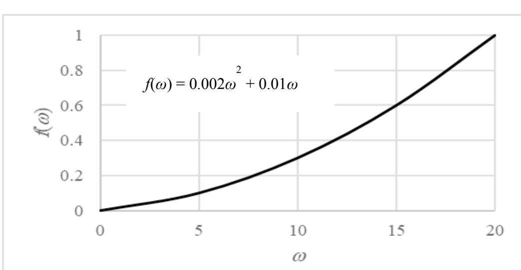


**Fig. 3.** Nonlinear deteriorating behaviour

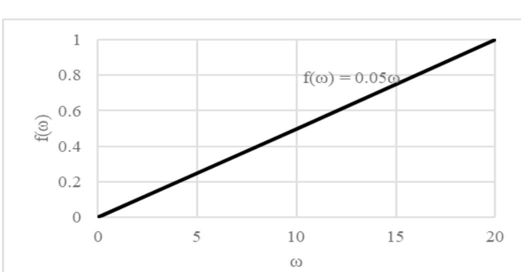


**Fig. 4.** linear deteriorating behavior

**Table 2. Production parameters of the 3-stage MS**

| Parameter | Stage 1 | Stage 2 | Stage 3 |
|---|---|---|---|
| $\alpha$ | 0.02 | 0.05 | 0.01 |
| $\beta$ | 0.01 | 0.1 | 0.01 |
| $I$ | 0.5 | 0.5 | 0.5 |
| $R_1$ | 5 | 0 | 0 |
| $R_2$ | 20 | 0 | 0 |
| $p$ | 10 | 20 | 5 |
| $S$ | 0 | -2 | 40 |
| $f_1$ | 0.2 | 0 | 0.5 |
| $f_2$ | 0.8 | 1 | 0.5 |

**Table 3. Vertices for the piecewise linear approximation**

| for | $f(\omega)$ | for | $u^2$ |
|---|---|---|---|
| $i$ | $(\tau_i,\tau'_i)$ | $i$ | $(\tau_i,\tau'_i)$ |
| 1 | (0,0) | 1 | (0,0) |
| 2 | (1,0.012) | 2 | (15,225) |
| 3 | (2,0.028) | 3 | (30,900) |
| 4 | (3,0.048) | 4 | (45,2025) |
| 5 | (4,0.072) | | |
| 6 | (5,0.1) | | |
| 7 | (6,0.132) | | |
| 8 | (7,0.168) | | |
| 9 | (8,0.208) | | |
| 10 | (9,0.252) | | |
| 11 | (10,0.3) | | |
| 12 | (11,0.352) | | |
| 13 | (12,0.408) | | |

**Table 4. The PM and Inspection activities (I) which are done in the SMSPL considering linear deteriorating**

| Period | Stage 1 | Stage 2 | Stage 3 |
|---|---|---|---|
| 1 | PM | PM | PM |
| 2 | I | PM | PM |
| 3 | I | PM | PM |
| 4 | I | PM | PM |
| 5 | I | PM | PM |
| 6 | I | PM | PM |
| 7 | I | PM | PM |
| 8 | I | PM | PM |
| 9 | I | PM | PM |
| 10 | I | PM | PM |
| 11 | I | PM | PM |
| 12 | I | PM | PM |

**Table 5. The PM and I activities which are done in the SMSPL considering non-linear deteriorating**

| Period | Stage 1 | Stage 2 | Stage 3 |
|---|---|---|---|
| 1 | PM | PM | PM |
| 2 | PM | I | PM |
| 3 | PM | I | PM |
| 4 | PM | I | PM |
| 5 | PM | PM | PM |
| 6 | PM | I | PM |
| 7 | PM | I | PM |
| 8 | PM | I | PM |
| 9 | PM | PM | PM |
| 10 | PM | I | PM |
| 11 | PM | I | PM |
| 12 | PM | I | PM |

## 6. CONCLUSIONS

This paper has developed a mixed-integer mathematical model for the integrated problem of preventive maintenance and inspection planning in a serial multi-stage manufacturing system. It has been assumed that each production stage is deteriorating nonlinearly in time and consequently the probability of a conforming item acquires a defect in each stage is increasing. The separable programming has been applied to linearize the model. The proposed model determines the optimum time and place for the preventive maintenance and inspection activities while minimizing the total manufacturing cost including repair, scrap, production, inspection, and preventive maintenance cost. A numerical example was solved for a serial 3-stage manufacturing system and the obtained results showed that in the nonlinear deterioration assumption, model decided to add and remove some of the PM and Inspection activities and this resulted in a cost increase around 12% in comparison with the linear deterioration assumption. However, 15% more conforming items was produced. In this paper, it has been considered that the life time of each manufacturing stage is certain and so the downtime is known. However, it is not a real assumption and the downtime shows a stochastic behavior. In this respect, it is strongly recommended to consider this stochastic behavior to the programming model as a future research direction.

## REFERENCES


Colledani, M., Tolio, T., Fischer, A., Iung, B., Lanza, G., Schmitt, R. and Vancza, J. (2014). Design and Management of Manufacturing Systems for Production Quality. *CIRP Annals - Manufacturing Technology*, 63(2), 773–796.

Galante, G. and Passannanti, G. (2007). Integrated approach to part scheduling and inspection policies for a job shop manufacturing system. *International Journal of Production Research*, 45(22), 5177–5198.

Gunter, I.S. and Swanson, A.L. (1985). Inspector location in convergent production units. *International Journal of Production Research*, 23(6), 1153–1169.

Kakade, V., Valenzuela, J.F. and Smith, J.S. (2004). An optimization model for selective inspection in serial manufacturing systems. *International Journal of Production Research*, 42(18), 3891–3909.

Kim, J. and Gershwin, B.S. (2008). Analysis of long flow lines with quality and operational failures. *IIE Transactions*, 40(3), 284–296.

Kogan, K., Raz, T., 2002. Optimal allocation of inspection effort over a finite planning horizon. *IIE Transactions*, 34(6), 515–527.

Mandroli, S.S., Shrivastava, A.K., and Ding, Y., 2006. A survey of inspection strategy and sensor distribution studies in discrete-part manufacturing processes. *IIE Transactions*, 38(4), 309–328.

Muchiri, P.N., Pintelon, L., Martin, H., and Chemweno, P., 2014. Modelling maintenance effects on manufacturing equipment performance: Results from simulation analysis. *International Journal of Production Research*, 52(11), 3287–3302.

Narahari, Y. and Khan, L.M. (1996). Modelling re-entrant manufacturing systems with inspection stations. *Journal of Manufacturing Systems*, 15(6), 367–378.

Park, H.Y., Park, H.E. and Ntuen, A.C. (1988). A study for optimal inspection policies in a flexible manufacturing cell. *Computer and Industrial Engineering*, 15(1-4), 307–314.

Penn, M. and Raviv, T. (2007) Optimizing the quality control station configuration. *Naval Research Logistics (NRL)*, 54(3), 301–314.

Penn, M. and Raviv, T. (2008) A polynomial time algorithm for solving a quality control station configuration problem. *Discrete Applied Mathematics*, 156(4), 412–419.

Raghavachari, M. and Tayi, G. (1991). Inspection configuration and reprocessing decisions in serial production systems. *International Journal of Production Research*, 29(5), 897–911.

Raz, T., Herer, Y.T. and Grosfeld-Nir, A. (2000). Economic optimization of off-line inspection. *IIE Transactions*, 32(3), 205–217.

Rezaei-Malek, M., Tavakkoli-Moghaddam, R., Siadat, A. and Dantan, J.-Y. (2017). Integrated preventive maintenance and inspection planning in a deteriorating serial multi-stage manufacturing system. *The proceeding of the 7th IESM Conference*, 352–357, October 10-13, Saarbrücken, Germany.

Rezaei-Malek, M., R. Tavakkoli-Moghaddam, A. Siadat and J.-Y. Dantan. (2018a). A novel model for the integrated planning of part quality inspection and preventive maintenance in a linear-deteriorating serial multi-stage manufacturing system. *The International Journal of Advanced Manufacturing Technology*. DOI: 10.1007/s00170-018-1751-1

Rezaei-Malek, M., Mohammadi, M., Dantan, J.-Y., Siadat, A. and Tavakkoli-Moghaddam, R. (2018b) A review on optimization of part quality inspection planning in a multi-stage manufacturing system. *International Journal of production Research* (to be apeared)

Sadegheih, A. (2007). Sequence optimization and design of allocation using GA and SA. *Applied Mathematics and Computation*, 186(2), 1723–1730.

Shiau, Y.R., Lin, M.H. and Chuang, W.C. (2007). Concurrent process/inspection planning for a customized manufacturing system based on genetic algorithm. *International Journal of Advanced Manufacturing Technology*, 33(7), 746–755.

Tayi, G.K., Ballou, D.P., 1988. An integrated production inventory model with reprocessing and inspection. *International Journal of production Research*, 26(8), 1299–1315.

Yum, B.J., and McDowell, E.D., 1987. Optimal inspection policies in a serial production system including scrap rework and repair: An MILP approach. *International Journal of Production Research*, 25(10), 1451–64.