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Study of actuation and energy reduction with an anthropomorphic knee on biped robot

Mathieu Hobon\textsuperscript{a}, Nafissa Lakbakbi Elyaaqoubi\textsuperscript{b}, Gabriel Abba\textsuperscript{b}

\textsuperscript{a}. Laboratoire de Conception Fabrication Commande, Ecole Nationale Supérieure d'Arts et Métiers CER Metz, 4 rue Augustin Fresnel, 57078, Metz
\textsuperscript{b}. Laboratoire de Conception Fabrication Commande, Ecole Nationale d'Ingénieur de Metz, 1 route d'Ars Laquenexy, 57078, Metz

Résumé :
Le genou des robots bipèdes possède habituellement un seul degré de liberté qui est une simple liaison rotoïde. Ce travail se concentre sur l'étude d'une cinématique de genou à contact roulant. Le genou est composé d'un fémur ayant une extrémité cylindrique, roulant sur une autre surface cylindrique représentant le tibia. Des études ont montré que ce type de structure permet de réduire les couples articulaires pendant la marche. Des séries d'optimisation énergétique de la marche ont été effectuées pour cette structure et pour différents rayons de cylindre. Les trajectoires articulaires générant la marche sont des fonctions splines cubiques. L'allure de marche est composée de phase de simple support suivi d'un impact. La minimisation du critère énergétique est réalisée à l'aide du Simplex. Les résultats montrent que des rayons élevés au niveau des tibias permettent de réduire l'énergie pendant la marche. L'étude de nouvelles surfaces de contact est une piste pour diminuer la consommation énergétique.

Abstract :
The knee of biped robots has usually one degree of freedom which one is a revolute joint. This work focuses on the study of rolling contact knee. The knee is composed of a cylinder in the extremity of the femur, rolling on another cylindrical surface which is the tibia. Studies have shown that this structure allows to reduce the torque during the gait. Energetic optimization series were made with this structure and for different cylinder radii. The angular trajectories describing the gait are generated by cubic splines functions. The gait is composed of single support phase followed by an impact. The minimization of energetic criterion is realized using the Simplex algorithm of Nelder-Mead. The results show that higher radii of the tibia permit to reduce energy consumption during the gait. The study of new contact surfaces is a way to decrease the energy consumption.

Mots clefs : Mechanical Design; Biped Robot; Gait Optimization

1 Introduction
The design of humanoid robot is a difficult problem. For many biped realized by researchers, the kinematic design is closed to the kinematic of human for the hip and the ankle, but is far from that of the human knee [1]. This article introduce a solution of rolling knee and their influence on the energy consumption.

Bio-mechanical studies relate the shape of the femur and tibia. [4] identifies the geometric shape and the movement of the knee. This shape identification was made on six cadaver knees with the MRI. The femur is represented by two parallel cylinder rolling on a flat surface of tibia. [5] presents a kinematic and dynamic analysis of the knee joint. Two dynamical studies were compared to the human knee flexion which one on the model proposed by [4] and a second on a solution proposing two elliptical surfaces rolling on contact. The radii of the circles are between 20 \textit{mm} and 35 \textit{mm}. The study gives results of the forces and moment applied on the knee in flexion.
In robotics, the knee is modelled by different structures. [2] proposes a four cross-bar knee and the results show a reduction of energy consumption and impact forces during the gait. [3] studies a rolling knee with two cylinders in contact and introduces a reduction of the torques, so a reduction of energy consumption during a cyclic gait composed of Single Support Phases (SSP) and an impact. This article is an extension of the work of [3].

The article is composed of a section on the kinematic and dynamic model of the biped with rolling knee (RK). In the section 2, the actuation control matrix function of the kinematic of the structure is discussed. Then, the optimization problem is described and the path generation is presented to calculate the energy consumption. Simulations were made on the HYDROID robot and the results are observed. Finally, the conclusions and future works are discussed.

2 Robot modelling

In this article, the mechanical design of the rolling knee is introduced. The geometric and dynamic models are presented. The detail of the actuation control matrix is analysed.

2.1 Biped robot models

The biped robot presented in the figure 1 is studied in the sagittal plane and makes a cyclic walk composed of Single Support Phases (SSP) followed by an impact. The robot contains seven bodies which are one trunk, two thighs, two legs and two feet. The 7-links robot is fully actuated on hips, knees and ankles by six actuators. The specificity of the knees is the movement of the cylindrical surface of femur rolling without sliding on the cylindrical surface of the tibia. The figures 2 show the geometrical description of the knee and also the torque applied at point C2 between the femur and the connecting rod C2C1. The revolute joint in CI is passive. The angle \( \gamma_1 \) is a coupling angle between \( q_1 \) and \( q_2 \). Its expression is:

\[
\gamma_1 = \frac{r_1 q_1 + r_2 q_2}{r_1 + r_2}
\]  

(1)

For the mobile knee, the expression of \( \gamma_2 \) is:

\[
\gamma_2 = \frac{r_1 q_4 + r_2 q_3}{r_1 + r_2}
\]  

(2)

The reference frame is \( \mathbb{R}_0 = (O_0, x_0, y_0, z_0) \). \( O_0 \) is defined by the projection of the point \( A_1 \) of the ankle axis on the ground. The direction of the walk is according to \( x_0 \). \( z_0 \) is the unit vector perpendicular to the ground. The radius of the tibia cylinder is the distance \( C_1K_1 \) called \( r_1 \) and the radius of the femur cylinder is the distance \( C_2K_1 \) called \( r_2 \) and \( K_1 \) is on the contact point. The biped shape is defined by the length of the tibia \( A_1K_1 = A_2K_2 = l_1 \), the length of the thigh \( K_1H = K_2H = l_2 \) and the length of the trunk \( HN = l_3 \). The dimension for the feet are resumed on the figure 1.

An explicite model is used to parametrize the biped robot and defined by the absolute angular coordinates \( q_i, \{i = 0 \ldots 6\} \) and the Cartesian position of the hip \((x_H, z_H)\) (see Fig.1). The geometric model is done for all the Cartesian positions of the biped. The Cartesian positions of the mobile foot are given by:

\[
x_{H_2} = x_H + l'_2 \sin q_3 + (r_1 + r_2) \sin \gamma_2 + l'_1 \sin q_4 - l_p \cos(q_5) + h_p \sin(q_5)
\]  

(3)

\[
z_{H_2} = z_H - l'_2 \cos q_3 - (r_1 + r_2) \cos \gamma_2 - l'_1 \cos q_4 - l_p \sin(q_5) - h_p \cos(q_5)
\]  

(4)

\[
x_{T_2} = x_H + l'_2 \sin q_3 + (r_1 + r_2) \sin \gamma_2 + l'_1 \sin q_4 - (l_p - L_p) \cos(q_5) + h_p \sin(q_5)
\]  

(5)

\[
z_{T_2} = z_H - l'_2 \cos q_3 - (r_1 + r_2) \cos \gamma_2 - l'_1 \cos q_4 - (l_p - L_p) \sin(q_5) - h_p \cos(q_5)
\]  

(6)

with \( l'_i = l_i - r_i \) for \( i = [1, 2] \).

The dynamic model is expressed to know the forces and the moments acting on the biped during the gait with one foot sole in contact with the ground. The Euler-Lagrange equations are used because all the contact phases can be modelled in the sagittal plane with this formalism. \( X \) is the state vector
composed of the absolute angular coordinates and of the Cartesian position of the hip. The size of this vector is \(9 \times 1\). The dynamic equation gives:

\[
D(X)\ddot{X} + H(\dot{X}, X) = B_{I}\Gamma + A_{c_1}(X)^{T}F_{1}
\]

(7)

with \(D(X)\) is the inertia matrix \(9 \times 9\), \(H(\dot{X}, X)\) is the vector of centrifugal, Coriolis and gravity effects \(9 \times 1\), \(B_{I}\) is the control matrix \(9 \times 6\), the torque vector is \(\Gamma = [\Gamma_1 \ldots \Gamma_6]^{T}\), \(A_{c_1}(X)\) is the Jacobian matrix \(3 \times 9\) of support foot constraint. \(F_{1}\) is the forces and moment applied on the ZMP point (see Fig. 1). It is composed of two forces \(F_{1x}\) and \(F_{1z}\) and one moment around the \(\vec{y}\) axis called \(C_y\). The joint torque vector \(\Gamma\) and is expressed by:

\[
\Gamma = \Gamma_m - F v \dot{\theta} - Cs \text{sign}(\dot{\theta})
\]

(8)

where \(\Gamma_m\) is the torque vector of actuators, \(\dot{\theta}\) is the joint speed vector expressed by \(\dot{\theta} = [\dot{q}_1 - \dot{q}_0, \dot{q}_2 - \dot{\gamma}_1, \dot{q}_6 - \dot{q}_2, \dot{q}_3 - \dot{\gamma}_2, \dot{q}_4 - \dot{\gamma}_3, \dot{q}_5 - \dot{\gamma}_4]^T\) and \(F v = \text{diag}(f v_1, f v_2, f v_3, f v_3, f v_2, f v_1)\) and \(Cs = \text{diag}(cs_1, cs_2, cs_3, cs_3, cs_2, cs_1)\) are the matrices of viscosity and Coulomb friction. All the robot actuator parameters are given in [3].

In addition to equation (7), the constraint of the contact between the foot sole and the ground can be two times derivated:

\[
A_{c_1}(X)^{T}\dddot{X} + H_{c_1}(X) = 0
\]

(9)
with the evolution of $\dot{X}$ satisfying eq. (9) during the gait.

## 2.2 Control matrix

The actuation matrix $B_\Gamma$ is calculated with the derivative of the articular position vector $\theta$ by the state vector $X$. The detail of this matrix is:

$$B_\Gamma = \begin{bmatrix} B_1 & 0_3 \\ 0_3 & B_1 \\ B_2 & B_3 \end{bmatrix}$$

(10)

with

$$B_1 = \begin{bmatrix} -1 & 0 & 0 \\ 1 - \frac{r_1}{r_1 + r_2} & 0 \\ 0 & \frac{r_1}{r_1 + r_2} & -1 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \times 3 \end{bmatrix}, B_3 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

(11)

In this matrix, the radii of the structure is equivalent to a transmission ratio. This matrix can allows to find the optimal placement of the actuator on the knees.

## 2.3 Impact model

During the gait, the feet touch the ground, alternatively the left foot and after the right foot. This is the impact phase. The impact phase separates two SSP. The impact phase between two rigid bodies, the foot and the ground, produces a mechanical energy dissipation phenomena [6]. We suppose that the restitution coefficient is equal to zero. This assumption ensures that we have no rebounds of the foot after the impact. The proposed model is given by:

$$D(X) \left( \dot{X}^+ - \dot{X}^- \right) = A_{cl}^T I_1$$

$$A_{cl}(X) \dot{X}^+ = 0$$

$$\Delta_i = D^{-1} A_{cl}^T I_1 = [\Delta_0, \Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_T]^T$$

(12)

(13)

(14)

This model is used to find the speed vector after the impact $\dot{X}^+$ from the configuration $X$ in relation with the speed vector before impact $\dot{X}^-$. $\Delta_i$ is the difference between the speed vector after and before the impact. It will be used to define the initial conditions of the trajectories in the following section. This model also gives the impact forces and torque $I_1 = [I_{F_1}, I_{F_2}, C_y]^T$ applied on the ankle.

## 3 Optimization and trajectories

The optimization process is based on a minimization of the energy criterion during one cyclic step. The energy criterion is an estimation of energy losses and its equation is:

$$C(p) = \frac{2}{d} \int_0^T k J \Gamma \Gamma + C_s |\dot{\theta}| + F \cdot \dot{\theta}^2 dt$$

(15)

with $\Gamma$ calculated from (7), $kJ$ represents the matrix of Joules quality factor and $|\dot{\theta}| = [\dot{\theta}_i]$ for $i = \{1 \ldots 6\}$, $d$ is the step length.

The optimization problem is a nonlinear minimizing problem under constraints and can be expressed:

$$\min_p C(p)$$

under $\Psi(p) \geq 0$

(16)

with $\Psi = [\Psi_1 \Psi_2 \Psi_3 \Psi_4 \Psi_5 \Psi_6 \Psi_7]^T$ are the constraints imposed by the initial configuration of the robot and the physical constraints. The constraints are:
- $\Psi_1 = F_{1z}$ define the vector of the force along the z-axis and must be positive,
- $\Psi_2 = x_{ZMP} + l_p$ and $\Psi_3 = -x_{ZMP} + (L_p - l_p)$ represent the limit position of the ZMP on x-axis (ZMP must stay in the foot support) to guaranty the stability of the robot with $x_{ZMP} = \frac{\Gamma_1 - h_p F_{1z} - m_p s_x g}{F_{1z}}$,
- $\Psi_4 = q_2 - q_1$ and $\Psi_5 = q_3 - q_4$ are the choice made to keep a gait human-like with no bend backward of knees,
- $\Psi_6 = z_{H_2}$ and $\Psi_7 = z_{T_2}$ are used to have the z-coordinates of heel and toes of the mobile foot above the ground during the SSP.

This problem is solved by using the Nelder-Mead simplex algorithm. This algorithm solves nonlinear problem without constraints. The constraints are also added in the criterion (15) as Lagrange multiplier. The equation becomes:

$$C_E(p) = C(p) + \frac{2}{d} \int_0^T \left( K \sum_{i=1}^7 \left( e(|\Psi_i - \Psi_i| - 1) \right) \right) dt + Err$$

(17)

with $\Psi_i$ represents the constraints already defined, $K$ is the multiplier of Lagrange equal to $10^6$ in our calculation and $Err$ can handle the errors due to the inverse geometric model (IGM). When a error of IGM is detected, $Err$ is activated and is equal to $10^6$. At the end of the optimization, we verify that all constraints are positive and $Err$ is equal to zero in all cases.

The trajectories $q_i$ used to make a step are cubic spline functions also used in [2], [3]. One trajectory is composed of two cubic spline functions during half period and the continuity between them is respected in angular position, velocity and acceleration. The velocity and acceleration are found by derivation. The parameter number is 17.

The procedure of the optimization problem is :
1. Generate optimal walking trajectories for the robot.
2. Calculate the torques and the forces with the dynamic model
3. Calculate the energy criterion $C_E$ and verify the constrains $\Psi_i$.

### 4 Simulations and Results

The simulations were made on the HYDROïD Robot. It measures 1.39 m and weight 45 kg. The body’s parameters are resumed in [3]. Firstly, the radii $r_1$ et $r_2$ are fixed at 0.05 m to calculate the energetic criterion of the biped. Two simulations are performed, one considering the dry friction of the geared motor and the second with zero friction. These results show the friction influence and the importance of the mechanical transmission quality.

Secondly, some optimizations were done with radii $r_1$ et $r_2$ fixed between 0.02 m et 0.1 m, lengths of the knee morphology of human. These simulations allow to know the radii of the circles best suited to reduce the energy consumption.

The figure 3 at left shows the criterias evolution versus the walking speeds with or without the dry frictions. At 0.2 m/s, the energy gain between the curves is around 85% and at 1 m/s, the gain is around 50%. With a better quality of transmission, the energy criterion will be lower and the energy gain 50% higher.

The evolution of the energy criterion in function of the radii length shows that the energy diminution appears with the radius $r_1$ bigger than the radius $r_2$. The convergence of the algorithm is slower and did not enough converge. These graphs suggest that the best structure is obtained when the radius $r_1$ tends to a flat surface as the approximation given in [4] with two cylinders on flat surface.

### 5 Conclusions

In this article, we propose a rolling knee kinematic for the biped robot to reduce the energy consumption during the gait. This kinematic give a natural transmission ratio inside the kinematic chain. The
optimization results show two points : first, the importance of mastering transmission quality of actuator to ensure the energy reduction. Secondly, the cylinder radius of the tibia must be the biggest than the cylinder radius of femur to reduce the energy consumption.

New designs of the contact surfaces between the tibia and the femur can be considered as bearing cylindrical of flat surfaces. The sliding between the contact surfaces can also be introduced in order to approximate a perfectly anthropomorphic knee. A first prototype is done and the figure 4 shows the view of this realization.

Références