The Scale Effect of Roughness on Hydrodynamic Contact Friction

I. DEMIRCI, S. MEZGHANI, M. YOUSFI, H. ZAHOUANI, and M. EL MANSORI

1 Arts et Métiers ParisTech, LMPF
Rue Saint Dominique, BP 508, 51006 Châlons en Champagne, France
2 Renault S.A.S, Direction de l’Ingénierie Mécanique
67 rue des bons raisons, 92500 Rueil Malmaison, France
3 Laboratoire de Tribologie et Dynamique des Systèmes
UMR CNRS 5513, Ecole Centrale de Lyon
36 avenue Guy de Collongue, 69131 Écully Cedex, France

Multistage abrasive finishing processes (grinding, polishing, honing, etc.) are commonly used to produce the geometrical properties of a surface to meet its technical functionalities in the operating characteristics of contacting parts in friction, relating to their durability and reliability (running-in performance, wear resistance, load-carrying capacity, etc.). Coarse abrasive grits followed progressively finer ones are used, which leads to a multiscalar stratified surface texture.

In this article, a numerical model of elastohydrodynamic (EHD) contact coupled to a multiscale surface texture model was developed that allows tracking the scale effect of surface features and their interactions on friction performance and lubricant flow under hydrodynamic lubrication conditions. Because the simulation model has as an input the surface topography and to overcome the variability in surface finish formation, textured surfaces at different stages of the finishing process were simulated (virtual texturing method). Surface topography can be decomposed into two principal components: superficial roughness and valleys. Superficial roughness was modeled using a fractal model and a scaling factor was introduced to model valley patterns. The results show the relationship between friction and surface scales.

KEY WORDS
- Roughness; Scale; Surface Texture; Elastohydrodynamic Lubrication; Friction

INTRODUCTION

To improve the tribological performance of mechanical components, such as load capacity, wear resistance, and friction coefficient, surface texturing is considered as a viable option in surface engineering. It consists of forming a reasonable texture on the surface. Different mechanisms are usually used to explain this improvement:

- Microhydrodynamic bearing by generation of additional hydrodynamic pressure to increase load-carrying capacity (Hamilton, et al. (1))
- Microreservoirs for the lubricant in cases of starved lubrication (Hamilton, et al. (1))
- A debris reservoir that allows trapping fragments to prevent surface wear

Various forms and techniques of texturing have been developed to generate a surface texture and improve tribological performance. The most common and earliest application of surface texturing consists of surface finishing processes such as grinding, honing, polishing, turning, shaving, and dimpling. More recently, various techniques such as laser beam texturing, etching techniques, and laser texturing have been employed. They are able to generate a deterministic and more controllable texture (Ellison (2)).

The principal statistical parameters of roughness, such as variances in height, slope, and curvature, are scale dependent (Fuller and Tabor (3); Mezghani, et al. (4)). Instrumets with different resolutions and scan lengths yield different values of these statistical parameters for the same surface. Therefore, it is important to characterize rough surfaces by intrinsic parameters that are independent of the sampling length or area (Fuller and Tabor (3)).

The scale effect has been well studied for elastic, elastoplastic, and plastic contacts between rough surfaces (Zahouani, et al. (5); Majumder and Bhushan (6); Ju and Farris (7)) but not for lubricated contacts under hydrodynamic and elastohydrodynamic (EHD) conditions.

For example, fractal geometry has been widely used in recent years and was applied to characterize surface topography and
contact mechanics (Ju and Farris (7); Liu, et al. (8); Gagnevin and Rixhe-Carmes (9); Majumder and Tien (10)).

In the hydrodynamic regime, studies have focused primarily on the effect of anisotropy (Ajati, et al. (11); Mezghani, et al. (12)), shapes and repartitions (Yu, et al. (13); Nakbi, et al. (14)), and deterministic systems (Sriramamurthy and Stephens (15)) of the textured surfaces. A majority of these studies have focused on surfaces achieved by laser surface texturing. The effects of roughness have been studied in an EHID regime. In particular, the effects of valleys and peaks were studied during the 1990s and 2000s. For transverse roughness in line contacts, Greenwood and Morales-Espejel (16) studied pressure and local friction fluctuations. In general, surface roughness components deform differently (Lupt and Morales-Espejel (17)) and some authors have investigated the amplitude reduction of transverse and longitudinal roughness (Wang, et al. (18); Vennner and Lubrecht (19)). Rough surface topography and orientation were also investigated in mixed lubrication (Zhu and Hu (20)). Friction was not discussed in any of these studies. The reduction in friction with partial laser surface texturing (Ryk and Eitison (21); Etison, et al. (22)), transitions in the lubrication regime (Kovalchenko, et al. (23)), and the effect on thin EHID lubrication film (Krupka and Hartl (24)) have been studied experimentally. In numerical analyses, it is common to use the concept of virtual texturing technology (Wang and Zhe (25)), which consists of producing surfaces numerically and solving the Reynolds equation. Different patterns and shapes can be studied using this technique. For example, the influence of geometrical shape effects (Yu, et al. (13)), microtextures in conformal contacts (Nabki, et al. (14)), and waviness amplitudes and different roughness orientations on pressure distribution (Ai and Cheng (26)) have been investigated.

However, none of these studies considered the scale effect. A global characterization of the area was used, which is considered unsuitable for a surface obtained through various stages of finishing and thus has a multiscale structure (Chen, et al. (27); Sanaremy, et al. (28); Sabri, et al. (29)).

In this article, the scale effects of roughness on friction in an elastohydrodynamic lubrication (EHID) regime were studied. To this aim, a numerical model was developed and a virtual texturing method was adopted. An advanced surface characterization was performed based on the decompositions of the surface topography into its two principal elements: superficial roughness (related to friction and wear) and valleys (related to lubricant circulation and reservoirs). These numerical surfaces were used on the numerical EHID contacts.

**DESCRIPTION OF THE ELASTOHYDRODYNAMIC MODEL**

A numerical model was developed to estimate the friction generated between rough surfaces. It takes into account the real topography of the surfaces. The scope of this model is to qualitatively predict the friction coefficient obtained when the groove characteristics of surfaces are varied in order to optimize performance.

**Elastohydrodynamic Equations**

The following dimensionless isothermal Reynolds equations (Reynolds (30)) were used to estimate the pressure distribution, film thickness, and friction coefficient. Effective viscosities are introduced to account for the effects of non-Newtonian lubricant behavior.

\[
\frac{d}{dX} \left( \frac{\rho H^2 \frac{dP}{dX}}{\lambda \sigma} \right) + \frac{d}{dY} \left( \frac{\rho H^2 \frac{dP}{dY}}{\lambda \sigma} \right) = \frac{\partial P}{\partial X} [1]
\]

The following boundary condition \( P - 0 \) and coevitation condition \( P(X, Y) \geq 0 \forall X, Y \) must be satisfied. To ensure this condition,
the Reynolds-Schlichting model was used. In this equation, \( \lambda \) is equal to \( \frac{\rho_{\infty}}{2\eta_{\infty}} \) and \( \eta \) is the effective viscosity. Considering that the variation in viscosity along the \( z \) direction can be ignored, the effective viscosity can be calculated considering the shear-thinning effect:

\[
\frac{1}{\eta} = \frac{1}{\eta_{\infty}} \sinh (\varphi) \tag{[1]}
\]

with \( \varphi = \frac{P}{\eta_{\infty}} \) where \( \varphi \) is a reference shear stress and \( \eta_{\infty} \) is the shear stress acting on the surface.

The lubricant's viscosity and density were assumed to be dependent on pressure, and we used the Dowson and Higginson formula (Dowson and Higginson [31]; Eq. [3]) and Reedsals law (Reedsals [32]; Eq. [4]):

\[
\bar{\rho}(P) = \left[ 1 + \frac{0.5 \times 10^{-6} \rho_{\infty} P}{1 + 1.7 \times 10^{-3} P} \right] \tag{[3]}
\]

where \( \bar{\rho} \) is the density at ambient pressure.

\[
\eta(\bar{\rho}) = \exp \left( 10 \eta_{\infty} \left[ 0.67 \left( 1 + \left( \frac{P_{\infty}}{P} \right)^{3/4} \right) \right] \right) \tag{[4]}
\]

where \( \eta(\bar{\rho}) \) is the viscosity at ambient pressure, \( \rho_{\infty} \) is a constant equal to \( 1.96 \times 10^{10} \), and \( \eta_{\infty} \) is the pressure viscosity index.

The film thickness equation is given in dimensionless form by the following equation:

\[
H(X, Y) = H_0 + \frac{X^2}{2} + \frac{Y^2}{2} + \bar{h}(X, Y) - Z(X, Y) \tag{[5]}
\]

\( Z \) is the height of the liner surface topography at each position \( (X, Y) \) and \( \bar{h}(X, Y) \) is the dimensioness surface deformation calculated by

\[
\bar{h}(X, Y) = \frac{X^2}{2} \int Y \int \frac{P(X, Y) dX dY}{\sqrt{X^2 + (Y - Y)^2}} \tag{[6]}
\]

The global force balance condition is given by:

\[
\frac{2\pi}{3} \int H(X, Y) dX dY \tag{[7]}
\]

### Numerical Procedure

In order to obtain the film pressure distribution, the Reynolds equation was solved using a finite difference method. A separate second-order backward scheme was used because it was the better scheme (Liu, et al. [31]) was used. The discretized equation was solved using Jacobi line relaxation (Vemmer and Lubrecht [34]). The full-scale mixed EHL approach developed by Hu and Zhu [35] was used in the present work. Either a multigrid method (Vemmer and Lubrecht [34]) or progressive mesh densification technique (Zhu [36]) was used in order to accelerate solution convergence. Elastic deformation calculation was conducted using the discrete convolution and fast Fourier transform method developed by Liu, et al. [8]. The solution domain was determined as \(-2.5 \leq X \leq 1.5 \) and \(-2.0 \leq Y \leq 2.0 \). The computational grid covering the domain consisted of \( 512 \times 512 \) or \( 1,024 \times 1,024 \) nodes equally spaced.

### Model Validation

The present model in a fully flooded smooth contacts case was compared to the non-Newtonian simulation done by Chapkov [37]. Table 1 provides the dimensionless central and minimum oil film thickness for the following dimensionless Moen and Vemmer parameters \( M = 500 \) and \( L = 3 \). The difference between our model and Chapkov's [37] work was less than 3%. This comparison confirmed the validity of the model presented in this article.

The mixed lubrication approach of our model was compared to the sinusoidal wave surface case in Hu and Zhu's [35] work. The same parameters of rolling velocity of \( 312.5 \, \text{mm/s} \) was chosen. Figure 1 shows the pressure profile and film thickness vs. the

<table>
<thead>
<tr>
<th></th>
<th>Chapkov [37]</th>
<th>Current Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central oil film thickness ( H_c )</td>
<td>0.05811</td>
<td>0.057</td>
</tr>
<tr>
<td>Minimum oil film thickness ( H_m )</td>
<td>0.0280</td>
<td>0.0279</td>
</tr>
</tbody>
</table>

![Fig. 1—Film thickness and pressure profile of the present work compared to Hu and Zhu's work (35).](color figure available online.)
X axis. Good agreement was found between the two models. This test confirmed the validity of the model presented in this article.

MULTISCALE SURFACES CHARACTERIZATION AND SYNTHESIS TEXTURE MODEL

Finshed surfaces are typically machined by more than one manufacturing process. The resulting surface topography exhibits topographical features over a wide range of scales from the nanometric scale to macroscopic features. Each element has a different role in the contact. The superficial roughness of the surface is an important role in load-carrying capacity and friction, whereas the valleys serve as lubricant reservoirs and distribution paths. Therefore, the surface can be analyzed separately (Deannciere and Jouslin (30)).

The objective of the scale effect of the surface texture, a mathematical model of each roughness component (plates and valleys) is presented.

Fractal Model of Superficial Surface Irregularities (Plateau Component)

The details of $Z(x)$ depend on the length scale; each $Z(x)$ was assumed to be a continuous but nondifferentiable function. This means that the presence of any small roughness elements may prevent it from being a satisfactory limit of $\frac{Z(x + \lambda) - Z(x)}{\lambda}$ as $\lambda \to 0$ [8].

A simple way to obtain this behavior for a function $Z(x)$ is to assume that the increment of $Z(x)$ is related to the self- affinity relation:

$[Z(x + \lambda) - Z(x)] \times \lambda^{-H}, 0 < H < 1$ as $\lambda \to 0$ [9]

$\lim_{\lambda \to 0} \lambda^{-H}$ is the only limit if $H = 1$. The derivative of $Z$ is proportional to the limit.

For $0 < H < 1$, this derivative is finite, although the function remains continuous. For $H = 0$, the function becomes discontinuous. So by varying $H$ from 0 to 1, the parameter $H$ characterizes the transition from a continuous to a discontinuous function, and the range $0 < H < 1$ corresponds to nondifferentiable functions that become smoother as $H$ increases. Thus, $H$ can be considered as an indicator of continuity (Monghini, et al. (4)). Because we are concerned with functions that can be considered to represent random processes, the self-affinity relation can be expressed in terms of variance in the increments of $Z$.

$\frac{[Z(x + \lambda) - Z(x)]^2}{\lambda^{2H}}, 0 < H < 1$ as $\lambda \to 0$ [10]

For random rough surfaces, the random displacement method provides one of the simplest algorithms to generate random fractal surfaces. For analysis purposes, it is particularly useful to have a simple algorithm that creates the same kind of morphological data arrays that most scanning acquisition methods (e.g., range images) create, namely an $x\times y$ array of $Z$ values: $Z(x, y)$. The algorithm for a random displacement method of rough surfaces is described in Monghini, et al. (4), Fickler (30), and Mandlebrot and Van Ness (40).

In order to study the influence of the Holder exponent of roughness on the friction parameter we propose generating rough surfaces parameterized in roughness by their Holder exponent. Each of those surfaces is generated by the random displacement model. Figure 2 shows the construction of three surfaces with different Holder exponents: $H = 0.1$, $H = 0.5$, $H = 0.8$. The surfaces generated were put into an elastic hydrodynamic contact under a pressure of 500 MPa with a rigid plane.

Multiscale Model of Surface Groove Patterns (Valley Component)

The hosed surfaces were generated by successive honing processes stages using increasingly finer abrasive grains. Honed surfaces are composed of profound valleys (lubrication reservoirs) and finer valleys (to reduce the contact area and friction). The scale effect is studied only for the finer valleys because they affect friction.

A multiscale model describing the essential characteristics of the finer valley pattern was developed. In this model, valley features (Fig. 3a) were modeled by the following $S_\alpha$ function:

$S_\alpha(x) = \begin{cases} S_\alpha, \alpha_x < 0 \\ 0, \alpha_x \geq 0 \end{cases}$ [11]

where $S_\alpha, \beta(x)$ is the second derivative of the Gaussian function (Fig. 3a) defined by

$\tilde{S}_\alpha, \beta(x) = \alpha_\beta(1 - \beta^2 x^2)^{-\alpha}$ [12]

$r = \frac{\beta}{\alpha_\beta}, \beta$ corresponds to the feature valley position, and $\alpha_\beta$ is the scale factor.

The modeled valley surface component takes into account the scale parameter $\alpha_\beta$ and the depth amplitude $\alpha_\beta$ as variables. To study the scale effect of this surface component, various surfaces were generated with varying $\alpha_\beta$ in the range of 0.5 to 2.1 mm and $\alpha_\beta$ from 7.5 to 10 mm. To more closely match realistic generated surfaces, two profound valleys with the coupled parameters $(\alpha_\beta)_{\max} = 3 \mu m$ and $(\alpha_\beta)_{\max} = 25 \mu m$ were added for each simulated surface.

Note that the effect of surface anisotropy is well known (A1 and Cheng (26)) and is not taken into account in this study. All grooves were oriented perpendicular to the sliding direction.

A spatial sampling step of 2 $\mu m$ was chosen. The total sampling area was therefore approximately 1.024 mm x 1.024 mm. Examples of the virtual surface with different values of $\alpha_\beta$ and $\alpha_\beta$ are presented in Fig. 4.

RESULTS AND DISCUSSION

Multiscale Effect of Roughness Irregularities (Plateau Component)

The sliding velocities varied between 2 and 52 m s$^{-1}$ for each of the four surfaces with different Holder exponents. The other parameters are shown in Table 2. The second surface was fixed. An isothermal model was used in the present work for simplicity, however, in pure sliding cases thermal effects are expected to have a significant impact on the frictional behavior of these
Fig. 2—Statistical self-affine surfaces with the same roughness amplitude (\(S_0 = 0.48 \mu m\)) and different Hölder dimension \(H\): (a) 0.1, (b) 0.5, and (c) 0.9. (color figure available online.)

Fig. 3.—(a) \(D_2 \mu G_2\) Second derivative of Gaussian function (\(D_2 \mu G_2\)) and (b) \(S_0\) valley feature (groove pattern).
contacts. However, qualitative comparison should also be done to study the influence of the H"older exponent on the friction coefficient.

Using the calculated pressure field, the friction coefficient was determined by the following equation:

\[ \mu = \frac{\mu}{p} \int_{\Omega} \nabla V \cdot \nabla X \, d^2 Y \]

Figure 5 shows the friction coefficient as a function of mean entainment velocities for surfaces with different H"older exponents (H). The smaller H is, the more irregular the surface is. For all surfaces, the friction coefficient decreased until a minimum value and increased when velocity increased. The results of this study show that the local scale of roughness modified the distribution of the contact pressure and friction. This clearly shows the influence of the fractal dimension of roughness on the friction coefficient in a wide range of sliding velocities. Two trends are visible in the Fig. 5. For low velocity, the friction coefficient is high for low H"older exponents. In other words, when the surface becomes more irregular, the friction coefficient increases. In this case, some asperity contacts appear in the mixed lubrication case. The more irregular the surface is (low H), the more friction coefficient is. In this situation, the contact surface increases when H decreases, which results in more asperity contacts, yielding a high friction coefficient.

Furthermore, for the elastohydrodynamic case (high velocity), the friction coefficient increased with sliding speed. The friction coefficient increased when the H"older exponent increased; that is, with a smoother surface. This indicates that the scale of superficial roughness irregularity has a beneficial effect on friction in a hydrodynamic regime. Rough surfaces engender microhydrodynamic bearing to increase load-carrying capacity, which improves the friction coefficient (Hisson, Yu, et al. [15]; Nakama, et al. [17]; Zhu and Hu [20]; Kovalcik, et al. [23]; Krupka and Hard [24]).

### Multiscale Effect of Surface Grooves (Valley Component)

The influence of the two parameters of the surface valleys texturing model (scale factor a_s and valley depth A_y) was studied.

---

**Table 2 — Operating Conditions and Lubricant Properties**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>F_N (N)</td>
<td>500</td>
<td>(u) (\text{Pa}^{-1})</td>
<td>22.00</td>
</tr>
<tr>
<td>(a_s) (\text{ms}^{-1})</td>
<td>4.0</td>
<td>(R_e) (\text{m})</td>
<td>0.4</td>
</tr>
<tr>
<td>(p_g) (\text{Pa.s})</td>
<td>0.04</td>
<td>(L_t) (\text{\textmu m})</td>
<td>240</td>
</tr>
<tr>
<td>(u)</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 5** — Friction coefficient vs H"older exponent for different velocities in the range of 2 to 52 m.s^{-1}.
The generated textured surfaces were used as inputs for the developed contact model. The operating conditions and lubricant parameters are presented in Table 3.

Figure 6 shows the pressure field obtained with the numerical models defined above. Two different film thicknesses with the same valley depth amplitude and 8- and 18-µm scale parameter values were used as the input parameters. The pressure fields were slightly different. The maximum pressure was higher for the low scale parameter (Fig. 6). The increase in scale, corresponding to a larger groove, led to great pressure variations.

Figure 7 shows the friction coefficient map (in percentage) for different values of valley depth amplitude (A1) and scale parameter (a_s) and by considering two different groove densities. We can clearly observe the existence of a critical scale parameter value of a_s_crit = 12 µm. Evolution of the friction coefficient differs depending on this critical value:

- Below this critical value (a_s ≤ 12 µm), the friction coefficient decreases when the valley depth (A1) increases. The increase in valley depth can reduce the friction coefficient by almost 6% (from 1.65 to 1.58%).
- However, above this value, groove depths have a negative effect on the friction coefficient, which is usually observed in the rough contacts (friction increases with increased roughness).

This result is in concordance with the results of some authors who found the existence of an optimum value of the depth-to-width ratio (Costa and Hutchings [ ]). However, there is considerable variation in values reported.

The same evolution was observed when the valley density increased (Fig. 7b). The critical scale value remained the same. This increase in groove density had a beneficial effect on the friction coefficient for low scale parameter values (below a_s_crit) compared to low density. However, for high scale parameter values (above a_s_crit), increasing the groove density led to higher friction.

Fig. 6—Pressure field obtained for two surfaces with the same groove depth (A1) and (a) 8-µm, (b) 18-µm scale parameter (a_s) values. (Color figure available online.)

Fig. 7—Effect of valley scale a_s and depth A1 on the friction coefficient (%) for density: (a) d1 and (b) d2. (Color figure available online.)
CONCLUSIONS

Finished surfaces are made with a succession of processes using different sizes of abrasive grits from coarser to finer, which lead to surface scale modifications. In this study, advanced characteristics were proposed and applied to investigate the scale effect in elastohydrodynamic contacts. Surface topography can be decomposed into three principal components: surface roughness and waviness. In this study, superficial roughness was modeled using a fractal model and a scaling factor was introduced to model valley patterns.

The results were as follows:

- A decrease in superficial surface irregularities (increase in Hölder exponent) reduces the coefficient of friction.
- There is a critical scale, the effect of valley depth is different on both sides of this critical value, below which the increase in valley depth amplitudes can reduce friction.

Finally, in the range of the simulation parameter, the results showed that in EHL contacts of a fine scale, the optimal friction coefficient can be achieved with showed superficial rough surfaces with high groove depth.

ACKNOWLEDGEMENT

The authors thank S. Legros for his dedication and computer support during this work.

REFERENCES