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Multiscale Modeling Approaches for Composites

George Chatzigeorgiou, Fodil Meraghni, Nicolas Charalambakis

Multiscale Modeling Approaches for Composites outlines the fundamentals of common multiscale modeling techniques and provides detailed guidance for putting them into practice. Various homogenization methods are presented in a simple, didactic manner, with an array of numerical examples.

The book starts by covering the theoretical underpinnings of tensors and continuum mechanics concepts, then progresses to actual micromechanics techniques for composite media and laminate plates. In the last chapters the book covers advanced topics in homogenization, including Green's tensor, Hashin-Shtrikman bounds, and special types of problems. All chapters feature comprehensive analytical and numerical examples (Python and ABAQUS scripts) to better illustrate the theory.

Key features:

- Bridges theory and practice, providing step-by-step instructions for implementing multiscale modeling approaches for composites and the theoretical concepts behind them
- Covers boundary conditions, data-exchange between scales, the Hill-Mandel principle, average stress and strain theorems, and more
- Discusses how to obtain composite properties using various full-field and mean-field approaches
- Includes access to a companion site featuring the numerical examples, Python, and ABAQUS codes

About the Authors

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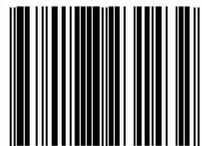
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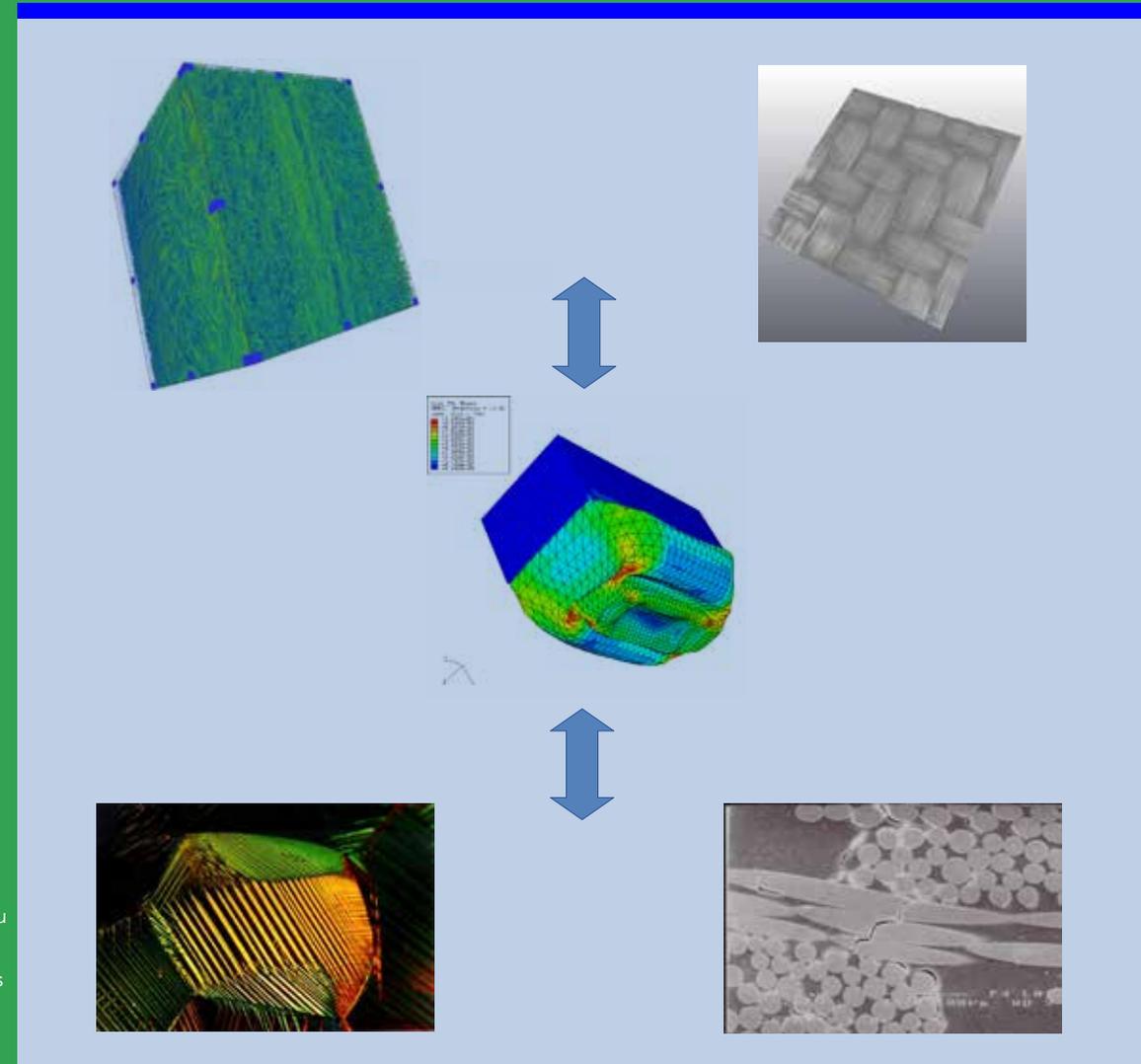
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