Stochastic Non Destructive Testing simulation: sensitivity analysis applied to material properties in clogging of nuclear power plant steam generators

O. Moreau[1,2], K. Beddek[1,2], S. Clénet[1,4] and Y. Le Menach[1,3]

[1]Laboratoire d'Analyse des Matériels Electriques (LAMEL)
[2]Electricité de France, 92141, Clamart, France
[3]L2EP, Université Lille1, 59655 Villeneuve d'Ascq, France
[4]L2EP, Arts et Métiers ParisTech, 8 Bd Louis XIV, 59046 Lille cedex

A Non destructive Testing (NDT) procedure is currently used to estimate the clogging of tube support plates in French nuclear power plant steam generators. A stochastic approach has been applied to Finite Element electromagnetic field simulation to evaluate the impact of material properties uncertainties on the monitoring signal. The Polynomial Chaos Expansion method makes it possible to easily derive the Sobol decomposition which measures how much the variability of each input parameter affects the model output

***Index Terms*—Stochastic Model, Sensitivity Analysis, Non Destructive Testing, Eddy Current, Finite Element Method.**

I. Introduction

Differences between experimental measurements, assumed reliable, and simulation results may have three origins:

- The mathematical model is not consistent with the investigated physics (wrong hypothesis)
- The numerical model fails (convergence and stability problems of the numerical schemes, discretisation error, ineffective resolution algorithms)
- Input data have not been properly chosen (intrinsically variable or badly known)

Nowadays, the predictive reliability of numerical models is limited by the relevancy of their input data. Unfortunately, geometry, material properties and sources would rather present uncertainties. Under those conditions, the output data (magnetic field distributions, global quantities like torque, flux, current…) become also uncertain. To improve model prediction, such input data have to be considered no longer as deterministic parameters but as random variables to describe intrinsic variability, manufacturing tolerance or aging effect. Uncertainty quantification approaches make it possible to quantify the variability of quantities of interest, depending on the input parameters variability.

The probabilistic methods based on polynomial chaos expansion come out as efficient methods enabling to derive the probability density function (PDF) of random output [1,2] which can stand for a response surface or a metamodel.

In addition, this method can easily provide a sensitivity analysis which measures how much the variability of each input parameter affect the model output. That makes it possible to focus, during the input data collecting step, on the properties which have to be accurately known with regard to the requested output.

A Non destructive Testing (NDT) procedure is currently used to estimate the clogging of tube support plates in French nuclear power plant steam generators. A stochastic approach has been applied to Finite Element electromagnetic field simulation to evaluate the impact of material properties uncertainties on the monitoring signal.

II. Probabilistic model

Let D be a spatial domain on which the permeability and conductivity are assumed to be random fields and denoted respectively $\mu(\mathbf{x},\theta)$ and $\sigma(\mathbf{x},\theta)$, where $\mathbf{x}$ denotes the spatial variable and θ the outcome belonging to the event space Θ. Therefore, the magnetic field $\mathbf{H}$, the magnetic flux density $\mathbf{B}$, the current source density $\mathbf{J_0}$ and the electric field $\mathbf{E}$ verify the Maxwell equations in the frequency domain, which can be written in quasi static approximation:

$$\begin{cases} \mathbf{curlH}(\mathbf{x},\theta) = \sigma(\mathbf{x},\theta)\mathbf{E}(\mathbf{x},\theta) + \mathbf{J}_0(\mathbf{x}) \\ \mathbf{curlE}(\mathbf{x},\theta) = -j\omega\mathbf{B}(\mathbf{x},\theta) \end{cases} \tag{1}$$

where ω is the angular velocity. We assume that some boundary conditions on $\mathbf{H}$ and $\mathbf{E}$ are prescribed in order to impose the uniqueness of the solution. Introducing the magnetic vector potential $\mathbf{A}(\mathbf{x},\theta)$ and the electric scalar potential $\varphi(\mathbf{x},\theta)$, the stochastic magneto-harmonic problem (1) can be rewritten :

$$\begin{aligned} &\mathbf{curl}\left(\mu^{-1}(\mathbf{x},\theta)\mathbf{curlA}(\mathbf{x},\theta)\right) \\ &+\sigma(\mathbf{x},\theta)\left(j\omega\mathbf{A}(\mathbf{x},\theta) + \mathbf{grad}\varphi(\mathbf{x},\theta)\right) = \mathbf{J}_0(\mathbf{x}) \end{aligned} \tag{2}$$

Applying the classical deterministic finite element method on the spatial dimension of the problem, the equation leads to solve, for each $\theta\in\Theta$ the linear system:

$$A(\theta)\,X(\theta) = B(\theta) \quad \forall\theta\in\Theta \tag{3}$$

We denote by N the total number of unknowns on the spatial dimension. Therefore, the matrix $A(\theta)$ is a $N\times N$ matrix with random coefficients. The linear system (3) can be solved, for instance, by a Monte Carlo Simulation Method to estimate some moments of the random solutions $\mathbf{A}(\mathbf{x},\theta)$ and $\varphi(\mathbf{x},\theta)$. Other techniques than sampling methods can also be used to solve numerically the problem (3), like those based on the polynomial chaos expansion.

1) Polynomial chaos expansion

Let consider the solution $X(\theta)$ of a stochastic model, having as input parameters the vector $\xi(\theta)=(\xi_1(\theta),\ldots,\xi_M(\theta))$ of M independent stochastic variables $\xi_i(\theta)$ with probability density function $f_{\xi i}$. It can be shown that if $X(\theta)$ has a finite variance then $X(\theta)$ can be written as a linear combination of multivariate polynomials $\Psi_\alpha(\xi(\theta))$:

$$X(\theta)=\sum_{\alpha\in\mathbb{N}^M} x_\alpha \Psi_\alpha\left(\xi(\theta)\right), \qquad (4)$$

where α is a M-tuple $(\alpha_1,..,\alpha_M)$ containing the order of the orthogonal univariate polynomials $\psi_{\alpha(i)}(\xi_i(\theta))$ with respect to the probability measure $f_{\xi i}$. The multivariate polynomials Ψ_α are orthogonal with respect to the joined probability measure:

$$f_\xi=\prod_{1\le i\le M} f_{\xi_i} \qquad (5)$$

that is to say:

$$E\left[\Psi_\alpha \Psi_\beta\right]=\int_{R^M}\Psi_\alpha\left(\xi(\theta)\right)\Psi_\beta\left(\xi(\theta)\right) f_\xi d\xi=\delta_{\alpha\beta} \qquad (6)$$

with E[.] the expectation operator and $\delta_{\alpha\beta}$ is equal to 1 if $\alpha=\beta$ and 0 if else. In practice, (4) is truncated up to the polynomial of orders p. If we denote $Z(M,p)$ the space of the M-tuples α which satisfy :

$$\sum_{1\le i\le M}\alpha_i\le p \qquad (7)$$

the total number of polynomials in the PC basis is equal to:

$$P=\frac{(M+p)!}{M!\,p!} \qquad (8)$$

Finally, to determine the PCE expansion of the solution $X(\theta)$, we have to compute the coefficients x_α. To achieve that, two major types of approach are available: non intrusive methods like L^2-projection [2] and intrusive ones like SSFEM, which has been applied in the present work.

2) Spectral Stochastic Finite Element Method (SSFEM)

With the SSFEM, the problem (3) is solved by applying weighted residual technique and the Galerkin method. The weak form of (3) is given by:

$$E\left[A(\theta)X(\theta),v(\theta)\right]=E\left[B(\theta),v(\theta)\right] \qquad (9)$$

where $v(\theta)$ are functions with finite variance. We apply the Galerkin method by considering that the solution $X(\theta)$ under the form (4), and by taking as test function $v(\theta)$ the polynomials chaos $\Psi_\alpha(\theta)$. The stochastic magneto harmonic problem leads therefore to solve the following linear system:

$$A^s X=B^s \qquad (10)$$

The size of the square matrix A^s is equal to $PN\times PN$, with N the total number of spatial unknowns and P the number of unknowns in the random dimension. It can be shown [1],[9] that the matrix A^s has a Kronecker structure which we can be used to solve efficiently the problem. Finally, a global quantity of interest $Y(\theta)$ can then be easily derived from the approximated solution of the problem and can be written under the form:

$$Y(\theta)=Y\left(\xi(\theta)\right)=\sum_{\alpha\in Z(M,p)} y_\alpha\Psi_\alpha\left(\xi(\theta)\right) \qquad (11)$$

Both numerical methods (L^2-projection and Galerkin) lead to a solution under the same form (11) but not necessarily to the same results.

III. Sensitivity analysis

Sensitivity analysis can be deduced from the Sobol or ANOVA decomposition [8] of the stochastic output Y:

$$\begin{aligned}Y\left(\xi(\theta)\right)=&Y_0\left(\xi(\theta)\right)+\sum_{1\le i\le M}Y_i\left(\xi_i(\theta)\right)\\&+\sum_{1\le i\le j\le M}Y_{ij}\left(\xi_i(\theta),\xi_j(\theta)\right)+\ldots+Y_{1\ldots M}\left(\xi(\theta)\right)\end{aligned} \qquad (12)$$

where : $Y_0\left(\xi(\theta)\right)=E\left[Y\left(\xi(\theta)\right)\right]$

The decomposition (12) is unique if the terms of the decomposition are orthogonal "term by term":

$$E[Y_\Omega(\theta)Y_\Sigma(\theta)]=0 \quad \text{if} \quad \Omega\ne\Sigma \qquad (13)$$

with Ω and Σ sets belonging to $\{1,\ldots,M\}$.

Due to the independence of the input random variables $\xi_1(\theta),\ldots,\xi_M(\theta)$, one can show that the variance V of Y can be written as:

$$V=E\left[Y(\theta)^2\right]=\sum_{1\le i\le M}V_i+\sum_{1\le i\le j\le M}V_{ij}+\ldots+V_{1\ldots M} \qquad (14)$$

where $V_{i\ldots j}$, $1\le i,j\le M$, are called partial or conditional variances and defined as (θ omitted hereafter):

$$V_{i\ldots j}=\int\ldots\int Y^2_{i\ldots j}\left(\xi_i,\ldots,\xi_j\right)d\xi_i\ldots d\xi_j \qquad (15)$$

The term $V_{i\ldots j}$ expresses the joint contribution of the random variables $\xi_i,\ldots,\xi_j$ to the total variability of the model. Due to the orthogonality of the decomposition (12), the total variance V is the sum of the partial variances of $Y(\xi)$. The Sobol indices are defined by:

$$S_{i\ldots j}=\frac{V_{i\ldots j}}{V} \qquad (16)$$

If $Y(\xi)$ is expressed as (11), the decomposition (14) becomes:

$$\begin{aligned}Y(\xi)=&Y_0(\xi)+\sum_{1\le i\le M}\sum_{\alpha\in Z(1,p)}y_\alpha\Psi_\alpha(\xi_i)+\\&\sum_{1\le i\le j\le M}\sum_{\alpha\in Z(2,p)}y_\alpha\Psi_\alpha\left(\xi_{\alpha(i)},\xi_{\alpha(j)}\right)+\ldots\end{aligned} \qquad (17)$$

The Sobol indices $S_{i,\ldots j}$ are then given by:

$$S_{i\ldots j}=\frac{1}{V}\sum_{\alpha\in Z^*(M,p)}y_\alpha^2 \qquad (18)$$

with $Z^*(M,p)$, the space of the M-tuple which verify:

$$\sum_{k=1}^{M}\alpha_k-\sum_{k=i}^{j}\alpha_k=0 \qquad (19)$$

IV. Industrial Background

The deposit of corrosion products in the foils of the tube support plates (TSP) in steam generators (SG) of nuclear power plants (Fig. 1) raises a safety concern and affects its operating conditions [3].

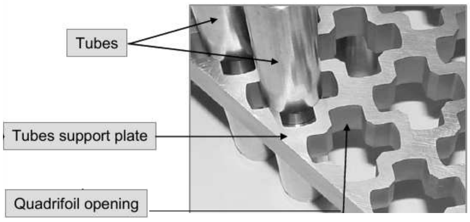

Fig. 1. Tube support plate (TSP) in nuclear power plant steam generator

To some extent, this phenomenon, called clogging (Fig. 2), may significantly affect the water, temperature distribution and steam circulation inside the SG. The resulting pressure drop can then cause flow-induced vibration instabilities leading to tube cracking risks. Clogging also compromises the thermal exchange efficiency between the primary and secondary circuits.

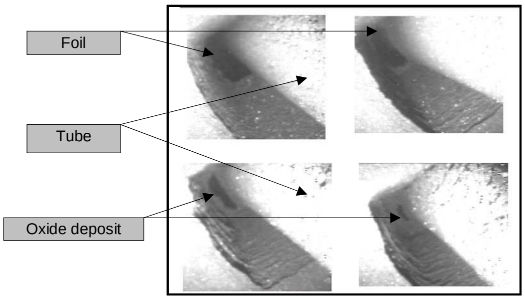

Fig. 2. Oxide buildup from almost completely open (upper left) to almost completely blocked (lower right).

For safety and optimum operating conditions, the oxide deposit build up has to be precisely evaluated to eventually :

- trigger a power reduction (preventive action)
- carry out a proper chemical cleaning (curative action)
- check the remedy efficiency

Therefore, in addition to the available techniques (video examination, load-loss based methods), EDF has developed a dedicated Eddy current NDT technique to evaluate the amount of clogging [2]. The R&D program intends to improve our knowledge of the sensitivity of this method with regard to:

- The amount of deposit for actual topologies of clogging
- Its ability to discriminate different shapes of deposit (layer on the tube outer walls, flakes in the foils, clogging at the TSP lower edge)
- The material properties of the deposit

V. Simulation Results

The eddy current inspection technique, called the SAX ratio, is based on detecting deposits in foils thanks to a NDT technique [3] usually used to assess the tube bundle integrity.

The principle consists (Fig. 3) in correlating the amount of deposit at the inlet foil to the variation, at each edge of the TSP, of the bobbin coils flux difference $\Delta\Phi$ of an axial probe, supplied at 100 kHz (20).

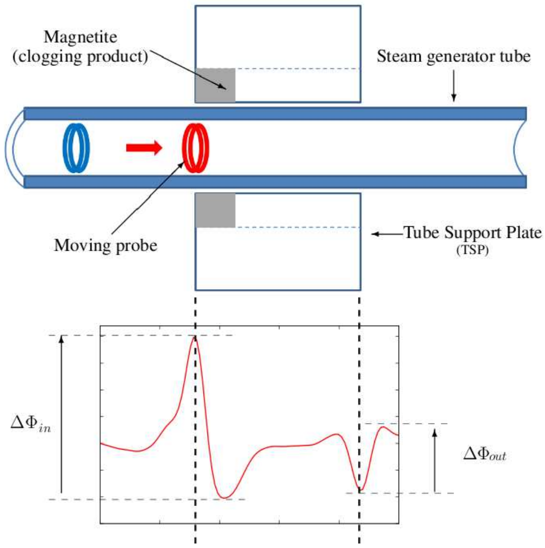

Fig. 3. Sax ratio principle

$$Ratio_{SAX} = \frac{\Delta\Phi_{in} - \Delta\Phi_{out}}{\Delta\Phi_{out}} \tag{20}$$

This method has been perfected on an experimental mock-up for canonical clogging configurations. As we intend to tackle realistic deposit topologies by numerical simulations, the first step was to validate the modelling approach with those experimental measurements [4]. The Finite Element electromagnetic field computation software Code_Carmel3D already in use at EDF in the qualification process for NDT eddy currents procedure [5] was chosen for the simulation program. The present work deals with the sensitivity analysis of the SAX ratio with regard to the material properties of the deposit. The first step to achieve that was to carry out stochastic simulations to compute the PDF of the SAX ratio due to the uncertainties of the conductivity and permeability of both the magnetite and the TSP. According to “expert saying” and experimental measurements, these properties have been chosen as random variables with uniform laws (see Table I).

TABLE I: Variability of the random parameters considered in the probabilistic model

	Relative Permeability	Conductivity (S.m)
Magnetite	U[1.3 ; 2.7]	U[45 ; 75]
Tube Support Plate	U[60 ; 100]	U[1710^5 ; 1810^5]

The material properties of the TSP and the probe operating frequency lead to a very thin skin depth in the TSP with regard to the geometrical dimensions of the device. To properly take into account the skin effect, TSP has been meshed with Surface Impedance Boundary Condition (SIBC) [6] enabling contact with conductive media like the magnetite.

The Whitney elements of the spatial mesh (Fig. 4) lead to 1.789.946 spatial unknowns whereas the Legendre polynomial chaos of order 4 has been chosen for the random dimension. Partial variances of $\Delta\Phi(\theta)$ and SAX ratio have been computed as a PCE form for each position of the probe in the tube.

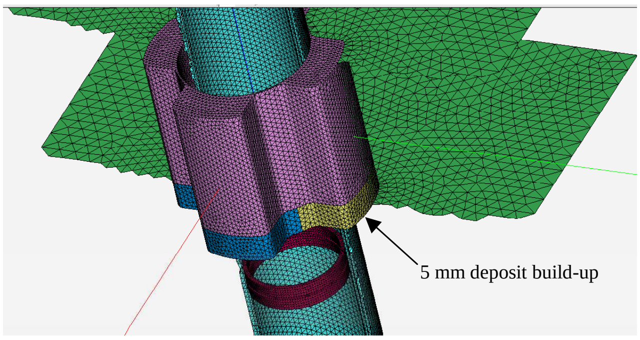

Fig. 4 – Finite element model of the NDT problem

Figure 5 (respectively 6) shows the partial variances of the real (respectively imaginary) part of the $\Delta\Phi(\theta)$ for each position of the probe with regard to each random variable. It can be noticed that partial variances for σ^m and σ^p are negligible (almost equal to zero) with regard to μ^m and μ^p.

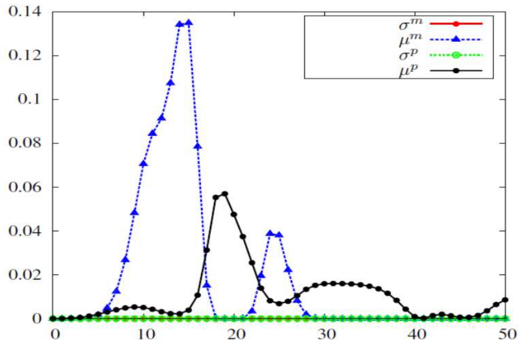

Fig. 5 - Partial variances of the real part of $\Delta\Phi(\theta)$ with regard to the position of the probe inside the tube

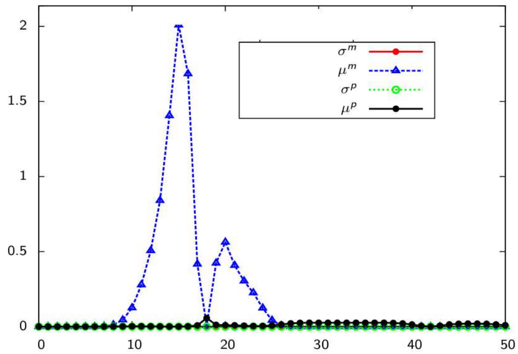

Fig. 6 - Partial variances of the imaginary part of $\Delta\Phi(\theta)$ with regard to the position of the probe inside the tube

In addition, Sobol indices of the SAX ratio are given in Table II for $\Delta\Phi(\theta)$ computed at position 15 and 35 which stand respectively for the lower side and the upper side of the tube support plate. It finally turns out that the permeability of the magnetite has the biggest influence on the SAX ratio especially its imaginary part. Moreover, we can notice that imaginary parts of $\Delta\Phi(\theta)$ and SAX ratio are almost not sensitive to the permeability of the TSP (Fig. 6 and Table II). Figure 7 shows the probability density function of the imaginary part of the Sax ratio. It is worth noticing that according to the variability of material properties, badly chosen threshold values for the SAX ratio can prevent from detecting the clogging.

TABLE II: SOBOL INDICES (IN %) OF THE SAX RATIO.

Sobol indices	σ^m	μ^m	σ^p	μ^p
Real(Sax ratio)	0.01	67.1	0.08	32.8
Imag(Sax ratio)	0.003	99.7	0.002	0.025

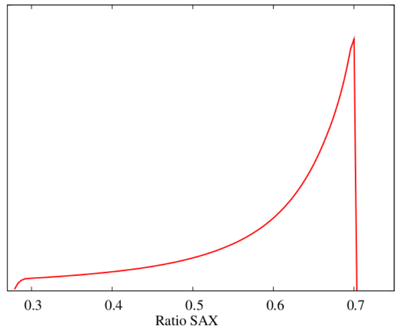

Fig. 7 – Probability Density Function of the imaginary part of the SAX ratio

VI. CONCLUSION

The Polynomial Chaos Expansion has been successfully applied to Finite Element electromagnetic field simulation to evaluate the impact of material properties uncertainties on the monitoring signal of steam generators clogging in nuclear power plant. Such approach makes it possible to derive an efficient sensitivity analysis, with regards to each random input data, for each global quantities of interest for each probe position. Generally speaking, such methods enable, on the one hand, to focus on the properties which have to be accurately known with regard to the requested output and, on the other hand, to select the output which is the less dependent on the other parameters.

VII. REFERENCES

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