Influence of global rotation and Reynolds number on the large-scale features of a turbulent Taylor–Couette flow

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We experimentally study the turbulent flow between two coaxial and independently rotating cylinders. We determined the scaling of the torque with Reynolds numbers at various angular velocity ratios (Rotation numbers) and the behavior of the wall shear stress when varying the Rotation number at high Reynolds numbers. We compare the curves with particle image velocimetry analysis of the mean flow and show the peculiar role of perfect counter-rotation for the emergence of organized large scale structures in the mean part of this very turbulent flow that appear in a smooth and continuous way: the transition resembles a supercritical bifurcation of the secondary mean flow. © *2010 American Institute of Physics*. [doi:10.1063/1.3392773]

I. INTRODUCTION

Turbulent shear flows are present in many applied and fundamental problems, ranging from small scales (such as in the cardiovascular system) to very large scales (such as in meteorology). One of the several open questions is the emergence of coherent large-scale structures in turbulent flows.[1] Another interesting problem concerns bifurcations, i.e., transitions in large-scale flow patterns under parametric influence, such as laminar-turbulent flow transition in pipes, or flow pattern change within the turbulent regime, such as the dynamo instability of a magnetic field in a conducting fluid,[2] or multistability of the mean flow in von Kármán or free-surface Taylor–Couette flows,[3,4] leading to hysteresis or nontrivial dynamics at large scale. In flow simulation of homogeneous turbulent shear flow it is observed that there is an important role for what is called the *background rotation*, which is the rotation of the frame of reference in which the shear flow occurs. This background rotation can both suppress or enhance the turbulence.[5,6] We will further explicit this in Sec. III.

A flow geometry that can generate both motions—shear and background rotation—at the same time is the Taylor–Couette flow which is the flow produced between differentially rotating coaxial cylinders.[7] When only the inner cylinder rotates, the first instability, i.e., deviation from laminar flow with circular streamlines, takes the form of toroidal (Taylor) vortices. With two independently rotating cylinders, there is a host of interesting secondary bifurcations, extensively studied at intermediate Reynolds numbers, following the work of Coles[8] and Andereck *et al.*[9] Moreover, it shares strong analogies with Rayleigh–Bénard convection,[10,11] which are useful to explain different torque scalings at high Reynolds numbers.[12,13] Finally, for some parameters relevant in astrophysical problems, the basic flow is linearly stable and can directly transit to turbulence at a sufficiently high Reynolds number.[14]

The structure of the Taylor–Couette flow, while it is in a *turbulent* state, is not so well known and only few measurements are available.[15] The flow measurements reported in Ref. 15 and other torque scaling studies only deal with the case where only the inner cylinder rotates.[12,13] In that precise case, recent direct numerical simulations suggest that vortexlike structures still exist at high Reynolds number ($\mathrm{Re} \gtrsim 10^4$),[16,17] whereas for counter-rotating cylinders, the flows at Reynolds numbers around 5000 are identified as "featureless states."[9] The structure of the flow is exemplified with a flow visualization in Fig. 1 in our experimental setup for a flow with only the inner cylinder rotating, counter-rotating cylinders, and only the outer cylinder rotating, respectively.

In the present paper, we extend the study of torques and flow field for independently rotating cylinders to higher Reynolds numbers (up to 10^5) and address the question of the transition process between a turbulent flow with Taylor vortices, and this "featureless" turbulent flow when varying the global rotation while maintaining a constant mean shear rate.

In Sec. II, we present the experimental device and the measured quantities. In Sec. III, we introduce the specific set of parameters we use to take into account the global rotation through a "Rotation number" and the imposed shear through a shear-Reynolds number. We then present torque scalings and typical velocity profiles in turbulent regimes for three particular Rotation numbers in Sec. IV. We explore the transition between these regimes at high Reynolds number varying the Rotation number in Sec. V and discuss the results in Sec. VI.

II. EXPERIMENTAL SETUP AND MEASUREMENT TECHNIQUES

The flow is generated between two coaxial cylinders (Fig. 2). The inner cylinder has a radius of $r_i=110 \pm 0.05$ mm and the outer cylinder of $r_o=120\pm0.05$ mm. The gap between the cylinders is thus $d=r_o-r_i=10$ mm and

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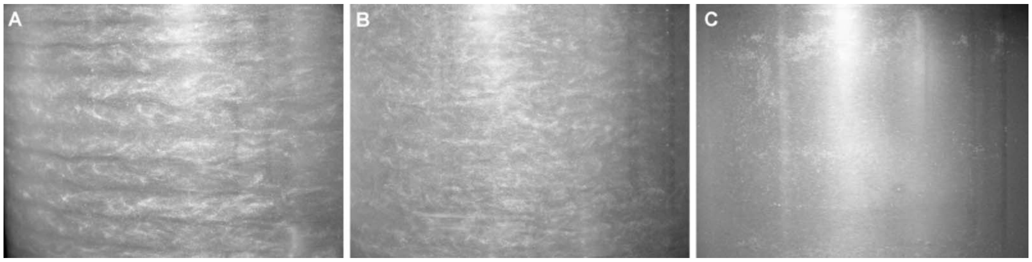

FIG. 1. Photographs of the flow at $\mathrm{Re}=3.6\times 10^3$. Left, A: only the inner cylinder rotating. Middle, B: counter-rotating cylinders. Right, C: only the outer cylinder rotating. The flow structure is visualized using microscopic mica platelets (Pearlessence).

the gap ratio is $\eta=r_i/r_o=0.917$. The system is closed at both ends, with top and bottom lids rotating with the outer cylinder. The length of the inner cylinder is $L=220$ mm (axial aspect ratio is $L/d=22$). Both cylinders can rotate independently with the use of two dc motors (Maxon, 250 W). The motors are driven by a homemade regulation device, ensuring a rotation rate up to 10 Hz, with an absolute precision of ± 0.02 Hz and a good stability. A LABVIEW program is used to control the experiment: the two cylinders are simultaneously accelerated or decelerated to the desired rotation rates, keeping their ratio constant. This ratio can also be changed while the cylinders rotate, maintaining a constant differential velocity.

The torque T on the inner cylinder is measured with a corotating torque meter (HBM T20WN, 2 N m). The signal is recorded with a 12 bit data acquisition board at a sample rate of 2 kHz for 180 s. The absolute precision on the torque measurements is ± 0.01 N m, and values below 0.05 N m are rejected. We also use the encoder on the shaft of the torque meter to record the rotation rate of the inner cylinder. Since that matches excellently with the demanded rate of rotation, we assume that the outer cylinder rotates at the demanded rate as well.

Since the torque meter is mounted in the shaft between driving motor and cylinder, it also records (besides the intended torque on the wall bounding the gap between the two cylinders) the contribution of mechanical friction such as in the two bearings, and the fluid friction in the horizontal (Kármán) gaps between tank bottom and tank top. While the bearing friction is considered to be marginal (and measured

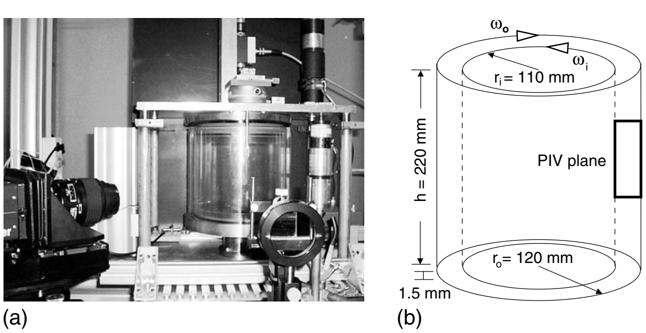

FIG. 2. Picture and sketch with dimensions of the experimental setup. One can see the rotating torque meter (upper part of the picture), the calibration grid displacement device (on top of the upper plate), one of the two cameras (left side), and the light sheet arrangement (right side). The second camera is further to the right.

so in an empty, i.e., air filled system), the Kármán-gap contribution is much bigger: during laminar flow, we calculated and measured this to be of the order of 80% of the gap torque. Therefore, all measured torques were divided by a factor of 2, and we should consider the scaling of torque with the parameters defined in Sec. III as more accurate than the exact numerical values of torque.

A constructionally more difficult, but also more accurate, solution for the torque measurement is to work with three stacked inner cylinders and only measure the torque on the central section, such as is done in the Maryland Taylor–Couette setup,[12] and (under development) in the Twente Turbulent Taylor–Couette setup.[18]

We measure the three components of the velocity by stereoscopic particle image velocimetry (PIV)[19] in a plane illuminated by a double-pulsed Nd:yttrium aluminum garnet laser. The plane is vertical (Fig. 2), i.e., normal to the mean flow: the in-plane components are the radial (u) and axial (v) velocities, while the out-of-plane component is the azimuthal component (w). It is observed from both sides with an angle of 60° (in air) using two double-frame charge coupled device cameras on Scheimpflug mounts. The light-sheet thickness is 0.5 mm. The tracer particles are 20 μm fluorescent (rhodamine B) spheres. The field of view is 11×25 mm^2, corresponding to a resolution of 300×1024 pixels. Special care has been taken concerning the calibration procedure, on which especially the evaluation of the plane-normal azimuthal component heavily relies. As a calibration target we use a thin polyester sheet with lithographically printed crosses on it, stably attached to a rotating and translating microtraverse. It is first put into the light sheet and traversed perpendicularly to it. Typically, five calibration images are taken with intervals of 0.5 mm. The raw PIV images are processed using DAVIS 7.2 by Lavision. They are first mapped to world coordinates, then they are filtered with a min-max filter, then PIV processed using a multipass algorithm, with a last interrogation area of 32×32 pixels with 50% overlap, and normalized using median filtering as postprocessing. Then, the three components are reconstructed from the two camera views. The mapping function is a third-order polynomial, and the interpolations are bilinear. The PIV data acquisition is triggered with the outer cylinder when it rotates in order to take the pictures at the same angular position as used during the calibration.

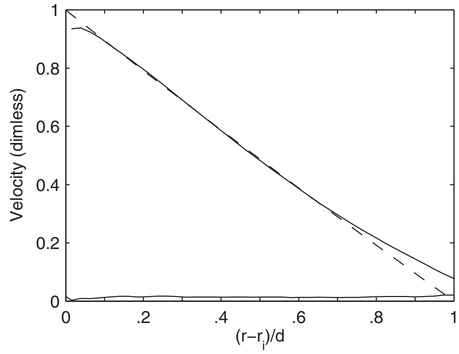

FIG. 3. Dimensionless azimuthal velocity profile $[w/(r_i\omega_i)$ vs $(r-r_i)/d]$ for $\mathrm{Ro}=\mathrm{Ro}_i$ at $\mathrm{Re}_S=90$ (see Sec. III for the definition of the parameters). Solid line: measured mean azimuthal velocity. Dashed line: theoretical profile. The radial component u, which should be zero, is also shown as a thin solid line.

To check the reliability of the stereoscopic velocity measurement method, we performed a measurement for a laminar flow when only the inner cylinder rotates at a Reynolds number as low as $\mathrm{Re}_S=90$ using an 86% glycerol-water mixture. In that case, the analytical velocity field is known: the radial and axial velocities are zero, and the azimuthal velocity w should be axisymmetric with no axial dependence, and a radial profile in the form $w(r)=\Omega r_i\eta(r_o/r-r/r_o)/(1-\eta^2)$.[8] The results are plotted in Fig. 3. The measured profile (solid line) hardly differs from the theoretical profile (dashed line) in the bulk of the flow $[0.1\lesssim(r-r_i)/d\lesssim 0.7]$. The discrepancy is, however, quite strong close to the outer cylinder $[(r-r_i)/d=1]$, which may be due to the refraction close to the curved wall that causes measurement errors. The in-plane components, which should be zero, do not exceed 1% of the inner cylinder velocity everywhere. In conclusion, the measurements are very satisfying in the bulk. Further improvements to the technique have been made since this first PIV test, in particular a new outer cylinder of improved roundness, and the measurements performed in water for turbulent cases are reliable in the range $[0.1\lesssim(r-r_i)/d\lesssim 0.85]$. The PIV measurements close to the wall could be further improved by using an outer cylinder that is machined in a block with flat outer interfaces normal to the cameras.

III. PARAMETER SPACE

The two traditional parameters to describe the flow are the inner (respectively, outer) Reynolds numbers $\mathrm{Re}_i=(r_i\omega_i d/\nu)$ [respectively, $\mathrm{Re}_o=(r_o\omega_o d/\nu)$], with the inner (respectively, outer) cylinder rotating at rotation rates ω_i (respectively, ω_o), and ν the kinematic viscosity.

We choose to use the set of parameters defined by Dubrulle *et al.*:[20] a shear Reynolds number Re_S and a Rotation number Ro,

$$\mathrm{Re}_S=\frac{2|\eta\mathrm{Re}_o-\mathrm{Re}_i|}{1+\eta}\mathrm{Ro}=(1-\eta)\frac{\mathrm{Re}_i+\mathrm{Re}_o}{\eta\mathrm{Re}_o-\mathrm{Re}_i}. \quad (1)$$

With this choice, Re_S is based on the laminar shear rate S: $\mathrm{Re}_S=Sd^2/\nu$. For instance, with a 20 Hz velocity difference in counter-rotation, the shear rate is around 1400 s^{-1} and $\mathrm{Re}_S\simeq 1.4\times 10^5$ for water at 20 °C. A constant shear Reynolds number corresponds to a line of slope η in the $\{\mathrm{Re}_o;\mathrm{Re}_i\}$ coordinate system (see Fig. 4).

The Rotation number Ro compares the mean rotation with the shear and is the inverse of a Rossby number. Its sign defines cyclonic (Ro>0) or anticyclonic (Ro<0) flows. The Rotation number is zero in case of perfect counter-rotation ($r_i\omega_i=-r_o\omega_o$). Two other relevant values of the Rotation number are $\mathrm{Ro}_i=\eta-1\simeq-0.083$ and $\mathrm{Ro}_o=(1-\eta)/\eta\simeq 0.091$ for, respectively, inner and outer cylinders rotating alone. Finally, a further choice that we made in our experiment was the value of $\eta=r_i/r_o$, which we have chosen as relatively close to unity, i.e., $\eta=110/120\simeq 0.91$, which is considered a narrow gap, and is the most common in reported experiments, such as Refs. 8, 9, 21, and 22, although a value as low as 0.128 is described as well.[4] A high η, i.e., $(1-\eta)\ll 1$, is special in the sense that for $\eta\to 1$ the flow is equivalent to a plane Couette flow with background rotation; at high η, the flow is linearly unstable for $-1<\mathrm{Ro}<\mathrm{Ro}_o$.[5,20,23]

In the present study we experimentally explore regions of the parameter space that, to our knowledge, have not been reported before. We present in Fig. 4 the parameter space in $\{\mathrm{Re}_o;\mathrm{Re}_i\}$ coordinates with a sketch of the flow states identified by Andereck *et al.*[9] and the location of the data discussed in the present paper. One can notice that the present range of Reynolds numbers is far beyond that of Andereck, and that with the PIV data we mainly explore the zone between perfect counter-rotation and only the inner cylinder rotating.

IV. STUDY OF THREE PARTICULAR ROTATION NUMBERS

In the experiments reported in this section, we maintain the Rotation number at constant values and vary the shear Reynolds number. We compare three particular Rotation numbers, Ro_i, Ro_c, and Ro_o, corresponding to rotation of the inner cylinder only, exact counter-rotation, and rotation of the outer cylinder rotating only, respectively. In Sec. IV A, we report torque scaling measurements for a wide range of Reynolds numbers—from base laminar flow to highly turbulent flows—and in Sec. IV B, we present typical velocity profiles in turbulent conditions.

A. Torque scaling measurements

We present in Fig. 5 the friction factor $c_f=T/(2\pi\rho r_i^2 LU^2)\propto G/\mathrm{Re}^2$, with $U=Sd$ and $G=T/(\rho L\nu^2)$, as a function of Re_S for the three Rotation numbers. A common definition for the scaling exponent α of the dimensionless torque is based on G: $G\propto\mathrm{Re}_S^{\alpha}$. We keep this definition and present the local exponent α in the inset in Fig. 5. We compute α by means of a logarithmic derivative, $\alpha=2+d\log(c_f)/d\log(\mathrm{Re}_S)$.

At low Re, the three curves collapse on a Re^{-1} curve. This characterizes the laminar regime where the torque is proportional to the shear rate on which the Reynolds number is based.

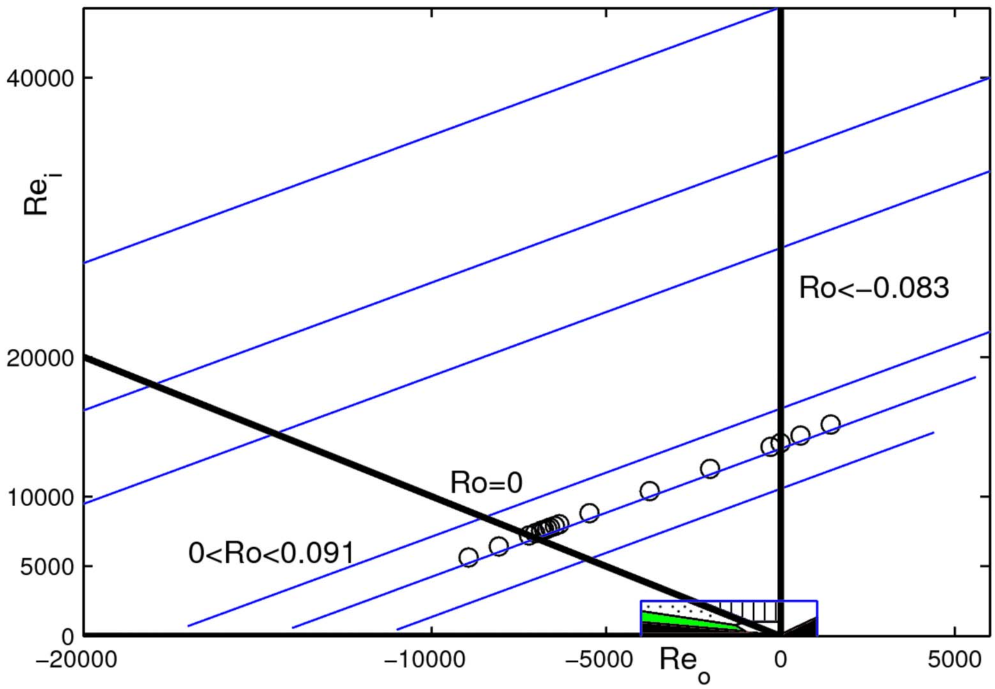

FIG. 4. (Color online) Parameter space in $\{\mathrm{Re}_o;\mathrm{Re}_i\}$ coordinates. The vertical axis $\mathrm{Re}_o=0$ corresponds to $\mathrm{Ro}=\mathrm{Ro}_i=-0.083$ and has been widely studied (Refs. 12, 13, 16, and 17). The horizontal axis $\mathrm{Re}_i=0$ corresponds to $\mathrm{Ro}=\mathrm{Ro}_o=0.091$. The line $\mathrm{Re}_i=-\mathrm{Re}_o$ corresponds to counter-rotation, i.e., $\mathrm{Ro}=\mathrm{Ro}_c=0$. The PIV data taken at a constant shear Reynolds number of $\mathrm{Re}_S=1.4\times 10^4$ are plotted with (○). The torque data with varying Ro at constant shear for various Re_S ranging from $\mathrm{Re}_S=3\times 10^3$ to $\mathrm{Re}_S=4.7\times 10^4$ are plotted as gray (blue online) lines. They are discussed in Fig. 8. We also plot the states identified at much lower Re_S by Andereck *et al.* (Ref. 9) as patches: black corresponds to laminar Couette flow, gray (green online) to "spiral turbulence," dotted zone to "featureless turbulence," and vertical stripes to an "unexplored" zone.

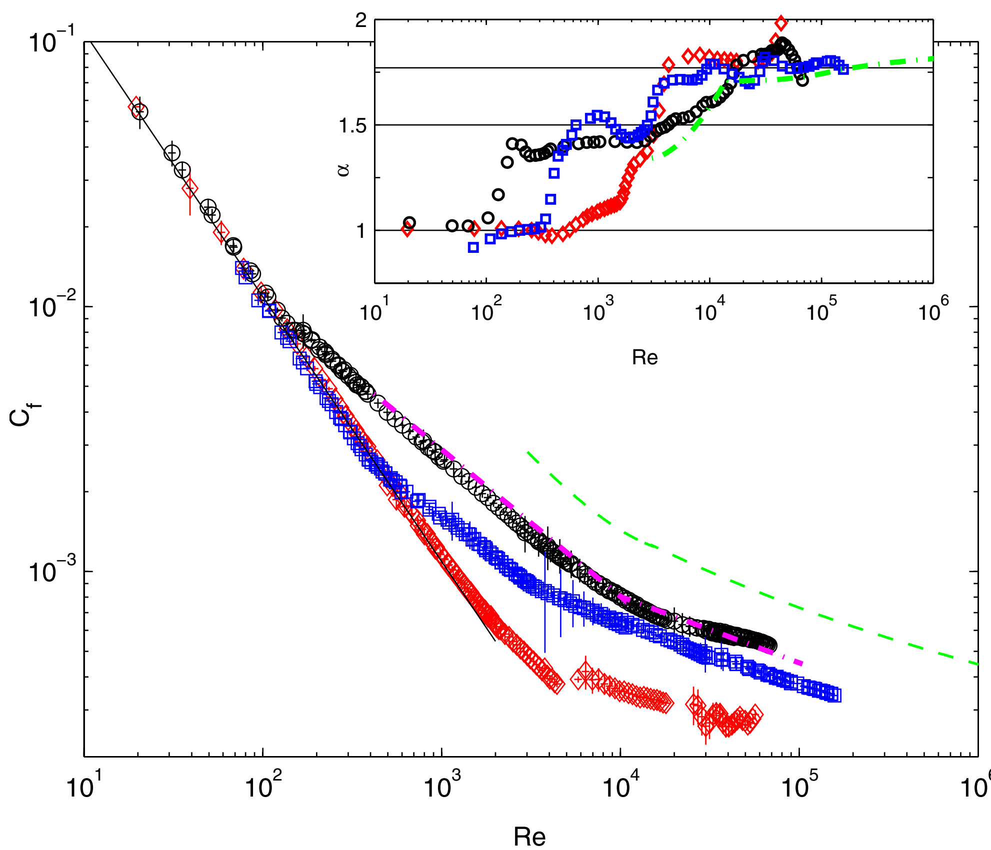

FIG. 5. (Color online) Friction factor c_f vs Re_S for $\mathrm{Ro}_i=\eta-1$ (black ○), $\mathrm{Ro}_c=0$ (blue □), and $\mathrm{Ro}_o=(1-\eta)/\eta$ (red ◇). Relative error on Re_S: ±5%; absolute error on torque: ±0.01 N m. Dashed (green online) line: Lewis' data [Ref. 13, Eq. (3)], for Ro_i and $\eta=0.724$. Dash-dotted (magenta online) line: Racina's data [Ref. 22, Eq. (10)]. Solid thin black line: laminar friction factor $c_f=1/(\eta\,\mathrm{Re})$. Inset: local exponent α such that $C_f\propto \mathrm{Re}_S^{\alpha-2}$, computed as $2+d\log(C_f)/d\log(\mathrm{Re}_S)$, for $\mathrm{Ro}_i=\eta-1$ (black ○), $\mathrm{Ro}_c=0$ (blue □), and $\mathrm{Ro}_o=(1-\eta)/\eta$ (red ◇). Dashed (green online) line: Lewis' data [Ref. 13, Eq. (3)] for Ro_i and $\eta=0.724$.

For $\mathrm{Ro}_i=\eta-1$, one can notice a transition to a different regime at $\mathrm{Re}_{ci}\simeq 140$ [the theoretical threshold is computed as $\mathrm{Re}=150$ (Ref. 23)]. This corresponds to the linear instability of the basic flow, leading in this case to the growth of laminar Taylor vortices. The friction factor is then supposed to scale as $c_f \propto \mathrm{Re}^{-1/2}$ ($\alpha=3/2$), which is the case here (see inset in Fig. 5). For exact counter-rotation ($\mathrm{Ro}_c=0$), the first instability threshold is $\mathrm{Re}_{cc}\simeq 400$. This is somewhat lower than the theoretical prediction $\mathrm{Re}_{cc}=515$,[23] which is probably due to our finite aspect ratio. Finally, the Taylor–Couette flow with only the outer cylinder rotating $[\mathrm{Ro}_o=(1-\eta)/\eta]$ is linearly stable whatever Re. We observe the experimental flow to be still laminar up to high Re; then, in a rather short range of Re numbers, the flow transits to a turbulent state at $4000 \lesssim \mathrm{Re}_{to} \simeq 5000$.

Further increase in the shear Reynolds number also increases the local exponent (see inset in Fig. 5). For $\mathrm{Ro}_i=\eta-1$, it gradually rises from $\alpha\simeq 1.5$ at $\mathrm{Re}\simeq 200$ to $\alpha\simeq 1.8$ at $\mathrm{Re}\simeq 10^5$. The order of magnitude of these values agrees with the results of Lewis *et al.*,[13] although a direct comparison is difficult, owing to the different gap ratios of the experiments. The local exponent is supposed to approach a value of 2 for increasing gap ratio. Dubrulle and Hersant[10] attribute the increase in α to logarithmic corrections, whereas Eckhardt *et al.*[11] attribute the increase in α to a balance between a boundary-layer/hairpin contribution (scaling as $\propto \mathrm{Re}^{3/2}$) and a bulk contribution (scaling as $\propto \mathrm{Re}^2$). The case of perfect counter-rotation shows a plateau at $\alpha\simeq 1.5$ and a sharp increase in the local exponent to $\alpha\simeq 1.75$ at $\mathrm{Re}_{tc}\simeq 3200$, possibly tracing back to a secondary transition. The local exponent then seems to increase gradually. Finally, for outer cylinder rotating alone (Ro_o) the transition is very sharp and the local exponent is already around $\alpha=1.77$ at $\mathrm{Re}\gtrsim 5000$. Note that the dimensional values of the torque at Ro_o are very small and difficult to measure accurately, and that these may become smaller than the contributions by the two Kármán layers (end effects) that we simply take into account by dividing by 2, as described in Sec. II. One can finally notice that at the same shear Reynolds number, for $\mathrm{Re}\geq 10^4$ the local exponents for the three rotation numbers are equal within ± 0.1 and that the torque with the inner cylinder rotating only is greater than the torque in counter-rotation, the latter being greater than the torque for only the outer cylinder rotating.

B. Velocity profiles at a high shear-Reynolds number

The presence of vortexlike structures at high shear-Reynolds number ($\mathrm{Re}_S \gtrsim 10^4$) in turbulent Taylor–Couette flow with the inner cylinder rotating alone is confirmed in our experiment through stereoscopic PIV measurements.[24] As shown in Fig. 6, the time-averaged flow shows a strong secondary mean flow in the form of counter-rotating vortices, and their role in advecting angular momentum (as visible in the coloring by the azimuthal velocity) is clearly visible as well. The azimuthal velocity profile averaged over both time and axial position w, as shown in Fig. 7, is almost flat, indicating that the transport of angular momentum is due mainly to the time-average coherent structures rather than by the correlated fluctuations as in regular shear flow.

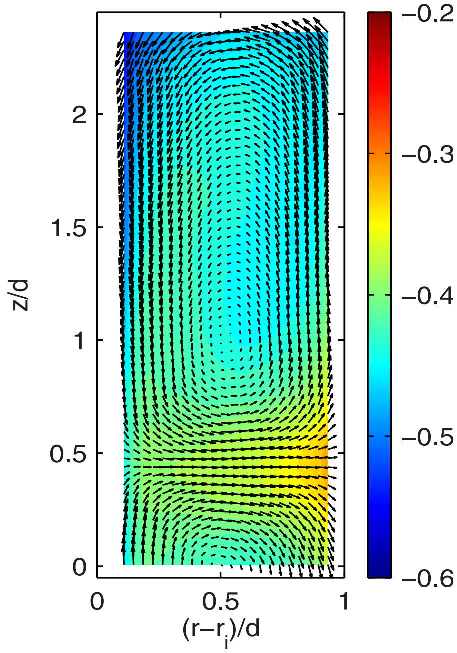

FIG. 6. (Color online) Secondary flow for $\mathrm{Ro}=\mathrm{Ro}_i$ at $\mathrm{Re}=1.4\times 10^4$. Arrows indicate radial and axial velocities; color indicates azimuthal velocity (normalized to inner wall velocity).

We then measured the counter-rotating flow at the same Re_S. The measurements are triggered on the outer cylinder position and are averaged over 500 images. In the counter-rotating case, for this large gap ratio and at this value of the shear-Reynolds number, the instantaneous velocity field is really disorganized and does not contain obvious structures like Taylor vortices, in contrast with other situations.[25] No peaks are present in the time spectra, and there is no axial dependency of the time-averaged velocity field. We thus average in the axial direction the different radial profiles; the

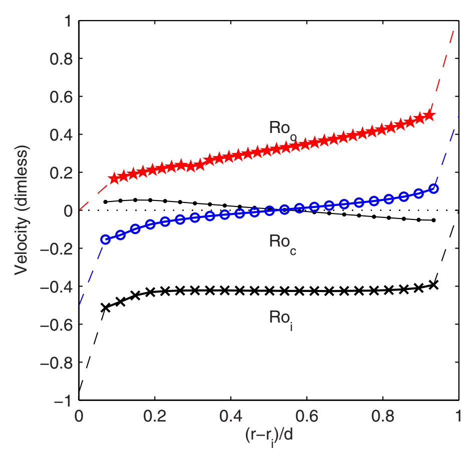

FIG. 7. (Color online) Profiles of the mean azimuthal velocity component for three Rotation numbers corresponding to only the inner cylinder rotating (×, black), perfect counter-rotation (○, blue), and only the outer cylinder rotating (⋆, red) at $\mathrm{Re}=1.4\times 10^4$. Thin line (·, black): axial velocity v (for $\mathrm{Ro}=\mathrm{Ro}_i$) averaged over half a period. The velocities are presented in a dimensionless form: $w/(Sd)$ with $Sd=2r_i(\omega_o-\omega_i)/(1+\eta)$.

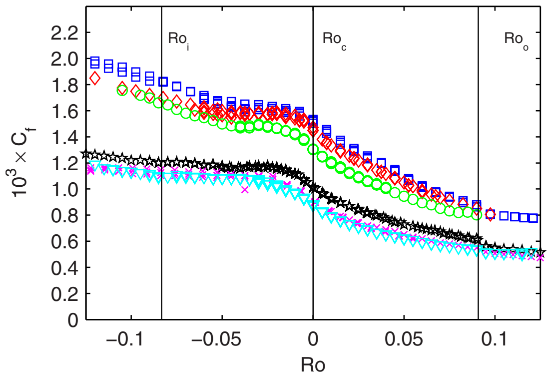

FIG. 8. (Color online) The friction factor c_f as a function of Ro at various constant shear Reynolds numbers: (blue □) $\mathrm{Re}=1.1\times 10^4$, (red ◇) $\mathrm{Re}=1.4\times 10^4$, (green ○) $\mathrm{Re}=1.7\times 10^4$, (black ★) $\mathrm{Re}=2.9\times 10^4$, (magenta ×) $\mathrm{Re}=3.6\times 10^4$, (cyan ▽) $\mathrm{Re}=4.7\times 10^4$.

azimuthal component w is presented in Fig. 7 as well. In the bulk, it is low, i.e., its magnitude is below 0.1 between $0.15 \lesssim (r-r_i)/d \lesssim 0.85$, that is 75% of the gap width. The two other components are zero within 0.002.

We finally address the outer cylinder rotating alone again at the same Re_S. These measurements are done much in the same way as the counter-rotating ones, i.e., again the PIV system is triggered by the outer cylinder. As in the counter-rotating flow, this flow does not show any large scale structures. The gradient in the average azimuthal velocity, again shown in Fig. 7, is much steeper than in the counter-rotating case, which can be attributed to the much lower turbulence, as it also manifests itself in the low c_f value for Ro_o.

V. INFLUENCE OF ROTATION ON THE EMERGENCE AND STRUCTURE OF THE TURBULENT TAYLOR VORTICES

To characterize the transition between the three flow regimes, we first consider the global torque measurements. We plot in Fig. 8 the friction factor or dimensionless torque as a function of Rotation number Ro at six different shear Reynolds numbers Re_S, as indicated in Fig. 4. We show three series centered around $\mathrm{Re}_S=1.4\times 10^4$ and three around $\mathrm{Re}_S=3.8\times 10^4$. As already seen in Fig. 5, the friction factor reduces with increasing Re_S. More interesting is the behavior of c_f with Ro: the torque in counter-rotation (Ro_c) is approximately 80% of $c_f(\mathrm{Ro}_i)$, and the torque with outer cylinder rotating alone (Ro_o) is approximately 50% of $c_f(\mathrm{Ro}_i)$. These values compare well with the few available data compiled by Dubrulle *et al.*[20] The curve shows a plateau of constant torque especially at the larger Re_S from $\mathrm{Ro}=-0.2$, i.e., when both cylinders rotate in the same direction with the inner cylinder rotating faster than the outer cylinder, to $\mathrm{Ro}\simeq -0.035$, i.e., with a small amount of counter-rotation with the inner cylinder still rotating faster than the outer cylinder. The torque then monotonically decreases when increasing the angular speed of the outer cylinder, with an inflexion point close or equal to Ro_c. It is observed that the transition is continuous and smooth everywhere and without hysteresis.

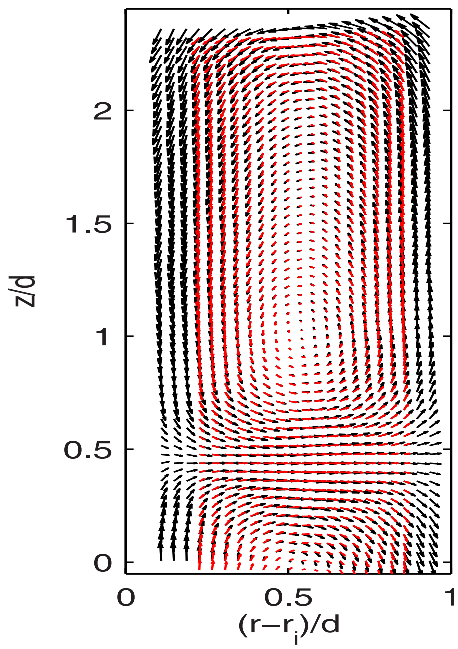

FIG. 9. (Color online) Overlay of measured time-average velocity field at $\mathrm{Ro}=\mathrm{Ro}_i$ [in gray (red online)] and the best-fit model velocity field (in black). The large third harmonic makes the radial flow being concentrated in narrow bands rather than sinusoidally distributed.

We now address the question of the transition between the different torque regimes by considering the changes observed in the mean flow. To extract quantitative data from the PIV measurements, we use the following model for the stream function Ψ of the secondary flow:

$$\Psi = \sin\left(\frac{\pi(r-r_i)}{d}\right) \times \left[A_1 \sin\left(\frac{\pi(z-z_0)}{\ell}\right) + A_3 \sin\left(\frac{3\pi(z-z_0)}{\ell}\right)\right], \quad (2)$$

with as free parameters A_1, A_3, ℓ and z_0. This model comprises a flow that fulfills the kinematic boundary condition at the inner and outer walls, r_i, r_i+d, and in between forms in the axial direction alternating rolls, with a roll height of ℓ. In this model, the maximum radial velocity is formed by the two amplitudes and given by $u_{r,\max}=(\partial\Psi/\partial z)_{\max}=\pi(A_1/\ell+3A_3/\ell)$. It is implicitly assumed that the flow is developed sufficiently to restore the axisymmetry, which is checked *a posteriori*. Our fitting model comprises a sinusoidal (fundamental) mode and its first symmetric harmonic (third mode), the latter which appears to considerably improve the matching between the model and the actual average velocity fields, especially close to Ro_i (see Figs. 9 and 10).

We first discuss the case $\mathrm{Ro}=\mathrm{Ro}_i$. A sequence of 4000 PIV images at a data rate of 3.7 Hz is taken, and 20 consecutive PIV images, i.e., approximately 11 cylinder revolutions, are sufficient to obtain a reliable estimate of the mean flow.[17] It is known that for the first transition the observed flow state can depend on the initial conditions.[8] When starting the inner cylinder from rest and accelerating it to 2 Hz in 20 s, the vortices grow very fast, reach a value with a velocity amplitude of 0.08 ms^{-1}, and then decay to become stabilized at a

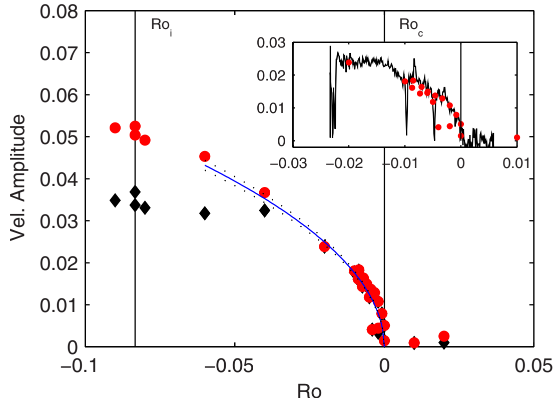

FIG. 10. (Color online) Secondary flow amplitude vs rotation number (Ro) at constant shear rate. Black (◇): model with fundamental mode only; red (○): complete model with third harmonic. Solid line is a fit of the form $A=a(-\mathrm{Ro})^{1/2}$. Inset: zoom close to counter-rotation combined with results from a continuous transient experiment (see text).

value around 0.074 ms^{-1} after 400 s. Transients are thus also very long in turbulent Taylor-vortex flows. For slower acceleration, the vortices that appear first are much weaker and have a larger length scale before reaching the same final state. The final length scale ℓ of the vortices for Ro_i is about 1.2 times the gap width, consistent with the data from Bilson *et al.*[16]

In a subsequent measurement we start from $\mathrm{Ro}=\mathrm{Ro}_i$ and vary the rotation number in small increments, while maintaining a constant shear rate. We allow the system to spend 20 min in each state before acquiring PIV data. We verify that the fit parameters are stationary and compute them using the average of the full PIV data set at each Ro. The results are plotted in Fig. 10. Please note that Ro has been varied both with increasing and decreasing values to check for a possible hysteresis. All points fall on a single curve; the transition is smooth and without hysteresis. For $\mathrm{Ro}\geq 0$, the fitted modes have zero or negligible amplitudes, since there are no structures in the time-average field.[24] One can notice that as soon as $\mathrm{Ro}<0$, i.e., as soon as the inner cylinder wall starts to rotate faster than the outer cylinder wall, vortices begin to grow. We plot in Fig. 10 the velocity amplitude associated with the simple model (single mode, ◇) and with the complete model (modes 1 and 3, ○). Close to $\mathrm{Ro}=0$, the two models coincide: $A_3\simeq 0$ and the mean secondary flow is well described by pure sinusoidal structures. For $\mathrm{Ro}\lesssim -0.04$, the vortices start to have elongated shapes with large cores and small regions of large radial motions in between adjacent vortices; the third mode is then necessary to adequately describe the secondary flow (Fig. 9). The first mode becomes saturated (i.e., it does not grow in magnitude) in this region. Finally, we give in Fig. 10 a fit of the amplitudes close to $\mathrm{Ro}=0$ of the form $A=a(-\mathrm{Ro})^{1/2}$. The velocity amplitude of the vortex behaves like the square root of the distance to $\mathrm{Ro}=0$, a situation reminiscent to a classical supercritical bifurcation, with A as order parameter and Ro as control parameter.

We also performed a *continuous transient* experiment, in which we varied the rotation number quasistatically from $\mathrm{Ro}=0.004$ to $\mathrm{Ro}=-0.0250$ in 3000 s, always keeping the Reynolds number constant at $\mathrm{Re}_S=1.4\times 10^4$. The amplitude of the mean secondary flow, computed on sequences of 20 images, is plotted in the inset of Fig. 10. The curve follows the static experiments (given by the single points), but some downward peaks can be noticed. We checked that these are not the result of a fitting error, and indeed correspond to the occasional disappearance of the vortices. Still, the measurements are done at a fixed position in space. Although the very long time-averaged series leads to well-established stationary axisymmetric states, it is possible that the instantaneous whole flow consists of different regions. Further investigation including time-resolved single-point measurements or flow visualizations needs to be done to verify this possibility.

VI. CONCLUSION

The net system rotation, as expressed in the Rotation number Ro, obviously has strong effects on the torque scaling. Whereas the local exponent evolves in a smooth way for inner cylinder rotating alone, the counter-rotating case exhibits two sharp transitions from $\alpha=1$ to $\alpha\simeq 1.5$ and then to $\alpha\simeq 1.75$. We also notice that the second transition for counter-rotation Re_{tc} is close to the threshold Re_{to} of turbulence onset for outer cylinder rotating alone.

The rotation number Ro is thus a secondary control parameter. It is very tempting to use the classical formalism of bifurcations and instabilities to study the transition between featureless turbulence and turbulent Taylor-vortex flow at constant Re_S, which seems to be supercritical; the threshold for the onset of coherent structures in the mean flow is Ro_c. For anticyclonic flows ($\mathrm{Ro}<0$), the transport is dominated by large scale coherent structures, whereas for cyclonic flows ($\mathrm{Ro}>0$), it is dominated by correlated fluctuations reminiscent to those in-plane Couette flow.

In a considerable range of Re_S, counter-rotation (Ro_c) is also close or equal to an inflexion point in the torque curve; this may be related to the crossover point, where the role of the correlated fluctuations is taken over by the large scale vortical structures. The mean azimuthal velocity profiles show that there is only a marginal viscous contribution for $\mathrm{Ro}\leq 0$ but of order 10% at Ro_o.[24] The role of turbulent versus large-scale transport (of angular momentum) should be further investigated from (existing) numerical or PIV velocity data. Since torque scaling with Ro as measured at much higher Re_S that is used for PIV does qualitatively not change, these measurements suggest that the large scale vortices are not only persistent in the flow at higher Re_S, but that they also dominate the dynamics of the flow. An answer to the persistence may be obtained from either a more detailed analysis of instantaneous velocity data or from torque scaling measurements at still higher Reynolds numbers in Taylor–Couette systems such as those under development.[18]

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