# Tolerance allocation under behavioural simulation uncertainty of a multiphysical system

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## ABSTRACT

The tolerancing process impacts the product quality, the production cost and scrap rate. Tight tolerances allow to assure product performance; loose tolerances to reduce production cost. The tolerance allocation of a complex system is performed under uncertainty. In fact, the accuracy of the behaviour simulation of the system significantly affects the tolerance analysis result, and thus the tolerance allocation result. Therefore, a method is proposed to perform tolerance allocation based on the Dempster Shafer theory, Monte-Carlo simulation and genetic algorithm. The application of the proposed framework is demonstrated through a complex case study.

## 1. Introduction

Each product design activity is performed under uncertainty [1] which incurs risk. This risk can impact the product performance(s), process scheduling, etc., including a significant cost effect of corresponding safety factors and mitigation actions. Particularly the definition of uncertainty has been covered extensively in literature, mostly distinguishing the two types: aleatory uncertainty (natural variability e.g. in manufacturing processes) and epistemic uncertainty resulting from a lack of knowledge (due to incomplete solutions, inaccurate models, etc.) [1]. As it is obvious that both types affect the design process in general, and all tolerancing tasks in particular, a clear strategy to address the corresponding differences is required.

Accordingly, and in light of the ever-increasing demand for high quality products and components, the topic of uncertainty has also impacted the development of tolerancing approaches. In fact, tolerancing has become the key concept for bridging the gap between design and manufacturing. It is not only essential for ensuring assemblability, suitable capabilities of the required manufacturing processes [2] and minimised costs, but also for achieving a high and consistent behaviour of multi-physical products [3].

Against this background, tolerance allocation has to take many, often uncertain, factors into consideration. One of the main tasks is hereby the assurance of "functional requirements", i.e. the verification of functional requirements after tolerances have been specified on each component. For the corresponding propagation of manufacturing imprecisions and the check whether functional requirements are achieved despite variation, a realistic and reliable representation of a product's behaviour is required. However, a behavioural simulation with a minimum of uncertainty is still a significant challenge for many engineering design tasks [3]. Following the above, this simulation is affected by model uncertainties or behavioural uncertainties, which impact the accuracy of the performance assessment. Therefore, it is necessary to propagate all of these uncertainties and to quantify their impact on the results of the tolerance allocation process.

This paper focuses on this challenge by proposing a tolerance allocation framework which integrates a heterogeneous uncertainty propagation approach. In this way, the framework will allow to allocate tolerances while considering several configurations of behavioural uncertainty. Based on these results, the designer can evaluate the impact of uncertainties and can specify the requested accuracy of the simulation model.

Moreover, the emergence of the Digital Twin concept offers new opportunities in the field of product behaviour simulation: the prediction of the behaviour and performance of the product, production and manufacturing processes without the need for costly and time-consuming physical mock-ups [4,5]. To mitigate the impact of the accuracy of this prediction [6], we propose to integrate it like the integration of the noise factors in the robust design approaches.

Section 2 is an overview of mathematical formulation of tolerance allocation and uncertainty management. Section 3 presents the framework to quantify the impact of the behavioural uncertainties on the results of tolerance allocation based on the probability-boxes and optimization. An application is demonstrated through an industrial case study in the last section. A surrogate model is used to assess the product performance according the geometrical deviations.

## 2. State of art-mathematical formulation of tolerance allocation & uncertainty management

In order to clarify the scope of tolerancing in design and uncertainty management, it is important to identify the impact of these uncertainties. Morse et al. [1] proposed an uncertainty management taxonomy, stating that: "*Uncertainty causes Risks, which are handled technically by Mitigations, which hopefully lead to desired Outcomes*". The authors agree that (i) despite the fact that tolerancing activity is

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affected by all types of uncertainty, (ii) available tolerancing approaches focus largely on aleatory uncertainty (manufacturing imprecision), resulting in (iii) the fact that tolerancing represents a wide spectrum of uncertainty management aspects, including uncertainty modeling (tolerance modeling, geometrical specification models and their mathematical formulations), uncertainty propagation (tolerance analysis), uncertainty quantification (metrology, verification, tolerance evaluation), and tolerance allocation.

Hereby, tolerance allocation is understood as the assignment of tolerance values according to the impact of uncertainties on product performance, and the engineering cost of uncertainty reduction vs the accuracy increase. Consequently, the three main objective functions and constraints of the tolerance optimization [2] are usually stated as the:

- tolerance cost which represents the manufacturing cost impacted by the tolerances. The cost model prediction is highly approximate, and its validity is strongly limited [7].
- non-conformance rate which represents the impacts of the manufacturing imprecisions on the functional requirements.
- process capabilities [2] which indicate how efficient a manufacturing process is at producing parts that meet the required tolerances.

The corresponding mathematical formulations, commonly used to allocate tolerances, are given below. Firstly, Eq. (1) summarizes the constrained optimization of the involved costs. In fact, the objective function is the cost of tolerances or the manufacturing cost impacted by the tolerances: $C(T)$. Several papers focus on the cost model [8−11], usually in form of an optimization problem subject to one constraint: the scrap rate must be less than $\alpha$, or the compliant rate $Prob(CP(T))$ must be greater than $1-\alpha$. In the case of worst case tolerance allocation, $\alpha$ is equal to 0. The behavioural simulation uncertainties affect the assessment of the respect of the constraint.

$$\min_T C(T) \quad s.t. \quad Prob(CP(T)) \geq 1 - \alpha \tag{1}$$

Secondly, Eq. (2) describes the optimisation of the process capabilities: $Cp(T)$ [2] with the same constraint than the previous optimization model. This optimization model is generally used when the production process is known. The tolerances are optimized such as the manufacturing processes have the best ability to produce products within specification limits. This optimization model is similar of the optimization of the quality loss function.

$$\max_T (\min_i Cp_i(T_i)) \quad s.t. \quad Prob(CP(T)) \geq 1 - \alpha \tag{2}$$

There exist several combinations of these two mathematical formulations. The factor "respect of functional requirements" or "compliant product(T)" is integrated into all mathematical formulations, and is unfortunately affected by the uncertainty of the behavioural simulation.

As this type of uncertainty may result from experiments, processing methods, material structure, and model parameters that support concurrent design of products, it is covered by several uncertainty classifications presented in literature [1].

With a focus on modelling and simulation, Walter et al. [12], for example, distinguish the aspects "phenomenological uncertainty", "uncertainty in human behaviour", "uncertainty in data" and "uncertainty in model and simulation". The behavioural uncertainty is thereby equivalent to "uncertainty in model and simulation" defined by Walter et al. as *"this uncertainty leads to deviations from conditions in reality and its framed reference due to model creation and its application within a simulation."*

Lastly, the impact of this uncertainty is therefore modelled by Eq. (3), where $N(X)$ represents the additional noise component due to this type of epistemic uncertainty [3].

$$Perf(X) = Perf_{simulation}(X) + N(X) \tag{3}$$

with

$X$: Values of geometrical deviations

$Perf(X)$: Real performance value according to the geometrical deviations

$Perf_{simulation}(X)$: Performance prediction through simulation

## 3. Proposed mathematical framework for tolerance allocation under behavioural simulation uncertainty

The conclusion of the previous state of art is that the tolerance allocation task is affected by several types of uncertainty; one of them being the behavioural simulation uncertainty, which affects the prediction of the non-conformance rate or the compliance rate. Therefore, we propose to take into account the impact of the additional noise due to this uncertainty.

The considered prediction (Eq. (4)) of the performance into the optimization model is composed of two corresponding terms: a performance prediction through simulation and the considered value of the noise according the geometrical deviations.

$$Perf_{prediction}(X) = Perf_{simulation}(X) + \check{N}(X) \tag{4}$$

with

$Perf_{prediction}(X)$: considered prediction of the performance

$\check{N}(X)$: considered value of the noise

Based on this formulation, the compliant rate can be expressed by Eq. (5). To generalize the mathematical expression, we consider that the product is compliant if the performance is greater than a limit c.

$$\begin{aligned} Prob(CP(T)) &= Prob(Perf_{prediction}(X) > c) \\ &= Prob(Perf_{simulation}(X) + N^{\vee}(X) > c) \end{aligned} \tag{5}$$

with

$X$: random vector which represents the geometrical deviations; the parameters of its probability distribution depend on the tolerances $T$.

The current practice of tolerance allocation considers the $\check{N}(X)$ equal to 0. To quantify the impact of this noise on the allocated tolerances, we propose two strategies: the first one is to consider the noise as a stochastic process; the second one is to consider the worst values of the noise according to the geometrical deviations. In other words, the first strategy considers the behavioural simulation uncertainty as an aleatory uncertainty; so that the conventional approaches of an aleatory uncertainty propagation such as Monte Carlo simulation can be used. The second strategy considers the behavioural simulation uncertainty as an epistemic uncertainty instead; which correspondingly requires the use of a heterogeneous uncertainty propagation technique for the quantification of the compliance rate [3].

For this purpose, interval analysis provides the simplest and most easily applicable approach to calculate the propagation of epistemic uncertainty. However, the assumption that all uncertain variables lie within certain interval bounds is overly conservative in many cases. Particularly for more complex systems, the use optimization to find the maximum and minimum values of the noises is advocated. Lastly, Dempster-Shafer Evidence theory [13] offers an efficient approach to tackle epistemic uncertainty based on a generalization of classical probability theory.

Dempster-Shafer evidence theory proposes to modelize the epistemic uncertain variables by sets of intervals. The intervals are then propagated to estimate the belief Cumulative Distribution Function (CDF) and plausibility Cumulative Distribution Function, as illustrated in Fig. 1 for belief and plausibility CDF of compliance. The estimation of these CDF is done by an approach which combines Monte carlo simulation for the aleatory uncertainty propagation and the optimization for identification of the worst values of noise [3].

Based on the two proposed strategies for the noise integration into the compliance probability estimation, four estimations of the probability (Eq. (5)) could be performed:

- the probability without noise - $\check{N}(X)=0$: $Prob_{Without}$
- the probability with noise as a stochastic process: $Prob_{Stat}$
- the probability with the lower value of noise: $Prob_{Pessimist}$
- the probability with the upper value of noise: $Prob_{Optimist}$

In fact, Plausibility CDF & Belief CDF represent the pessimistic and the optimistic case (Fig. 1).

The factor of cost or process capability is added to the decision making criteria as the objective function of the tolerance allocation. The new formulation of tolerance allocation (Eq. (6)) integrates the impact of the behavioural simulation uncertainty.

$$\min_T \ldots (T) \quad s.t. \quad Prob\left( Perf_{simulation}(X) + \check{N}(X) > c \right) \geq 1 - \alpha \qquad (6)$$
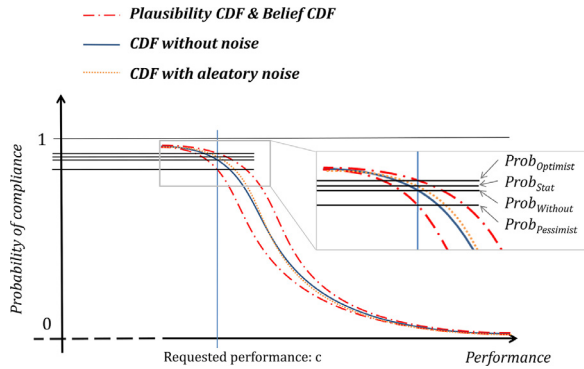


**Fig. 1.** Probability of compliance.

where the noise $\check{N}(X)$ could be considered in four ways

In order to solve the tolerance allocation under uncertainty, the algorithm must be capable of obtaining the optimal or near-optimal solution within a reasonable time. A genetic algorithm (GA) is utilized to cope with this challenging issue and to solve the proposed optimization problem [6,14−16]. A penalty function is used to reformulate the original constrained optimization, and the assessment of the constraint satisfaction is performed by Monte-Carlo simulation coupled with optimization. We use a linear non-convex multi-parent crossover operator [14] and an aleatory mutation; the mutation rate is 20% and the crossover rate is 80%; the population size is 50 and the generation number is 3000.

## 4. Case study: external gear pump

To illustrate the impact of the behavioural simulation uncertainty, the tolerance allocation is performed on an external gear pump (Fig. 2). The pump efficiency depends on different backlashes. These
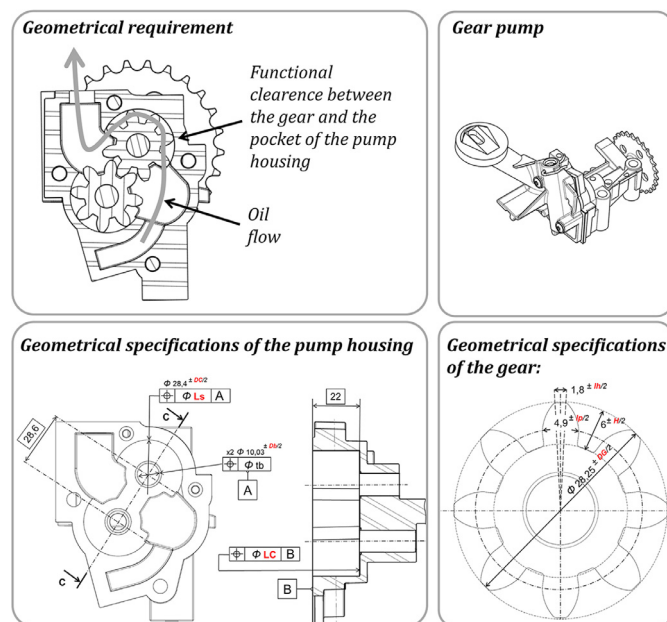


**Fig. 2.** Overview of the case study.

backlashes are between the gears and the housing as well as between the gears and shafts. The behavioural simulation uncertainty is mainly due to the used surrogate model to predict the leakage rate according to the backlashes. To estimate the statistical distribution of these backlashes, the used geometrical model and tolerance analysis technique of this external gear pump were detailed in [17] and the heterogeneous uncertainty propagation technique in [3].

Table 1 summarizes all considered geometrical characteristics: gear tolerances, housing tolerances and shaft tolerances.

**Table 1**
Geometrical specification description.

| Component | Geometrical characteristics | Id of tolerance |
|---|---|---|
| Gear | Head length of the teeth | lh |
| | Primitive length of the teeth | lp |
| | Base length of the teeth | lb |
| | Gear thichness | LG |
| | Gear external diameter | DG |
| | Tooth depth | H |
| | Gear root diameter | Dr |
| Pump housing | Localization- Depth of the housing pocket for the gear | LC |
| | Diameter of the housing pocket for the gear | DC |
| | Bearing diameter | Db |
| Shaft | Shaft diameter | Ds |
| | Localization - Shaft length | Ls |

In this case study, the used objective function (Eq. (7)) is: to minimize the cost weighted quality [7] each cost component is estimated based on the ABC method, and the performance requirement is a minimum oil flow of $4.35 \times 10^{-4}$ m³/s.

$$\min_T C(T) \qquad s.t. Prob\left( Q_{simulation}(X) + N^\vee(X) > 4.35 \times 10^{-4} \right) \geq 1 - \alpha$$
$$(7)$$

The noise assessment is performed based on the confidence intervals of each parameter of the surrogate model (Table 2), which are estimated from experimental results and results of finite element simulations. The accuracy of this predictive model is average (Table 2-percentage prediction error). The statistical distributions of the oil flow are calculated from the statistical distribution of all functional backlashes and from the surrogate model. The sample size of the Monte Carlo simulation is equal to $10^6$. The confidence interval of each conformance rate estimation by the Monte Carlo simulation is equal to $\pm$ 8 ppm, and the considered value of $\alpha$ is equal to 1000 ppm. Therefore, the accuracy of the prediction of the Monte Carlo simulation is negligible.

**Table 2**
Surrogate model information.

| Model Parameter | Estimated value | Confidence interval | percentage prediction error |
|---|---|---|---|
| a | 2.75 | [2.72, 2.78] | 2.18% |
| b | 235 | [234.6, 235.4] | 0.34% |
| c | 500,000 | [497,000,503,000] | 1.20% |
| d | 19.8e-3 | [19.3e-3, 20.3e-3] | 3.03% |

Fig. 3 summarizes the results of this tolerance allocation:

- the optimist tolerances which are estimated in the optimistic configuration of noise,
- the pessimist tolerances which are estimated in the pessimistic configuration of noise,
- the statistical tolerances which are estimated with noise as a stochastic process,
- the classical tolerances which are estimated without noise.

These results highlight the impact of the strategy of the noise consideration and the behavioural simulation uncertainty. In this case, the differences are not negligible. The ratios between the two extreme allocations are between 2,6 and 5,8. These ratios are due to
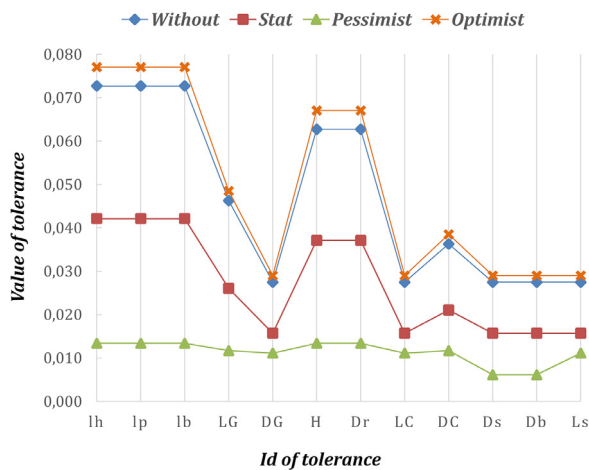
**Fig. 3.** Tolerance allocation results ($\alpha$=0.001).

the impact of behavioural simulation uncertainty (prediction error of the surrogate model) around the desired conformance probability (Fig. 1). According to these results, designers could visualize and analyse the impact of the predictive model accuracy and they could define the requested accuracy.

The behavioural simulation uncertainty has little impact on the allocation scheme, i.e., the upper tolerances are relative to the same geometrical characteristics, idem for the lower tolerances.

Moreover, Fig. 4 highlights the impact of the risk $\alpha$ on the results of tolerance allocation. These results allow to find a compromise between risk and process capabilities.
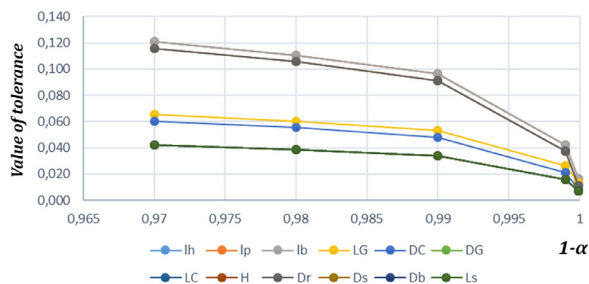


**Fig. 4.** Tolerance allocation results according $\alpha$.

## 5. Conclusion

An adequate Digital simulation, Digital twin or Hybrid Twin for geometric quality management must allow for the efficient prediction of the behaviour and performance of product and manufacturing process as well as for making decisions on the product and the manufacturing process without the need for costly and time-expensive physical mock-ups. Regarding the deployment of Digital simulation, Digital or Hybrid twin in the context of tolerancing, this paper highlights the impact of the performance predictive model accuracy on the tolerance allocation, and, to quantify this impact it proposes an approach based on Dempster Shafer theory, Monte-Carlo simulation and genetic algorithm.

Based on the proposed approach, designers could analyze the impact of the behavioural simulation uncertainty - predictive model accuracy. Therefore, they could specify the requested accuracy of the predictive model and answer to the question: How to decide which models and simulation tools to use for tolerance allocation?

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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