Damage size quantification using lamb waves by analytical model identification

William Briand, Marc Rébillat, Mikhail Guskov, Nazih Mechbal

Processes and Engineering in Mechanics and Materials laboratory (PIMM) ENSAM, CNRS, CNAM, HESAM Université, 151 Boulevard del'Hôpital, 75013 Paris, France

ABSTRACT

Thanks to their mechanical properties, composite materials are widely used in the aeronautic industry. However, they are subject to internal damages like delamination that can threaten structural integrity while being invisible to the naked eye. Structural Health Monitoring (SHM) allows to ensure in real-time that aircraft substructures can still perform their function. Among all the technologies used in SHM, the emission/reception of Lamb waves makes it possible to obtain a lot of information regarding the state of the structure since one knows the input signal in addition to the output signal. Algorithms using Lamb waves for damage detection and localization already exist in the literature but damage size estimation is still an open problem. In this paper, we propose a baseline-free approach to quantify delamination damage size by relying on an analytical scattering model. The structure considered is a plate equipped with piezoelectric transducers (PZT) acting both as actuators and sensors. We use the framework of the Mindlin-Kane plate theory to describe S0-mode Lamb wave propagation. We make the assumption that the S0 mode can be assimilated to an extensional-compressional wave. The damage considered is a cylindrical inhomogeneity where the mechanical properties are different from the rest of the plate. The analytical model derived takes into account the signal emission by a PZT, the scattering by the damage, and the reception by a sensor PZT. This model is then used in an identification process to estimate the size of the damage by minimizing a dedicated cost function. The proposed approach is applied on simulation data using aluminum plate.

INTRODUCTION

In order to reduce their maintenance costs, airlines are increasingly interested in predictive maintenance systems. This type of maintenance consists in immobilizing an aircraft once a condition indicator exceeds a threshold, rather than wait for a pre-determined period of use. This indicator reflects the actual state of degradation of the monitored structure. To build this indicator, it is necessary to know the size of the detected damages to be able to estimate the remaining life of the structure. This is why there is a strong need for reliable and robust quantification algorithms in SHM field. In this paper, we will address the quantification step of SHM by a physic based analytical model inversion.

Yet the literature is rich in articles on the different physical phenomena involved in the process of guided waves SHM. Indeed, the propagation of Lamb waves is well known in isotropic materials, transverse isotropic materials and laminated composites that are of particular interest for aerospace applications. Similarly, analytical models of the effect of a piezoelectric actuator on its support exist. Crawley and de Luis proposed a model based on Euler-Bernoulli strain distribution and shear lag theory [1]. This approach is detailed for a 1D piezoelectric element glued to the structure through a bonding layer transmitting the shear produced by the actuator to the host structure. Shear lag theory is based on the assumptions that the adhesive only carries shear stress and adherends (transducer and plate) only deforms axially [2]. Giurgiutiu extend this work for 2D circular and rectangular transducers [3]. In both cases the dynamic of the actuator is neglected thus, this model is only valid for low frequency range. If the bonding layer is enough stiff and thin we can consider the piezoelectric wafer as ideally bonded to the structure. In this limit case, the actuator only produces shear at its tips, or for a circular actuator, on its perimeter. This simplified model is called pin-force model and is widely use in the literature. Piezoelectric sensors are easier to model [4]. Indeed, the output voltage is proportional to the in-plane normal surface strain. Finally, some analytical damage models exist. In [5], the author proposed an analytical model of scattering for blind hole in an isotropic plate. He compared the results obtained with zeroth-order plate theory for extensional and flexural waves to the ones obtained with 2D wave equations. The scattered wave is written as a Fourier-Bessel series and the unknown coefficients are determined from continuity conditions at damage interface. The comparison shown that both models have similar results at low frequency range. Several articles investigated the model based on plate theory by using different plate theory and damage shape. The authors of [6] studied the conversion of modes for extensional and flexural waves induced by a blind hole in an isotropic plate. Good agreement was find between the proposed model and experimental results. The same kind of analysis has been made for blind holes on each side of the plate that scatter extensional and flexural waves [7]. In [8], the authors used a higher plate theory for extensional waves called Kane-Mindlin theory and described a model for extensional wave scattered by a cylindrical inhomogeneity with a different thickness from the plate for transverse isotropic material.

However, even if these models are known and used to qualitatively dimension the PZT network (size and position of the transducers, choice of excitation signal), there is no comprehensive analytical model of a signal received after the actuator-damage-sensor path in the literature. The advantages of such an analytical model are multiple: calculation of the signal induced by the wave reflected by the damage without reference state, speed of calculation and easy study of the influential parameters. Up to now, this kind of results can only be obtained by finite element simulation, which is computationally expensive and requires a careful selection of the simulation parameters (time step, mesh size) to ensure convergence.

This complete analytical model can also be used to determine the size of the damage. Let say we have the output signals of transducers mounted on a plate in an unknown state of damage. Then we can solve the inverse problem which consists in estimating the size and the severity of the damage by minimizing the difference between the experimental received signal and the signal obtained with the model. To do this, we can use an optimization algorithm that will minimize a cost function by varying the parameters of interest. The speed of calculation of the modeled signal is crucial for this approach because it requires a large number of evaluations of the model: the use of a finite element model is therefore unfeasible.

The topic of this paper is to build an analytical model that takes as input the material and geometrical parameters of the problem as well as the input signal to return the signal received by the sensor after reflection on a damage. We will first build this model and validate it on data from finite element simulation. Then, we will apply this model to estimate the size of a damage in a plate from simulation data.

ANALYTICAL MODEL

In this section, we propose to develop a physics based analytical model that takes the excitation signal send to the actuator as input and returns the electrical signal received by the sensor after the incident wave has been scattered by a damage. The plate is made of an isotropic material, piezoelectric elements are considered perfectly bonded to the plate and the damage is modeled as a through thickness cylindrical inhomogeneity. Inside the damage boundaries, the material parameters are different from the rest of the plate. Fig.01 illustrates the overall configuration of the problem.

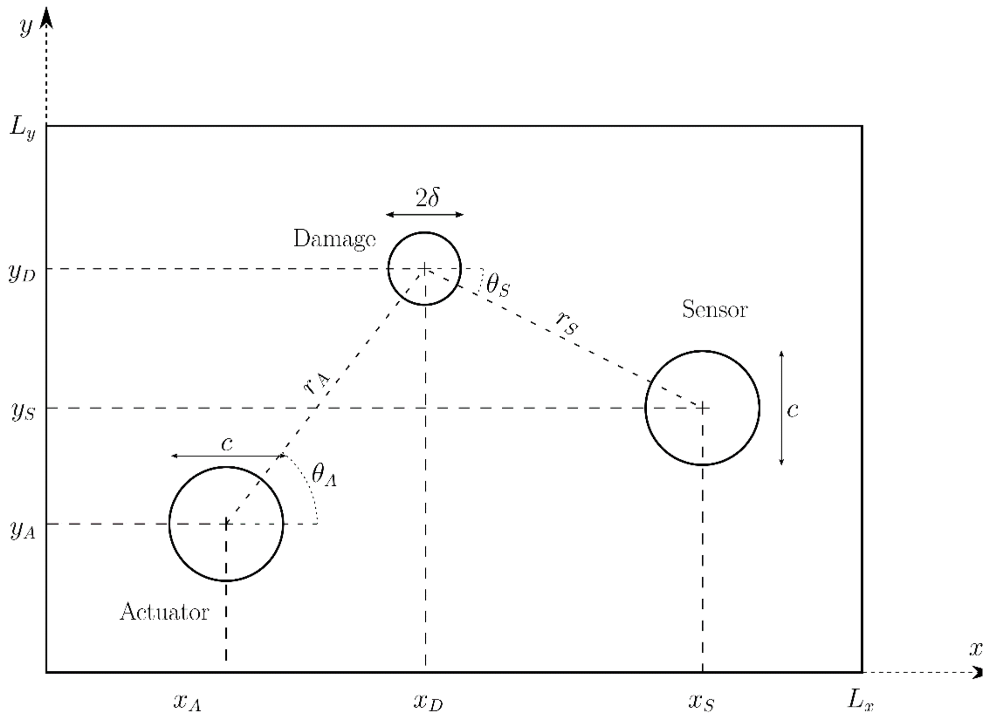

Figure 1. Configuration of the scattering problem.

In the following, we will only consider the symmetrical fundamental mode S0 because it travels faster and is then easy to isolate from A0 mode and wave reflections.

Actuator model

Since we limit our study to low frequencies, that is to say frequency range where only fundamentals Lamb wave modes exists, we will use the pin-force actuator model to represent the exciting PZT [3]. The expression of the displacement radial component for S0 mode is

$$u_r = \frac{\pi i \tau_0}{4\mu} c J_1(kc) \frac{N_s(k)}{D'_s(k)} H_1^{(2)}(kr) = K(k)\, H_1^{(2)}(kr) \qquad (1)$$

where μ is the shear modulus of the plate material, c is the radius of the PZT, τ_0 is the interfacial shear stress and k is the wavenumber of the S0 mode. $N_s(k)$ and $D'_s(k)$ expressions are given in the reference paper.

Scattering model

Since we are interested in the low frequency excitation range, the extension-compression plate waves are a good approximation of the S0 mode of Lamb. In this paper we follow the method proposed by Wang and Chang [8] which relies Kane-Mindlin theory [9] where the displacement is written

$$u_x = v_x(x,y), \qquad u_y = v_y(x,y), \qquad u_z = \frac{z}{h} v_z(x,y) \tag{2}$$

The equation of motion then becomes

$$h\mu\nabla^2 \underline{v} + h(\lambda+\mu)\nabla\big(\nabla.\underline{v}\big) + \lambda\,\kappa\,\nabla v_z = \rho h \ddot{\underline{v}} \tag{3}$$

$$\frac{\mu h^2}{3}\nabla^2 v_z - \kappa^2(\lambda+2\mu)v_z - h\lambda\kappa\,\nabla.\underline{v} = \frac{\rho h^2}{3}\ddot{v}_z \tag{4}$$

where $\underline{v} = v_x(x,y)\underline{x} + v_y(x,y)\underline{y}$, $v_z = v_z(x,y)$ and κ is a correction factor with $\kappa^2 = \frac{\pi^2}{12}$. The purely extensional thickness mode of the plate can be displayed by taking $v_x = v_y = 0$ and $v_z = e^{-i\omega_\mathrm{t} t}$. This exhibits the mode frequency ω_t, which will appear in the following

$$\omega_\mathrm{t} = \sqrt{\frac{3\kappa^2(\lambda+2\mu)}{h^2\rho}}\,. \tag{5}$$

According to Helmholtz decomposition, the displacement field can be separated into two scalar field and one vector field

$$\underline{v}(x,y) = \nabla\phi_1(x,y) + \nabla\phi_2(x,y) + \nabla\wedge\psi(x,y) \tag{6}$$

By replacing $\underline{v}$ by its new expression, we can decompose the equation of motion into three independent Helmholtz equations

$$\nabla^2\phi_1 + k_1^2\phi_1 = 0,\quad \nabla^2\phi_2 + k_2^2\phi_2 = 0,\quad \nabla^2\psi + k_3^2\psi = 0. \tag{7}$$

We exhibit three wavenumbers k_1, k_2 and k_3 which correspond respectively to first extensional mode, second extensional mode and first shear horizontal mode and have the expression

$$k_1^2 = B + \sqrt{B^2 - C} \tag{8}$$

$$k_2^2 = B - \sqrt{B^2 - C} \tag{9}$$

$$k_3^2 = \frac{\omega^2}{c_3^2} \tag{10}$$

where

$$\boldsymbol{B} = \frac{\boldsymbol{\rho\omega^2}}{\boldsymbol{2(\lambda+2\mu)}} + \frac{\boldsymbol{\rho}}{\boldsymbol{2\mu}}(\boldsymbol{\omega^2 - \omega_t^2}) + \frac{\boldsymbol{3\kappa^2\lambda^2}}{\boldsymbol{2h^2\mu(\lambda+2\mu)}} \tag{11}$$

$$C = \frac{\rho^2\omega^2}{\mu(\lambda+2\mu)}(\omega^2 - \omega_t^2),\ c_3 = \sqrt{\frac{\mu}{\rho}} \tag{12}$$

.

The displacement field coordinates can then be written in the cylindrical coordinate system as

$$\boldsymbol{v_r} = \left(\frac{\boldsymbol{\partial\phi_1}}{\boldsymbol{\partial r}} + \frac{\boldsymbol{\partial\phi_2}}{\boldsymbol{\partial r}} + \frac{\boldsymbol{1}}{\boldsymbol{r}}\frac{\boldsymbol{\partial\psi}}{\boldsymbol{\partial\theta}}\right)\boldsymbol{e^{-i\omega t}} \tag{13}$$

$$\boldsymbol{v_\theta} = \left(\frac{\boldsymbol{1}}{\boldsymbol{r}}\frac{\boldsymbol{\partial\phi_1}}{\boldsymbol{\partial\theta}} + \frac{\boldsymbol{1}}{\boldsymbol{r}}\frac{\boldsymbol{\partial\phi_2}}{\boldsymbol{\partial\theta}} - \frac{\boldsymbol{\partial\psi}}{\boldsymbol{\partial r}}\right)\boldsymbol{e^{-i\omega t}} \tag{14}$$

$$v_z = (\sigma_1\phi_1 + \sigma_2\phi_2)e^{-i\omega t} \tag{15}$$

We consider an incoming wave generated by the actuator. We assume a pure compressive wave thus the wave potentials associated to the other wave mode are null. For the following calculation, we need to derive an expression of the potential ϕ_1^{i} under Fourier series form. From equation (1), we get

$$\phi_1^i = -K(k_1)H_0^{(2)}(k_1r) \sim -K(k_1)\frac{1+i}{\sqrt{\pi k_1 r}}\sum_{n=-\infty}^{\infty} i^n J_n(-k_1r)e^{in\theta} \tag{16}$$

$$\phi_2^{\mathrm{i}} = \psi^{\mathrm{i}} = 0 \tag{17}$$

The Helmholtz equation governing all wave potentials equation is solved using the separation of variables technique. Thus, the governing equation lead to two independent single-variable ordinary differential equations: the angle variable θ must satisfy the harmonic oscillator equation whereas the radius r must satisfy the Bessel's equation. The solutions of this last equation are a linear combination of Bessel function of different order. Nevertheless, the only pair of functions that satisfies the physics of the problem (finite potential at $r = 0$ and outgoing wave toward $r \to +\infty$) and that guarantees linear independence are $H_n^{(1)}$ and J_n which are Hankel function of the first kind and Bessel function of the first kind at order n, respectively. Finally, the wave potentials are written as

$$\phi_1^{\mathrm{s}} = \sum_{n=-\infty}^{\infty} A_n i^n H_n^{(1)}(k_1r)e^{in\theta} \qquad \phi_2^{\mathrm{s}} = \sum_{n=-\infty}^{\infty} B_n i^n H_n^{(1)}(k_2r)e^{in\theta}$$

$$\psi^{\mathrm{s}} = \sum_{n=-\infty}^{\infty} C_n i^n H_n^{(1)}(k_3r)e^{in\theta} \tag{18}$$

$$\phi_1^{\mathrm{t}} = \sum_{n=-\infty}^{\infty} D_n i^n J_n(k_1^* r) e^{in\theta} \qquad \phi_2^{\mathrm{t}} = \sum_{n=-\infty}^{\infty} E_n i^n J_n(k_2^* r) e^{in\theta}$$
$$\psi^{\mathrm{t}} = \sum_{n=-\infty}^{\infty} F_n i^n J_n(k_3^* r) e^{in\theta} \tag{19}$$

where A_n, B_n, C_n, D_n, E_n and F_n are the unknown coefficients at order n related to each potential field. The terms with star superscript are properties inside the damage region. To determine the numerical value of each of these coefficients, we write the continuity conditions at damage interface

$$\begin{array}{ll} v_r^{\mathrm{i}}(\delta,\theta) + v_r^{\mathrm{s}}(\delta,\theta) = v_r^{\mathrm{t}}(\delta,\theta) & N_{rr}^{\mathrm{i}}(\delta,\theta) + N_{rr}^{\mathrm{s}}(\delta,\theta) = N_{rr}^{\mathrm{t}}(\delta,\theta) \\ v_\theta^{\mathrm{i}}(\delta,\theta) + v_\theta^{\mathrm{s}}(\delta,\theta) = v_\theta^{\mathrm{t}}(\delta,\theta) & N_{r\theta}^{\mathrm{i}}(\delta,\theta) + N_{r\theta}^{\mathrm{s}}(\delta,\theta) = N_{r\theta}^{\mathrm{t}}(\delta,\theta) \\ v_z^{\mathrm{i}}(\delta,\theta) + v_z^{\mathrm{s}}(\delta,\theta) = v_z^{\mathrm{t}}(\delta,\theta) & R_{rz}^{\mathrm{i}}(\delta,\theta) + R_{rz}^{\mathrm{s}}(\delta,\theta) = N_{rz}^{\mathrm{t}}(\delta,\theta) \end{array} \tag{20}$$

where the generalized stresses expression are

$$N_{rr} = 2h\left((\lambda + 2\mu)\frac{\partial v_r}{\partial r} + \lambda\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} + \frac{v_z}{h}\right)\right) \tag{21}$$
$$N_{r\theta} = 2h\mu\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right) \tag{22}$$
$$R_{rz} = \frac{2h^2\mu}{3}\frac{\partial v_z}{\partial r}. \tag{23}$$

Sensor model

The expression of the voltage V produced by a thin PZT in sensing mode and considered as a voltage generator is

$$V = \frac{d_{31} Y_{\mathrm{s}}^{\mathrm{E}} h_{\mathrm{s}}}{\pi c^2(\varepsilon_3^T(1-\nu_{\mathrm{s}}) - 2d_{31}^2\, Y_{\mathrm{s}}^{\mathrm{E}})}\int (\varepsilon_{rr} + \varepsilon_{\theta\theta})\ \mathrm{d}A \tag{24}$$

where $Y_{\mathrm{s}}^{\mathrm{E}}$, ν_{s}, d_{31} and ε_3^T are respectively the Young's modulus in $0xy$ plane, Poisson's ratio, the xz-directional piezoelectric coefficient and the electric permittivity of the transducer's material. h_{s}, and c are the thickness and the radius of the sensing PZT. ε_{rr} and $\varepsilon_{\theta\theta}$ are the in-plane normal surface strains and can be replaced by their expressions derived from the displacement field (13), (14) and the wave potentials (18). The resulted voltage generated by the scattered wave is

$$H(\omega) = \frac{d_{31} Y_{\mathrm{s}}^{\mathrm{E}} h_{\mathrm{s}}}{\pi c^2\left(\varepsilon_3^T(1-\nu_{\mathrm{s}}) - 2d_{31}^2\, Y_{\mathrm{s}}^{\mathrm{E}}\right)}\left(\frac{A_0}{H_0^{(1)}(k_1\delta)}\int_{r_{\mathrm{s}}-c}^{r_{\mathrm{s}}+c}\left(k^2 H_2^{(1)}(k_1 r) - \frac{2k}{r}H_1^{(1)}(k_1 r)\right)\Theta r\ \mathrm{d}r - \sum_{\substack{n=-N \\ n\neq 0}}^{n=N}\frac{2i^n A_n e^{in\theta_{\mathrm{s}}}}{nH_n^{(1)}(k_1\delta)}\left(\int_{r_{\mathrm{s}}-c}^{r_{\mathrm{s}}+c} k_1^2 H_{n+2}^{(1)}(k_1 r) - \frac{2k_1(n+1)}{r}H_{n+1}^{(1)}(k_1 r)\right)\sin\left(\frac{n\Theta}{2}\right) r\ \mathrm{d}r\right) \tag{25}$$

where N is the truncation order of the Bessel-Fourier series. Finally, to obtain the time signal received from the sensor we compute the inverse Fourier transform of $U(\omega)H(\omega)$ with U the Fourier transform of the excitation signal.

IDENTIFICATION ALGORITHM

Now we have the expression of the received signal after wave scattered by a damage, we will use this physic model for damage size quantification purpose. We first define a cost function that we will seek to minimize. This cost function is based on the difference of Damage Index (DI) between the theoretical physic model and the experimental signal. This experimental signal is calculated as the difference between the pristine state and the current damaged state. The DI selected here is the maximum amplitude of the wave packet because this feature is greatly sensitive to the size of the damage. This difference is squared and computed over each actuator-sensor paths.

$$J(\delta) = \sum_{i=1}^{M_{\mathrm{PZT}}} \sum_{\substack{j=1 \\ j\neq i}}^{M_{\mathrm{PZT}}} \left|\mathrm{DI}_{ij}^{\mathrm{xp}} - \mathrm{DI}_{ij}^{\mathrm{th}}(\delta)\right|^2 \tag{26}$$

Since the physical model only, account for the S0 mode, we isolate the first wave packet of the experimental signal using an appropriate signal. The size of the damage is estimated by minimizing J according to the parameter δ. We also set boundaries to constrain the results between a lower bound (0 mm) and an upper bound (50 mm) to improve the performance. The selected minimization algorithm is the interior-point method already implemented in the MATLAB function fmincon. This method is particularly suited for nonlinear cost function with bound constraints, which we are interested in here. We denote δ^* the estimated damage radius.

$$\delta^* = \underset{\delta}{\operatorname{argmin}}\ J\,(\delta) \tag{27}$$

APPLICATION ON SIMULATION DATA

The quantification algorithm proposed in this paper is applied on data coming from FEM simulation. The structure considered is a 600 mm by 600 mm aluminum and 2.4 mm thickness plate meshed with 2 mm square Mindlin plate elements. 5 PZT of 25 mm diameter are ideally bonded on the surface and meshed with piezoelectric elements. The simulation is done in two cases : a pristine state and a damage state where a cylindrical inhomogeneity –a region where the Young’s modulus is lower than in the main structure- is introduced. Here the damage has a 5 mm radius and a Young’s modulus equal to 90% of the plate Young’s modulus. The time step chosen is 200 ns. The excitation signal is a 5-cycle tone burst of central frequency 150 kHz and 10 V amplitude. Each PZT is actuated sequentially.

The proposed quantification method is applied on the simulation data. It finds a damage radius of 4.3 mm which is very close to the real size 5 mm, even for a low severity damage as in the studied case. It means that the algorithm seems very sensitive to the damage presence and can be exploited to quantify small and emerging damage with low severity.

TABLE 1. PZT AND DAMAGE POSITIONS

	PZT 1	PZT 2	PZT 3	PZT 4	PZT 5	Damage
x [mm]	473	287	103	456	265	356
y [mm]	166	251	331	376	456	343

CONCLUSION

In this article we proposed a damage size quantification method using the inversion of a physics model. We presented the theoretical model that enable to calculate the signal received by a sensor PZT after an incident wave generated by an actuator PZT had been scattered by a cylindrical inhomogeneity. Then we described the identification method used for damage size quantification purposes. This algorithm relies on the minimization of a cost function to estimate the radius of the damage. The proposed approach has been applied on simulation data and show very promising results even for a small damage size and a low severity.

The future work will focus on the extension to composite material on one hand, and on the application to experimental data on the other hand. Besides, we will extend the current identification method to also estimate the severity of the damage along with its size.

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