A Novel Method for Accelerating the Analysis of Nonlinear Behaviour of Power Grids using Normal Form Technique

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Abstract— **Today's power systems are strongly nonlinear and are becoming more complex with the large penetration of power-electronics interfaced generators. Conventional Linear Modal Analysis does not adequately study such a system with complex nonlinear behavior. Inclusion of higher-order terms in small-signal (modal) analysis associated with the Normal Form theory proposes a nonlinear modal analysis as an efficient way to improve the analysis. However, heavy computations involved make Normal Form method tedious, and impracticable for large power system application. In this paper, we present an efficient and speedy approach for obtaining the required nonlinear coefficients of the nonlinear equations modelling of a power system, without actually going through all the usual high order differentiation involved in Taylor series expansion. The method uses eigenvectors to excite the system modes independently which lead to formulation of linear equations whose solution gives the needed coefficients. The proposed method is demonstrated on the conventional IEEE 9-bus system and 68-bus New England/New York system.**

Index Terms–**Fast computation, Higher Order Analysis, Modal analysis, Nonlinear, Normal Form Technique, Normal Form coefficients, Small Signal Analysis.**

I. INTRODUCTION

Power system is composed of several generators working in parallel to supply common load. It is nonlinear and usually modelled by a set of nonlinear differential algebraic equations. An inherent problem associated with interconnected power systems is the presence of oscillations that could have dangerous effects on the system. The multiplication of distributed generation units usually composed of renewable-energy-based generators coupled with the increase of energy exchanges through long distance lead to highly stressed power systems and consequent poor damping and inter-area oscillations [1], [2]. As a result of the nonlinearity increase in the system, the conventional small signal (modal) analysis (SSA) can be deceptive since its validity becomes very small [3].

The addition of higher-order terms in SSA has led to Nonlinear Modal Analysis tools which proved to be more efficient than their linear counterpart. The two currently used techniques for nonlinear modal analysis are method of Normal Form and Modal Series method [4]. One major limitation common to these methods is the heavy computation required. Due to the computational burden, most works reported are based on inclusion of second order term of Taylor series expansion. Though, few works have been recently reported on inclusion of third order terms [4]–[8]. One of the difficulties in computation with these nonlinear analysis tools is the need to compute numerous coefficients even with system of small size. These coefficients increase rapidly with increase in the system size. In order to reduce the computational burden, it was suggested in [9], [10] that accurate determination of the interacting modes by higher order spectra (HOS) and prony analyses will significantly reduce the computational burden associated with the method of Normal Form since several of the computations can be restricted to the interacting modes. This suggestion calls for a convenient method for selectively computing these coefficients. Normal Form computations are generally uninteresting irrespective of the size because many results of the computation are not very useful in the end. In the study of modal interaction for instance, even though all these coefficients are computed, only few of them reveal the interacting modes at the end. The traditional way to get these coefficients in power system is to perform higher order derivatives on the system equations in order to build the higher order Hessian matrices either by **symbolic tools** or manually defining the derivatives for the Hessian matrices in advance. Symbolic computations are generally slow. The challenge of defining the Hessian derivatives in advance is that when a model is modified the whole process is overhauled to account for the new model. Moreover, modal interaction depends on the operating condition making it difficult to pre-define the derivatives for the needed coefficients (assuming only some coefficients are needed) since the interaction can change with the change in operating condition. The authors in [3], [11] acknowledged the challenges in getting these coefficients.

This paper proposes an easy way to compute the nonlinear coefficients of a power system nonlinear model in modal coordinate without all the needed Taylor series expansions and associated Hessian matrices. The rest of the paper is organized as follows. Section II is a review of the Normal Form technique. Section III details the proposed method used in this paper to compute the needed coefficients. The proposed method is demonstrated on the IEEE 9-bus and IEEE 68-bus systems in section IV. Finally, a conclusion section shows the interesting perspectives proposed by the method.

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II. REVIEW OF NORMAL FORM TECHNIQUE

Normal Form technique transforms a set of highly coupled nonlinear differential equation up to a desirable order into a set of simplified differential equations. This is achieved by sequential nonlinear coordinate transformations which simplify the equations. The system is then said to be in their simple form (Normal Form) [12]. As in SSA, a nonlinear function is Taylor-expanded around a stable operating point (SEP); however the expansion is performed beyond first-order. Consider a power system modelled as

$$\mathbf{M}\dot{\mathbf{X}} + \mathbf{f_s}(\mathbf{X}) = \mathbf{P_m} \tag{1}$$

Where $\mathbf{M}, \mathbf{X}$ and $\mathbf{P_m}$ are matrix of inertia, vector of states, and vector of mechanical powers respectively. Though, $\mathbf{M}$ and $\mathbf{Pm}$ do not apply to all the equations if the system is in first order differential equations, the idea is to put element of $\mathbf{M} = 1$ and element of $\mathbf{P_m} = 0$ where they do not apply. This representation is chosen to facilitate the understanding of the proposed method. Equation (1) is the same as

$$\dot{\mathbf{X}} + \mathbf{f}(\mathbf{X}) = \mathbf{P_T}. \tag{2}$$

where $\mathbf{f}(\mathbf{X}) = \mathbf{M^{-1}f_s}(\mathbf{X}), \mathbf{P_T} = \mathbf{M^{-1}P_m}$.
Assuming a perturbation $\mathbf{X} = \mathbf{X_0} + \mathbf{x}$, the system in third order Taylor expansion can be written in Einstein notation as

$$\dot{x_i} + A_{ij}x_j + F2_{ijk}x_jx_k + F3_{ijkl}x_jx_kx_l = P_i. \tag{3}$$

where $A_{ij} = \frac{\partial f_i}{\partial X_j}|_{\mathbf{X}=\mathbf{X_0}}, F2_{ijk} = \frac{1}{2}\frac{\partial^2 f_i}{\partial X_j \partial X_k}|_{\mathbf{X}=\mathbf{X_0}}, P_i = \Delta P_{T_i}$
$F3_{ijkl} = \frac{1}{6}\frac{\partial^3 f_i}{\partial X_j \partial X_k \partial X_l}|_{\mathbf{X}=\mathbf{X_0}}(i,j,k,l = 1,2,\ldots N)$.
$\mathbf{A}$, $\mathbf{F2}$, and $\mathbf{F3}$ are respectively the Jacobian of size $(N \times N)$, second order Hessian of size $(N \times N \times N)$ and third order Hessian of size $(N \times N \times N \times N)$ corresponding to first, second and third order terms evaluated at the initial operating point vector $\mathbf{X_0}$. Equation (3) can be expressed compactly as

$$\dot{\mathbf{x}} + \mathbf{Ax} + \beta(\mathbf{x}) = \mathbf{P}. \tag{4}$$

Where $\beta(\mathbf{x})$ is a collection of all the nonlinear parts.

Assuming $\mathbf{\Lambda}$, $\mathbf{U}$ and $\mathbf{V}$ are respectively matrices of eigenvalue, right and left eigenvectors of $\mathbf{A}$, using a linear transformation of (5) in (4) and pre-multiplying the resulting equation by the left vector yields:

$$\mathbf{x} = \mathbf{Uy} \tag{5}$$

$$\dot{\mathbf{y}} + \mathbf{V^T AUy} + \mathbf{f_{NL}}(y_1, y_2, \ldots, y_N) = \mathbf{V^T P}. \tag{6}$$

Or

$$\dot{y}_p + \lambda_p y_p + \sum_{k,l=1}^{N} C^p_{qr} y_q y_r + \sum_{q,r,s=1}^{N} D^p_{qrs} y_q y_r y_s = v_{pi} P_i \tag{7}$$

Where $\mathbf{f_{NL}} = \mathbf{V^T}\beta$, $D^p_{qrs} = F3_{ijkl}v_{ip}u_{jq}u_{kr}u_{ls}$, $C^p_{qr} = F2_{ijk}v_{ip}u_{jq}u_{kr}$, λ_p is the p^{th} element of $\mathbf{\Lambda}$, C^p_{qr} and D^p_{qrs} are the 2nd and 3rd order nonlinear coefficients in modal coordinate. In power system modal analysis, if the governor of the machine is not modelled, the mechanical power is assumed to be constant for small disturbance and therefore, the right hand side of (6) is usually assumed zero but as will be seen later, the proposed method assumes this quantity is not zero even without governor model.

From (7), it is observed that the linear part is decoupled and simplified but the second and third order terms are yet coupled. Also the system is now in a new coordinate (modal coordinate). Normal Form theory requires simplification of the system by introducing nonlinear transformation given as

$$y_p = z_p + \sum_{q=1}^{N}\sum_{r=1}^{N} h2^p_{qr} z_q z_r + \sum_{q=1}^{N}\sum_{r=1}^{N}\sum_{s=1}^{N} h3^p_{qrs} z_q z_r z_s. \tag{8}$$

The vector $\mathbf{z}$ is the state vector in the new coordinate, **h2** and **h3** are respectively complex valued quadratic and cubic polynomials in $\mathbf{z}$. If $h2^p_{qr}$ and $h3^p_{qrs}$ coefficients are determined and certain conditions apply, the second and third order terms can be annihilated. Then the system maybe simplified but in another coordinate (Normal Form coordinate) as in (9).

$$\dot{z_p} = \lambda_p z_p + g_p(z) \tag{9}$$

The term $g_p(z)$ are called *resonant* terms. Resonance is defined in following lines. The quadratic and cubic Normal Form coefficients in (8) are given by [12]

$$h2^p_{qr} = \frac{C^p_{qr}}{\lambda_q + \lambda_r - \lambda_p}, h3^p_{qrs} = \frac{D^p_{qrs}}{\lambda_q + \lambda_r + \lambda_s - \lambda_p}. \tag{10}$$

From (10) it can be seen that if the denominators are very close to zero assuming the magnitude of the numerators are not also very near to zero , the values of the coefficients will be high. Resonance occurs where eigenvalue combinations lead to zero denominator or very small divisor (near resonance). Under such condition, the related terms cannot be removed by Normal Form transformation. Therefore, large values of Normal Form coefficients indicate how close a set of modes are and hence potential strong interactions among the modes. This information is very important in achieving a proper control design and siting of power system stabilizers PSS [9]. The system can be reversed to its original coordinate by an inverse Normal Form transformation.

The whole work in applying Normal Form transformation to study power system revolves around finding the so called Normal Form coefficients $h2^p_{qr}$ and $h3^p_{qrs}$. However, manipulations to get coefficients in (7) are very cumbersome to follow due to multivariate higher order differentiation involved. For every model, one has to either work out the expressions for all the needed derivatives in advance or use symbolic tools or resort to multivariate numerical differentiation where possible. Multivariate numerical differentiation accumulates substantial error as the orders of differentiation increases.

For linear mode basis of size N (i.e N differential equations), the number of coefficients is given by

$$N_c = \frac{N^4}{6} + \frac{5N^2}{6} + N^3. \tag{11}$$

The above equation clearly shows that the computational burden will increase in the power of four.

III. THE PROPOSED METHOD

In this paper we propose a method that utilizes only the information from the state matrix to obtain simultaneously the coefficients of the higher order terms in (7). Similar idea was introduced in [13] and used in [14] for studying geometric nonlinearities of second order mechanical systems with the help of finite element method. To the author's best knowledge, this is the first time the method is being formulated in order to study power system in first order. In the following paragraphs, step by step derivation of the proposed method is presented.

Step 1 - *Power flow and linear analysis:* The equations for the nonlinear model are written, power flow performed and linear analysis done to obtain the state matrix (**A**), the right eigenvector (**U**) and the left eigenvector (**V**).

Step 2 - *Change of variables and transformation to modal coordinate (5):* The nonlinear equation is written as a combination of linear and nonlinear part (see equation (4)) and then transformed to Jordan form (equation (6)).

Step 3 - *Prescription of modal deviations in the original system (2):* From (6) we know the expression for $\mathbf{f_{NL}}$ for say r^{th} equation to be

$$f^r_{NL}(y_1,y_2,...y_N)=\sum_{j=1}^{N}\sum_{k=1}^{N}C^r_{jk}y_jy_k+\sum_{j=1}^{N}\sum_{k=1}^{N}\sum_{l=1}^{N}D^r_{jkl}y_jy_ky_l. \tag{12}$$

Where $r=(1,2,...N)$.
But avoiding the Hessian evaluations, we can find f^r_{NL} by prescribing small deviation in modal space to the original nonlinear system under static considerations (i.e all derivative = 0) and estimating the forcing term $\mathbf{P_T}$. The assumption made for this method is that the mechanical input power is no longer constant. But this assumption is only to compute the coefficients. That is to say the system is disturbed by varying the state variables arbitrarily and the forcing vector that would have caused such variation is estimated. For instance, using (2) let us prescribe a very small deviation $\mathbf{x}$; then

$$\mathbf{f}(\mathbf{X_0}+\mathbf{x})=\mathbf{P_T}\approx\mathbf{f}(\mathbf{X_0})+\mathbf{A}\mathbf{x}+\beta(\mathbf{x})=\mathbf{P_0}+\mathbf{P_L}+\mathbf{P}_{NL}. \tag{13}$$

$\mathbf{P_T}$ is the total forcing vector estimated which can cause such deviation $(\mathbf{X_0}+\mathbf{x})$, $\mathbf{P_0}$ is the original value at equilibrium, $\mathbf{P_L}$ is linear contribution of the forcing vector while $\mathbf{P_{NL}}$ is the nonlinear contribution.

To ensure that the deviation is in the modal space, the value of this deviation is assigned using the linear eigenvector. We choose arbitrary small amplitude of deviation say α and multiply with right eigenvector since any multiple of eigenvector is still an eigenvector. Hence $\mathbf{x}=\mathbf{U}\alpha$; where α is the amplitude of the deviation (*note that α means arbitrary value of y*).

Step 4 - *Evaluation of the nonlinear contribution:* From (13) we can write

$$\mathbf{P_{NL}}=\mathbf{P_T}-\mathbf{P_L}-\mathbf{P_0}. \tag{14}$$

Step 5 - *Formulation of linear equations:* Passing $\mathbf{x}=\mathbf{U}_i\alpha_i$ to linear and nonlinear static models will have effect only on the node ii since only mode i will be excited, $\mathbf{x}=\mathbf{U}_i\alpha_i+\mathbf{U}_j\alpha_j$ will have effect on nodes ii, jj and ij since both mode i and j will be excited. By prescribing series of $\mathbf{x}$ each time getting the linear and nonlinear static contributions, a set of linear equations is formulated and solved simultaneously to get the coefficients.
For example, if C^r_{11} and $D^r_{111}(r=1,2,N)$ are needed, we can prescribe $\mathbf{x}=+\mathbf{U}_1\alpha_1$ and $\mathbf{x}=-\mathbf{U}_1\alpha_1$. The essence of the negative part is to create two equations in order to solve simultaneously. By prescribing as above and solving for linear and nonlinear static solutions, $\mathbf{P_{NL}}$ is obtained from (14). Then from (6) and (12) we can write set of linear equations as

$$\begin{cases}\mathbf{V_r^T}\mathbf{P}^+_{NL}=\mathbf{V_r^T}\beta(+\mathbf{U_1}\alpha_1)=C^r_{11}\alpha_1\alpha_1+D^r_{111}\alpha_1\alpha_1\alpha_1\\ \mathbf{V_r^T}\mathbf{P}^-_{NL}=\mathbf{V_r^T}\beta(-\mathbf{U_1}\alpha_1)=C^r_{11}\alpha_1\alpha_1-D^r_{111}\alpha_1\alpha_1\alpha_1\end{cases}. \tag{15}$$

Or generally as $\mathbf{A_c}\mathbf{X_c}=\mathbf{B_c}$.
Where
$\mathbf{A_c}=\begin{bmatrix}\alpha_1^2 & \alpha_1^3\\ \alpha_1^2 & -\alpha_1^3\end{bmatrix},\mathbf{X_c}=\begin{bmatrix}C^r_{11}\\ D^r_{111}\end{bmatrix},\mathbf{B_c}=\begin{bmatrix}\mathbf{V_r^T}\mathbf{P}^+_{NL}\\ \mathbf{V_r^T}\mathbf{P}^-_{NL}\end{bmatrix}.$
In this way all $C_{11}\&D_{111}$ coefficients are computed. The approach is same for all C_{22} and D_{222} only that $\mathbf{x}$ becomes $\mathbf{x}=\mathbf{U_2}\alpha_2$. All C_{ii} and D_{iii} can be calculated this way. For coupled terms such as $C^r_{ij},D^r_{iij}\&D^r_{jji}$, $\mathbf{x}$ may be

$$\mathbf{x}=\begin{cases}\pm\mathbf{U_i}\alpha_i\\ \pm\mathbf{U_j}\alpha_j\\ \mathbf{U_i}\alpha_i+\mathbf{U_j}\alpha_j\\ \mathbf{U_i}\alpha_i-\mathbf{U_j}\alpha_j\\ -\mathbf{U_i}\alpha_i+\mathbf{U_j}\alpha_j\end{cases}.$$

Following the same approach, we get similar expression $\mathbf{A_c}\mathbf{X_c}=\mathbf{B_c}$, with

$$\mathbf{A_c}=\begin{bmatrix}\alpha_i^2 & \alpha_i^3 & 0 & 0 & 0 & 0 & 0\\ \alpha_i^2 & -\alpha_i^3 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \alpha_j^2 & \alpha_j^3 & 0 & 0 & 0\\ 0 & 0 & \alpha_j^2 & -\alpha_j^3 & 0 & 0 & 0\\ \alpha_i^2 & \alpha_i^3 & \alpha_j^2 & \alpha_j^3 & \alpha_i\alpha_j & \alpha_i^2\alpha_j & \alpha_i\alpha_j^2\\ \alpha_i^2 & \alpha_i^3 & \alpha_j^2 & -\alpha_j^3 & -\alpha_i\alpha_j & -\alpha_i^2\alpha_j & \alpha_i\alpha_j^2\\ \alpha_i^2 & -\alpha_i^3 & \alpha_j^2 & \alpha_j^3 & -\alpha_i\alpha_j & \alpha_i^2\alpha_j & -\alpha_i\alpha_j^2\end{bmatrix}$$

$$\mathbf{X_c}=\begin{bmatrix}C^r_{ii} & D^r_{iii} & C^r_{jj} & D^r_{jjj} & C^r_{ij} & D^r_{iij} & D^r_{jji}\end{bmatrix}^T$$

$$\mathbf{B_c}=\begin{bmatrix}\mathbf{V_r^T}\mathbf{P}^+_{NL_i}\mathbf{V_r^T}\mathbf{P}^-_{NL_i}\mathbf{V_r^T}\mathbf{P}^+_{NL_j}\mathbf{V_r^T}\mathbf{P}^-_{NL_j}\mathbf{V_r^T}\mathbf{P}^{++}_{NL_{ij}}\mathbf{V_r^T}\mathbf{P}^{+-}_{NL_{ij}}\mathbf{V_r^T}\mathbf{P}^{-+}_{NL_{ij}}\end{bmatrix}^T$$

For coupled terms such as $D^r_{ijk},k\neq i\neq j$, $\mathbf{x}$ can be prescribed as: $\mathbf{x}=\mathbf{U_i}\alpha_i+\mathbf{U_j}\alpha_j+\mathbf{U_k}\alpha_k$ and then the size of $\mathbf{A}_c$ increases accordingly.

With the nonlinear coefficients determined, (10) is then used to determine the Normal Form coefficients $h2^p_{qr}$ and $h3^p_{qrs}$. With the Normal Form coefficients and Normal Form initial conditions known, the indices defined in [9] can reveal many information about the system behaviour under stress.

IV. APPLICATION TO IEEE TEST SYSTEMS

The proposed method is demonstrated on a 3-machine 9-bus power system [15] in Fig. 1 and 16-machine 68-bus power system [16] in Fig. 2. The system of Fig.1 is represented in two-axis model with 4 sate variables per

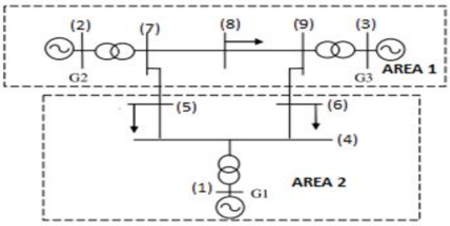

Fig. 1. IEEE 9-Bus Power System

machine 16-19.

$$\dot{\delta}_i = \omega_s(\omega_i - 1) \tag{16}$$

$$T'_{d0}\dot{E'_{q_i}} = -E'_{q_i} - (x_{d_i} - x'_{d_i})I_{d_i} + E_{fd_i} \tag{17}$$

$$T'_{q0}\dot{E'_{d_i}} = -E'_{d_i} + (x_{q_i} - x'_{q_i})I_{q_i} \tag{18}$$

$$M_i\dot{\omega}_i = P_{m_i} - (E'_{q_i}Iq_i + E'_{d_i}Id_i) - D_i(\omega_i - 1) \tag{19}$$

Where ω_{s_i} is the rated speed of the synchronous generato, ω_i is the rotor speed deviation in p.u, D_i is damping coefficient, M_i is the inertial constant, δ_i is the rotor angle.

If one of the generators is used as a reference, the number of differential equations will be 11. From equation 11, it implies we expect at least 3,872 coefficients. In this paper we show randomly some of the coefficients in order to compare with the usual direct Hessian approach.

The quadratic and cubic coefficients of r^{th} equation of system 16-19 using the proposed method are presented in Table II and Table IV. The results from the conventional direct Hessian approach are also presented (Table I and Table III) for comparison. As could be seen from the tables, the two results highly agree.

TABLE I

QUADRATIC COEFF.HESSIAN METHOD

r^{th}	C_{11}	C_{23}	C_{45}
1	0.3249 + 0.2378i	0.5124 + 0.3663i	0.6631 - 3.0947i
4	-0.1323 - 0.0191i	0.0641 + 0.1335i	0.3234 - 0.8295i
7	-0.1193 + 0.0151i	0.0264 - 0.0059i	-0.0597 + 0.0051i
8	0.1065 +0.0439i	-0.0052 - 0.0020i	-0.1101 - 0.0258i
11	0.0140 +0.0099i	-0.0469 +0.0075i	-0.2191 - 0.0653i

TABLE II

QUADRATIC COEFF.PROPOSED METHOD

r^{th}	C_{11}	C_{23}	C_{45}
1	0.3249 + 0.2376i	0.5124 + 0.3661i	0.6631 - 3.0948i
4	-0.1323 - 0.0189i	0.0641 + 0.1337i	0.3234 - 0.8293i
7	-0.1191 - 0.0151i	0.0264 - 0.0059i	-0.0597 + 0.0051i
8	0.1065 + 0.0439i	-0.0051 - 0.0020i	-0.1100 - 0.0258i
11	0.1088 + 0.0439i	-0.0468 + 0.0075i	-0.2191 - 0.0653i

The maximum error taking into account all the coefficients were computed as $\xi_2 = max|C2^j_{kl_{Hessian}} - C2^j_{kl_{Proposed}}|$ and $\xi_3 = max|D3^j_{pqr_{Hessian}} - D3^j_{pqr_{Proposed}}|$ for quadratic and cubic coefficients respectively. These errors were found to be 0.024% and 1.8% for quadratic and cubic coefficients respectively.

TABLE III

CUBIC COEFF. HESSIAN METHOD

r^{th}	D_{222}	$D_{4,8,8}$	$D_{9,10,11}$
1	-0.3191 - 0.3763i	-0.0015 - 0.0188i	-0.2008 - 1.0589i
4	0.0034 + 0.0091i	0.0077 - 0.0214i	0.0210 - 0.2150i
7	-0.0046 - 0.0104i	0.0104 + 0.0015i	0.0633 + 0.0000i
8	-0.0162 - 0.0334i	0.0189 + 0.0004i	0.1378 + 0.0000i
11	-0.0022 - 0.0067i	0.0013 - 0.0000i	0.0071 + 0.0000i

TABLE IV

CUBIC COEFF. PROPOSED METHOD

r^{th}	D_{222}	$D_{4,8,8}$	$D_{9,10,11}$
1	-0.3191 - 0.3763i	-0.0015 - 0.0188i	-0.2010 - 1.0584i
4	0.0034 + 0.0091i	0.0077 - 0.0213i	0.0209 - 0.2150i
7	-0.0046 - 0.0104i	0.0104 + 0.0015i	0.0632 + 0.0000i
8	-0.0162 - 0.0334i	0.0189 + 0.0004i	0.1378 + 0.0000i
11	-0.0022 - 0.0067i	0.0013 - 0.0000i	0.0071 + 0.0000i

The time cost was investigated on an Inter CoreTM i7-3520M 2.9GHz desktop computer, where the nonlinear coefficients computation using the Symbolic Math Toolbox in MATLAB takes about 40.7 seconds while the proposed method takes about 10.5seconds.

As earlier stated, the proposed method was also applied to 16-machine 68-bus New England/New York power system shown in Fig.2. Classical representation of generator was used and one generator used as a reference, making the total number of differential equations 31. It follows from equation (11) that there are 184,512 different coefficients to be computed. The direct Hessian approach was performed with the help of Symbolic MATLAB Toolbox in the same computer as before and the total time for all the coefficients was 21,924.095 seconds ($\approx$ 6hrs). This same coefficients were all computed with proposed method in about 3,540 seconds ($\approx$1hr) with same computer. No time optimization was considered in coding the algorithm. All coefficients were selectively computed each time as if there had not been any previous computations. Hence, several computations were repeated. Much time will be saved if it is programmed to use the previously computed coefficients in the linear equations. We will optimize the time of computation in future as this work progresses.

As shown in Table V and Table VI, the results are highly in agreement. We have stated that amplitude of the modal deviation is chosen arbitrarily, however, it should neither be too small nor too big. Very small value of α does not

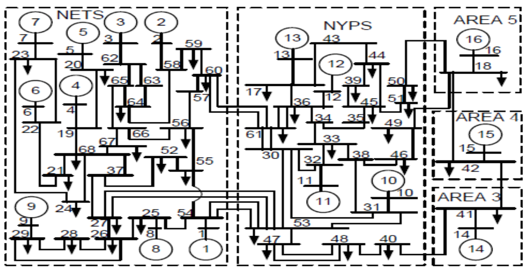

Fig. 2. IEEE 68-Bus Power System

TABLE V

QUADRATIC & CUBIC COEFF.HESSIAN METHOD

r^{th}	$C_{14,14}$	$C_{1,20}$	$D_{2,10,16}$	$D_{30,30,30}$
1	- 0.0260i	- 0.0154i	0.0073i	0.0655i
5	- 0.0085i	0.0006i	0.0013i	- 0.0015i
7	- 0.0060i	0.0025i	0.0004i	0.0007i
20	- 0.0024i	- 0.0520i	0.0015i	- 0.0019i

TABLE VI

QUADRATIC & CUBIC COEFF.PROPOSED METHOD

r^{th}	$C_{14,14}$	$C_{1,20}$	$D_{2,10,16}$	$D_{30,30,30}$
1	- 0.0261i	- 0.0153i	0.0075i	0.0655i
5	- 0.0090i	0.00061i	0.0014i	- 0.0016i
7	- 0.0061i	0.0025i	0.0004i	0.0007i
20	- 0.0025i	- 0.0510i	0.0016i	- 0.0020i

trigger the nonlinearity very well; hence, the system is more or less linear. On the other hand too large value of α leads to higher nonlinearity, hence the domain of validity of 3rd order approximation is exceeded. A value in the range of $0.001 \leq \alpha < 0.9$ gives good result.

Fig. 3 shows the evolution of some coefficients with variation of α. As pointed out earlier, the coefficients are

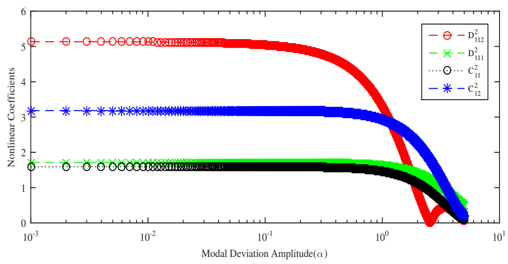

Fig. 3. Sensitivity of Modal deviation amplitude (α)

consistent for α within certain range as seen in Fig. 3.

The benefits of the proposed method are highlighted as follows:

1) Both 2^{nd} and 3^{rd} order nonlinear coefficients are computed simultaneously.
2) With good knowledge of the coefficients of interest, they can be selectively computed rapidly.
3) It requires only the state matrix used for the linear analysis. No further differentiation needed.
4) It is easy to implement and can easily adapt to model variations.
5) It maybe convenient to integrate in a commercial software.

V. CONCLUSION

In this paper a method for selectively determining the nonlinear coefficients of power system represented with third order Taylor expansion has been proposed. The method is easy requiring only the state matrix of the system and direct substitution of arbitrary values in the static equations of the system. The proposed method has been demonstrated on conventional IEEE 9-bus and IEEE 68-bus systems but can be extended to other types of grid. The method is convenient and fast with results in agreement with the conventional direct Hessian approach.

Since many commercial time-domain simulation software can give the state matrix and the static computations correspond to system's steady state after the small modal deviation introduced, we will investigate in future work the possibility of obtaining the static computation directly from commercial time domain simulation software. Future focus is to investigate a criterion for determining a priori the most relevant coefficients and then use the proposed method to compute just these coefficients in order to apply Normal Form to some selected modes in a large system.

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