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Computation of dynamic transmission error for gear transmission systems using modal decomposition and Fourier series

Eddy Abboud^{1,2}  · Aurélien Grolet¹ · Hervé Mahé² · Olivier Thomas¹

Abstract

In this paper, a method for computing the dynamics of a geared system excited by its static transmission error is proposed. The method is based on the iterative spectral method (ISM) and on the harmonic balance method (HBM). It is shown that the dynamic transmission error (DTE) can be obtained in the frequency domain by solving a linear system of equations, which in turn allows the computation of the modal and physical coordinates of the system.

Berechnung des dynamischen Übertragungsfehlers bei Zahnradübertragungssystemen mit Hilfe von Modalzerlegung und Fourier-Reihen

Zusammenfassung

In diesem Beitrag wird eine Methode vorgestellt, die die Dynamik eines durch dessen statischen Übertragungsfehler angeregten Getriebesystems ausgearbeitet. Die Methode basiert auf der iterativen Spektralmethode (ISM) und der harmonischen Gleichgewichtsmethode (HBM). Es wird gezeigt, dass der dynamische Übertragungsfehler (DTE) im Frequenzbereich durch Lösen eines linearen Gleichungssystems ermittelt werden kann, was wiederum die Berechnung der modalen und physikalischen Koordinaten des Systems ermöglicht.

1 Introduction

The use of electric motors generates noises that were previously masked by the sound of internal combustion engines, such as the gear noises produced by the gearbox.

The primary source of this noise is gear vibration caused by transmission errors, defined by Harris in 1958 [1]. Notably, Velex et al. [2] introduced an analytical method for modeling the excitations of a gear system. In this method, beginning with the instantaneous contact between teeth, new equations of motion are proposed in terms of transmission errors. Analytical and numerical solutions to these equations are proposed. By contrast, Garambois and Perret-

Liaudet [3–5] proposed another method of solving the equations of motion, which in turn allows the simulation of the vibro-acoustic responses of the gear system, called the iterative spectral method (ISM). In this method, the equations of motion are projected onto the modal basis. Using Fourier transforms, an equation is obtained to express the DTE in the frequency domain (thus the name spectral), which is then solved using fixed point iteration (thus the name iterative). In this paper, we propose an adoption of the ISM to solve the DTE when the input signals (STE and $k(t)$) are periodic. The method is based on the ISM, but instead of using a Fourier transform, Fourier series will be used. An equation that governs the amplitude of the harmonics of the DTE is obtained from the modal equation and it is solved using a single linear system inversion instead of fixed point iterations. The method is applied on a simple example of two spur gears mounted on flexible shafts to illustrate its application and performance.

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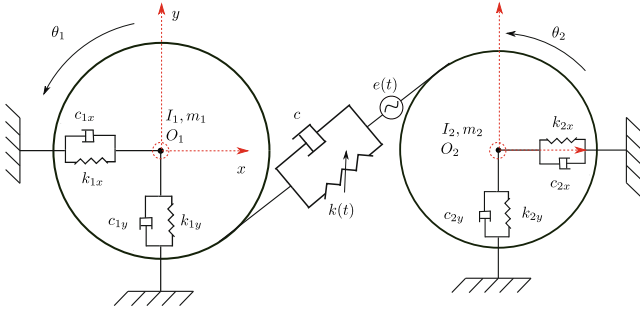


Fig. 1 Model of two spur gears mounted on flexible shafts (6 DOF model)

2 General system

In order to represent the method, we consider a system of two geometrically perfect spur gears (NLTE = 0¹) mounted on two flexible shafts (see Fig. 1). Note, however, that the method is applicable to more complicated systems (e.g helical, with NLTE, etc.). If we restrict ourselves to 2D motion (in the xy plane) the gears are represented as two rigid bodies each having three degrees of freedom (two translations (x_i and y_i) and one rotation θ_i for $i \in [1,2]$).

The meshing interaction occurring between the two gears is modeled by a time varying stiffness (mainly related to the number of teeth in contact at a given time) in series with an imposed displacement $e(t)$ (related to the static transmission error). The aim is to predict the behavior of the system and to define the modes that are most excited by the internal force (STE).

We define the geometrical vector R , which allows the effect of the contact force (on the gear teeth) to be transported onto the gear centers.

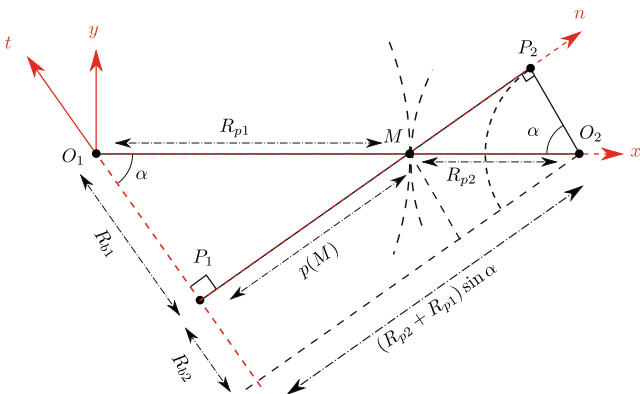


Fig. 2 Geometric description of the contact

¹ NLTE: No Load Transmission Error.

In the simple case of Fig. 2, the geometrical vector R , can then be written as²:

$$\underline{R} = \begin{pmatrix} \underline{n} \\ O_1 \underline{M} \wedge \underline{n} \\ -O_2 \underline{M} \wedge \underline{n} \end{pmatrix} \text{ with } \underline{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}_{\underline{n},t,z} = \begin{pmatrix} \sin \alpha \\ \cos \alpha \\ 0 \end{pmatrix}_{\underline{x},y,z} \quad (1)$$

where α is the pressure angle. This leads to:

$$\underline{R}^T = (\sin \alpha \quad \cos \alpha \quad R_{b1} \quad -\sin \alpha \quad -\cos \alpha \quad R_{b2})_{(\underline{x},\underline{y},\underline{z})} \quad (2)$$

One should note that $\underline{\bullet}$ is used to represent a vector and $\underline{\bullet}$ a matrix. Using the vector of degrees of freedom associated with both gear centers and the geometrical vector R , one can derive the expression of the transmission error expressed along the line of action as:

$$\Delta = \underline{R}^T \underline{X} \quad (3)$$

where $\underline{X}^T = [x_1 y_1 \theta_1 x_2 y_2 \theta_2]$.

3 Dynamic equation

3.1 Equation of motion

The equation of motion of the system in Fig. 1 can be obtained by applying Newton's second law to each of the gears (resulting in three equations per gear). The equation of motion can be written under matrix form as:

$$\underbrace{\begin{pmatrix} \underline{M}_1 & 0 \\ 0 & \underline{M}_2 \end{pmatrix}}_{\underline{M}_{FE}} \begin{pmatrix} \ddot{\underline{X}}_1 \\ \ddot{\underline{X}}_2 \end{pmatrix} + \underbrace{\begin{pmatrix} \underline{K}_1 & 0 \\ 0 & \underline{K}_2 \end{pmatrix}}_{\underline{K}_{FE}} \begin{pmatrix} \underline{X}_1 \\ \underline{X}_2 \end{pmatrix} + \underbrace{\begin{pmatrix} \underline{F}_{2 \rightarrow 1} \\ \underline{F}_{1 \rightarrow 2} \end{pmatrix}}_{\underline{F}_{int}} = \underbrace{\begin{pmatrix} \underline{F}_1 \\ \underline{F}_2 \end{pmatrix}}_{\underline{F}_{ext}} \quad (4)$$

where $\underline{F}_{2 \rightarrow 1}^T = [F_{x1} \ F_{y1} \ C_1]$ is the action of gear 2 on gear 1 (and $\underline{F}_{1 \rightarrow 2}^T = [-F_{x1} \ -F_{y1} \ C_2]$ the action of gear 1 on gear 2). In Eq. (4), $\underline{M}_i, \underline{K}_i$ are the mass and stiffness matrices respectively, with $\underline{M}_i = \text{diag}(m_i, m_i, I_i)$ and $\underline{K}_i = \text{diag}(k_{ix}, k_{iy}, 0)$.

² It is important to note that in 3D, the geometric vector R is formed by 12 non null elements which represents the degrees of freedom. In 2D, taking into account that there are only two translations and one rotation, it will be made of the 2 center nodes.

The vector $\underline{F}_{\text{ext}}$ contains the external applied force. Using the geometrical vector \underline{R} and the displacement associated with the static transmission error it can be shown that the internal force corresponding to the parametric excitation (due itself to the varying stiffness) can be written as follows [3]:

$$\underline{F}_{\text{int}} = \underbrace{k(t)\underline{R}\underline{R}^T}_{\underline{K}_{\text{mesh}}} \left[\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} - \begin{pmatrix} X_{1s} \\ X_{2s} \end{pmatrix} \right] \quad (5)$$

where $k(t) = k_0 + g(t)$ is the time-varying stiffness obtained from the meshing process (e.g from numerical analysis with commercial software). With the gears considered geometrically perfect, the no-load transmission error is zero, and the static transmission error is thus expressed as [3, 6]:

$$\Delta^s(t) = \underline{R}^T \begin{pmatrix} X_{1s} \\ X_{2s} \end{pmatrix} \quad (6)$$

The meshing stiffness $\underline{K}_{\text{mesh}}$ is formed of a constant and a variable part. At the level of the FEM model, one can define a mean stiffness matrix:

$$\underline{K}_{\text{mesh}} = k_0 \underline{R}\underline{R}^T + g(t)\underline{R}\underline{R}^T \quad (7)$$

$$\underline{K}_{\text{AV}} = \underline{K}_{\text{FE}} + k_0 \underline{R}\underline{R}^T \quad (8)$$

Substituting Eqs. (5), (7) and (8) into Eq. (4), the final dynamic equation of the system can be written:

$$\underline{M}_{\text{FE}} \ddot{\underline{X}} + \underline{K}_{\text{AV}} \underline{X} + g(t)\underline{R}\underline{R}^T \underline{X} = \underline{F}_{\text{ext}} + k(t)\Delta^s(t)\underline{R}. \quad (9)$$

3.2 Projection onto the modal basis

The natural angular frequency ω_n and the associated mode shapes ϕ_n of the coupled system can be calculated using $\underline{M}_{\text{FE}}$ and $\underline{K}_{\text{AV}}$. Using the modal basis, the vector of DOFs \underline{X} , can be written as

$$\underline{X}(t) = \sum_{n=1}^{N_{\text{DOF}}} \underline{\Phi}_n q_n(t) \quad (10)$$

with q_n the modal amplitude of mode n . After the projection of the equation of motion onto the modal basis, it becomes:

$$\begin{aligned} \ddot{q}_n(t) + 2\xi_n \omega_n \dot{q}_n(t) + \omega_n^2 q_n(t) + g(t)r_n \sum_m r_m q_m(t) \\ = k(t)r_n \Delta^s(t) \end{aligned} \quad (11)$$

where ω_n is the angular frequency of mode n , ξ_n is the modal damping of mode n and $r_n = \underline{\Phi}_n^T \underline{R}$ is the projection of the geometrical vector onto the mode shape $\underline{\Phi}_n$. As seen in Eq. (11), r_n is a multiplying factor of both applied forces (direct and parametric). Therefore, it gives an indication

of the most excited mode (the higher the value of r_n , the higher the response of the system).

Eq. (11) contains a set of coupled linear differential equations with periodic coefficients. Considering that the time-varying stiffness and the excitation are periodic with fundamental pulsation Ω the harmonic balance method can be used to solve the set of differential equations [7]. The periodic signals are represented using Fourier series:

$$\begin{aligned} k(t)\Delta^s(t) &= \sum_k \hat{f}_k e^{ik\Omega t}; \quad q_n(t) = \sum_k \hat{q}_{nk} e^{ik\Omega t}; \\ g(t) &= \sum_k \hat{g}_k e^{ik\Omega t} \end{aligned} \quad (12)$$

One should note that the dynamic transmission error is defined as follows in the frequency domain:

$$\hat{\Delta}_k = \sum_m r_m \hat{q}_{m,k} \quad (13)$$

The balancing of each harmonic leads to the following set of algebraic equations for $n \in [1, N_{\text{mode}}]$ and $k \in [-H, H]$:

$$\hat{q}_{nk} + H_{n,k} r_n \sum_l \hat{g}_{k-l} \hat{\Delta}_l = H_{n,k} r_n \hat{f}_k \quad (14)$$

with $H_{n,k}^{-1}(\Omega) = \omega_n^2 - (k\Omega)^2 + 2i\xi_n \omega_n (k\Omega)$. Multiplying Eq. (14) by r_n and summing over n leads to:

$$\sum_n r_n \hat{q}_{nk} + \sum_n H_{n,k} r_n^2 \sum_l \hat{g}_{k-l} \hat{\Delta}_l = \sum_n H_{n,k} r_n^2 \hat{f}_k \quad (15)$$

with the DTE previously defined in Eq. (13), Eq. (15) becomes:

$$\hat{\Delta}_k + T_k(\omega) \sum_l \hat{g}_{k-l} \hat{\Delta}_l = T_k(\omega) \hat{f}_k \quad (16)$$

where $T_k = \sum_n H_{n,k} r_n^2$. The k -th harmonic components of the DTE (Δ_k) can be obtained by solving the linear system in Eq. (16). It is then possible to calculate the modal coordinates by introducing the obtained values for Δ_k in Eq. (14) and solving for \hat{q}_{nk} . The coordinates in the physical basis can then be calculated using Eq. (10) and Eq. (12).

4 Application on a simple example

Using Masta [8], a system of two spur gears system is modeled (see Table 1). The static transmission error (STE) and the meshing stiffness ($k(t)$) are calculated using the built-in tools of Masta. Using these values and Eq. (16),

Table 1 Geometrical characteristics of the two spur gears

	Symbol	Pinion	Wheel
Number of teeth	Z	35	48
Mass	M (kg)	0.5152	1.0409
Inertia	I (kg m ²)	0.0001928	0.0006687
Pressure angle	α°	20	
Modulus	m (mm)	2	
Facewidth	b (mm)	20	

the DTE is calculated. Then, using Eq. (10) and (14), both the modal and physical coordinates are computed.

As previously mentioned, r_n is the projection of the geometrical vector onto the mode shape. Therefore, it represents the spatial sensitivity localized at the meshing (an essential factor of the excitation forces, see Eq. 14). Based on Table 2a, the 2nd mode has the highest r_n value. This indicates that it is the most excited mode by the internal excitation element. This can also be seen in the presentation of the modal coordinates (see Fig. 3c). As for the DTE

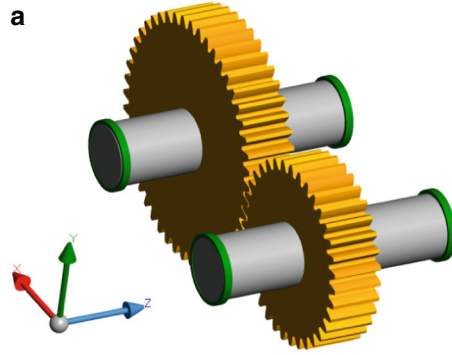
Table 2 The eigenvalues, eigenmodes and their corresponding r_n values

Mode	Freq(KHz)	r_n	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
1	0	0	0	0.04	1.18	0.42	0	-0.02
2	5.92	-1.8	0	0.12	-0.42	1.17	0	-0.08
3	15.9	0	-26.57	-35.35	0	3.49	0	-1.63
4	16.4	1.38	0	-0.01	0	0.02	-0.85	0.30
5	20.96	0	0	-0.03	0	0.06	0.31	0.85
6	21.18	-1.07	19.44	-14.45	0	1.4261	0	-0.67

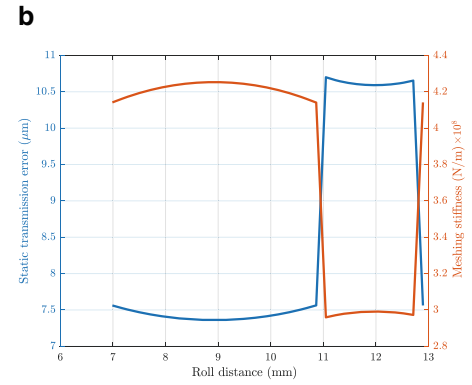
(a) Natural frequencies/ r_n

(b) Modal shapes of the system

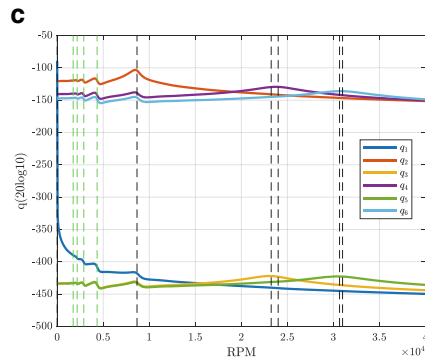
Fig. 3 A general presentation of the input data (macro-geometry, STE and meshing stiffness) and the output signals (DTE and modal coordinates) calculated using the described method³. (a) 3D view of the generated gear model, (b) STE(t) and $k(t)$ signals for $T = 100N.m$, (c) Modal coordinates of the system, (d) Dynamic transmission error



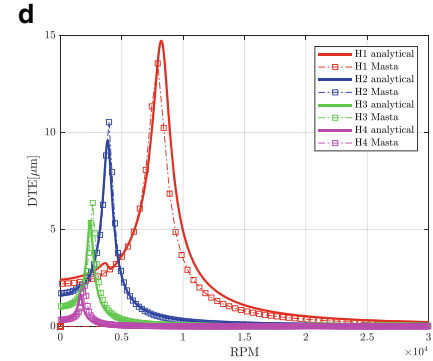
3D view of the generated gear model



STE(t) and $k(t)$ signals for $T = 100N.m$



Modal coordinates of the system



Dynamic transmission error

calculation, it is shown in Fig. 3d³ that the proposed method gives the same values as those obtained with Masta.

Although the method described in this article was applied to a spur gear system, it can easily be applied in the case of a helical gear case while considering other important elements for the study (NLTE, external forcing, etc.).

5 Conclusion

A novel method for the calculation of the dynamic transmission error of a gear system has been presented in this paper. The method is rooted in the iterative spectral method originally proposed by Perret-Liaudet, but instead uses a Fourier series expansion technique in the place of a Fourier transform. The equations derived in this paper present a simple and effective way of calculating the dynamic transmission error along with the modal and physical coordinates. Such a method is interesting in that it provides engineers and researchers with a tool for the study of gear vibrations. In replacing values for the stiffness and the static transmission error, one can easily predict the general vibration behavior

of any given gear system. This is a first step toward a complete gearbox simulation which can be done, for example, using model reduction techniques (e.g. Craig-Bampton).

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³ One should note that the DTE values were calculated for an idealized system whereby the shafts have been reduced to simple springs. The primary purpose is to demonstrate the exactitude of the calculation method in a basic example.