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Modeling of Rolling Knee Biped Robot

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Abstract :

This report presents the dynamic modeling of a planar biped robot. The robot has seven bodies and 9 DOF. Two kinematic configurations are investigated. The first has only revolute joints on all articulation. The second differs by the presence of rolling contact on the knees. All matrices involved in the model are given in explicit form. All the possibilities of contact between the feet and the ground are considered.

1 Keywords : *Biped robot, rolling knee, dynamic model, contact model.*

2 Introduction

The study concerns a planar biped robot with seven bodies namely a trunk, two thighs, two shins and two feet. Two different kinematic configurations are considered for the biped. The first named CK is the classical configuration with only revolute joints on all articulations. The second named RK has rolling contact joint on the knee. The other joints are revolute joints. Figures 1 and 2 define the notations used thereafter. These robots are controlled by six torques applied on the axis of rotation of each joint. For the RK robot, this means that the knee torques are applied on a bar connected between the thigh and the shin (see Fig.).

3 Dynamic model of a Biped Robot with 7 bodies

The two configurations are defined in the sagittal plane by the absolute angular coordinates named $X_e = [q^\top \ x_H \ z_H]^\top$ with (x_H, z_H) the cartesian coordinates of the hip in R_0 and $q^\top = [q_0, q_1, q_2, q_3, q_4, q_5, q_6]$ the vector of absolute joint angles. The joint torque vector is defined by $\Gamma = [\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6]^\top$ as represented on Fig. 1 and 2. Defining $\theta = [q_1 \quad q_0, q_2 \quad q_1, q_6 \quad q_2, q_6 \quad q_3, q_3 \quad q_4, q_4 \quad q_5]^\top$ the vector of joint angular position.
The dynamic model is determined by the application of the Newton's second law. A direct explicit relation is obtained by the Euler-Lagrange equations. For the sake of simplicity, we consider a simple point-mass model for all the bodies. This hypothesis reduces the parameters defining the model for each body which are thus the mass m_i and the positions $s_i = dist(O_i Cg_i)$ of center of mass Cg_i. Furthermore, we assume that its interaction with the environment is limited to contact between the ground and the foot soles.

$$D(X_e)\ddot{X}_e + H(\dot{X}_e, X_e) + Q(X_e) = B\Gamma + A_{cL}(X_e)^\top F_L + A_{cR}(X_e)^\top F_R \tag{1}$$

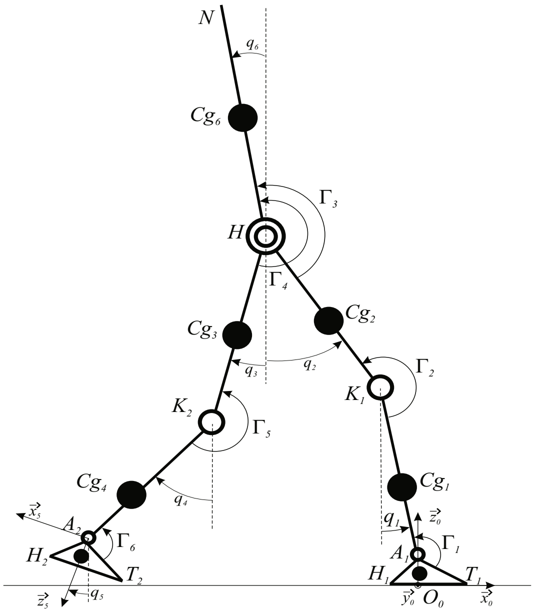

FIGURE 1 – Biped robot with revolute joint knees named CK

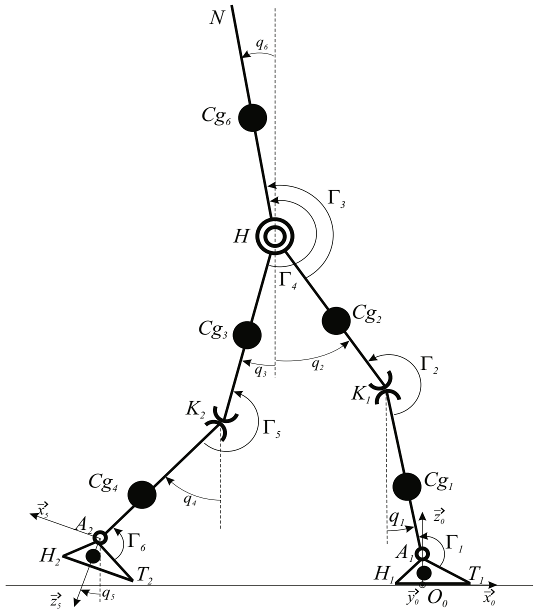

FIGURE 2 – Biped robot with rolling joint knees named RK

with $D(X_e)$ the $9 \in 9$ inertia matrix, $H(\dot{X}_e, X_e)$ the $9 \in 1$ vector of centrifugal and Coriolis forces, $Q(X_e)$ the $9 \in 1$ vector due to the gravity, B the $9 \in 6$ control matrix, Γ the $6 \in 1$ vector of torques, A_{cL} and A_{cR} the $3 \in 9$ Jacobian matrix of external forces and F_L and F_R the vector of external wrench respectively on the left and right feet.
Inertia matrix D is expressed :

$$D = \begin{bmatrix} D_{11} & D_{12} & D_{13} & 0 & 0 & 0 & 0 & D_{18} & D_{19} \\ D_{21} & D_{22} & D_{23} & 0 & 0 & 0 & 0 & D_{28} & D_{29} \\ D_{31} & D_{32} & D_{33} & 0 & 0 & 0 & 0 & D_{38} & D_{39} \\ 0 & 0 & 0 & D_{44} & D_{45} & D_{46} & 0 & D_{48} & D_{49} \\ 0 & 0 & 0 & D_{54} & D_{55} & D_{56} & 0 & D_{58} & D_{59} \\ 0 & 0 & 0 & D_{64} & D_{65} & D_{66} & 0 & D_{68} & D_{69} \\ 0 & 0 & 0 & 0 & 0 & 0 & D_{77} & D_{78} & D_{79} \\ D_{81} & D_{82} & D_{83} & D_{84} & D_{85} & D_{86} & D_{87} & D_{88} & 0 \\ D_{91} & D_{92} & D_{93} & D_{94} & D_{95} & D_{96} & D_{97} & 0 & D_{99} \end{bmatrix} \quad (2)$$

For the CK robot, the terms of matrix D are defined as follows.

$$
\begin{aligned}
D_{11} &= D_{66} = s_z^2 m_0 + s_x^2 m_0 + I_0 \\
D_{12} &= D_{21} = m_0 l_1 (s_x \sin(\theta_1) \quad s_z \cos(\theta_1)) \\
D_{13} &= D_{31} = \quad m_0 l_2 (s_x \sin(q_0 \quad q_2) \quad s_z \cos(q_0 \quad q_2)) \\
D_{22} &= D_{55} = I_1 + m_0 l_1^2 + m_1 s_1^2 \\
D_{23} &= D_{32} = l_2 \cos(\quad \theta_2)(m_0 l_1 + m_1 s_1) \\
D_{33} &= D_{44} = m_0 l_2^2 + I_2 + m_1 l_2^2 + m_2 s_2^2 \\
D_{45} &= D_{54} = l_2 \cos(\theta_5)(m_0 l_1 + m_1 s_1) \\
D_{46} &= D_{64} = m_0 l_2 (\cos(q_3 \quad q_0) s_z + s_x \sin(q_3 \quad q_0)) \\
D_{56} &= D_{65} = m_0 l_1 (s_x \sin(\theta_6) + s_z \cos(\theta_6)) \\
D_{18} &= D_{81} = \quad m_0 (s_x \sin(q_0) \quad s_z \cos(q_0)) \\
D_{19} &= D_{91} = m_0 (s_z \sin(q_0) + s_x \cos(q_0)) \\
D_{28} &= D_{82} = (m_0 l_1 + m_1 s_1) \cos(q_1) \\
D_{29} &= D_{92} = (m_0 l_1 + m_1 s_1) \sin(q_1) \\
D_{38} &= D_{83} = (m_0 l_2 + m_1 l_2 + m_2 s_2) \cos(q_2) \\
D_{39} &= D_{93} = (m_0 l_2 + m_1 l_2 + m_2 s_2) \sin(q_2) \\
D_{48} &= D_{84} = (m_0 l_2 + m_1 l_2 + m_2 s_2) \cos(q_3) \\
D_{49} &= D_{94} = (m_0 l_2 + m_1 l_2 + m_2 s_2) \sin(q_3) \\
D_{58} &= D_{85} = (m_0 l_1 + m_1 s_1) \cos(q_4) \\
D_{59} &= D_{95} = (m_0 l_1 + m_1 s_1) \sin(q_4)) \\
D_{68} &= D_{86} = \quad m_0 (\quad \cos(q_5) s_z + s_x \sin(q_5)) \\
D_{69} &= D_{96} = m_0 (s_x \cos(q_5) + s_z \sin(q_5)) \\
D_{77} &= I_3 + m_3 s_3^2 \\
D_{78} &= D_{87} = \quad m_3 s_3 \cos(q_6) \\
D_{79} &= D_{97} = \quad m_3 s_3 \sin(q_6) \\
D_{88} &= D_{99} = 2m_0 + 2m_1 + 2m_2 + m_3
\end{aligned}
$$

For the RK robot, the terms of matrix $D = [D_{ijRK}],\ (i,j) \in [1 \times\times\times 6]$ are defined as follows.

$$
\begin{aligned}
D_{11RK} &= D_{66RK} = D_{11} \\
D_{12RK} &= D_{21RK} = D_{12} + m_0 r_1 (s_z(\cos\alpha_1 - \cos\theta_1) + s_x(\sin\alpha_1 - \sin\theta_1)) \\
D_{13RK} &= D_{31RK} = D_{13} + m_0 r_2 (s_z(\cos\alpha_1 - \cos(q_2 - q_0)) + s_x(\sin\alpha_1 - \sin(q_2 - q_0))) \\
D_{22RK} &= D_{22} + 2r_1((m_1(r_1 - s_1) - m_0 l'_1) + (m_1(s_1 - r_1) + m_0 l'_1)\cos(\alpha_3)) \\
D_{23RK} &= D_{32RK} = D_{23} + (m_1 l'_2 + m_0 l'_2) r_1 \cos\alpha_2 + (m_0 l'_1 + m_1(s_1 - r_1)) r_2 \cos\alpha_3 \\
&\quad - ((m_1 + m_0) r_1 l'_2 + m_0(l_1 + s_1) r_2)\cos(q_1 - q_2) + (m_1 + m_0) r_1 r_2 \\
D_{33RK} &= D_{33} + 2r_2((m_1 + m_0) l'_2 \cos(\alpha_2) - l'_2) \\
D_{44RK} &= D_{44} + 2r_2((m_1 + m_0) l'_2 \cos(\alpha_5) - l'_2) \\
D_{45RK} &= D_{54RK} = D_{54} + (m_1 r_2 (s_1 - r_1) + m_0 r_2 l'_1)\cos\alpha_4 + (m_0 + m_1) l'_2 r_1 \cos\alpha_5 \\
&\quad - (m_0(l_2 r_1 + l'_1 r_2) + m_1(r_2 s_1 + l'_2 r_1))\cos(q_4 - q_3) + (m_1 + m_0) r_1 r_2 \\
D_{46RK} &= D_{64RK} = D_{64} - m_0 r_2 (s_z(\cos(q_3 - q_5) - \cos\alpha_6) + s_x(\sin(q_3 - q_5) - \sin\alpha_6) \\
D_{55RK} &= D_{55} + 2r_1((m_1(r_1 - s_1) - m_0 l'_1) + (m_1(s_1 - r_1) + m_0 l'_1)\cos(\alpha_4)) \\
D_{56RK} &= D_{65RK} = D_{56} + m_0 r_1 (s_x(\sin\alpha_6 - \sin\theta_6) + s_z(\cos\alpha_6 - \cos\theta_6) \\
D_{18RK} &= D_{81RK} = D_{18} \\
D_{19RK} &= D_{91RK} = D_{19} \\
D_{28RK} &= D_{82RK} = D_{28} - r_1(m_0(\cos q_1 - \cos\gamma_1) + m_1(\cos q_1 - \cos\gamma_1)) \\
D_{29RK} &= D_{92RK} = D_{29} + r_1(m_1(\sin\gamma_1 - \sin q_1) + m_0(\sin\gamma_1 - \sin q_1)) \\
D_{38RK} &= D_{83RK} = D_{38} + r_2(m_1(\cos\gamma_1 - \cos q_2) + m_0(\cos\gamma_1 - \cos q_2)) \\
D_{39RK} &= D_{93RK} = D_{39} - r_2(m_1(\sin q_2 - sin\gamma_1) + m_0(\sin q_2 - \sin\gamma_1)) \\
D_{48RK} &= D_{84RK} = D_{48} - r_2(m_0(\cos q_3 - \cos\gamma_2) + m_1(\cos q_3 - \cos\gamma_2)) \\
D_{49RK} &= D_{94RK} = D_{49} - r_2(m_1(\sin q_3 - \sin\gamma_2) + m_0(\sin q_3 - \sin\gamma_2)) \\
D_{58RK} &= D_{85RK} = D_{58} + r_1(m_0(\cos\gamma_2 - \cos q_4) + m_1(\cos\gamma_2 - \cos q_4)) \\
D_{59RK} &= D_{95RK} = D_{59} + r_1(m_0(\sin\gamma_2 - \sin q_4) + m_1(\sin\gamma_2 - \sin q_4)) \\
D_{68RK} &= D_{86RK} = D_{68} \\
D_{69RK} &= D_{96RK} = D_{69} \\
D_{77RK} &= D_{77} \\
D_{78RK} &= D_{87RK} = D_{78} \\
D_{79RK} &= D_{97RK} = D_{79} \\
D_{88RK} &= D_{99RK} = D_{88}
\end{aligned}
$$

with

$$
\begin{aligned}
l'_1 &= l_1 - r_1 \\
l'_2 &= l_2 - r_2 \\
\alpha_1 &= (-q_0 r_1 - q_0 r_2 + r_1 q_1 + r_2 q_2)/(r_1 + r_2) \\
\alpha_2 &= r_1(q_1 - q_2)/(r_1 + r_2) \\
\alpha_3 &= r_2(q_1 - q_2)/(r_1 + r_2) \\
\alpha_4 &= r_2(-q_3 + q_4)/(r_1 + r_2) \\
\alpha_5 &= r_1(-q_3 + q_4)/(r_1 + r_2) \\
\alpha_6 &= (r_1 q_4 + r_2 q_3 - q_5 r_1 - q_5 r_2)/(r_1 + r_2)
\end{aligned}
$$

The terms of Coriolis and centrifugal forces for both robots are calculated by Christoffel symbols. We obtain :

$$
\begin{aligned}
H_1 &= m_0 l_1 \left[s_x \cos(\theta_1) - s_z \sin(\theta_1)\right] \dot{q_1}^2 + m_0 l_2 \left[s_x \cos(q_2 - q_0) - s_z \sin(q_2 - q_0)\right] \dot{q_2}^2 \\
H_2 &= -m_0 l_1 \left[s_x \cos(\theta_1) - s_z \sin(\theta_1)\right] \dot{q_0}^2 + (m_1 s_1 + m_0 l_1)\, l_2 \sin(\theta_2) \dot{q_2}^2 \\
H_3 &= -m_0 l_2 \left[s_x \cos(q_0 - q_2) + s_z \sin(q_0 - q_2)\right] \dot{q_0}^2 + (m_0 l_1 + m_1 s_1)\, l_2 \sin(\theta_2) \dot{q_1}^2 \\
H_4 &= -m_0 l_2 \left[s_x \cos(q_3 - q_5) - s_z \sin(q_3 - q_5)\right] \dot{q_5}^2 + (m_0 l_1 + m_1 s_1)\, l_2 \sin(\theta_5) \dot{q_4}^2 \\
H_5 &= -m_0 l_1 \left[s_x \cos(\theta_6) - s_z \sin(\theta_6)\right] \dot{q_5}^2 - (m_0 l_1 + m_1 s_1)\, l_2 \sin(\theta_5) \dot{q_3}^2 \\
H_6 &= m_0 l_2 \left[s_x \cos(q_5 - q_3) + s_z \sin(q_5 - q_3)\right] \dot{q_3}^2 + m_0 l_1 \left[s_x \cos(\theta_6) - s_z \sin(\theta_6)\right] \dot{q_4}^2 \\
H_7 &= 0 \\
H_8 &= -D_{19}\dot{q_0}^2 - D_{29}\dot{q_1}^2 - D_{39}\dot{q_2}^2 - D_{49}\dot{q_3}^2 - D_{59}\dot{q_4}^2 - D_{69}\dot{q_5}^2 - D_{79}\dot{q_6}^2 \\
H_9 &= D_{18}\dot{q_0}^2 + D_{28}\dot{q_1}^2 + D_{38}\dot{q_2}^2 + D_{48}\dot{q_3}^2 + D_{58}\dot{q_4}^2 + D_{68}\dot{q_5}^2 + D_{78}\dot{q_6}^2
\end{aligned}
$$

For the RK robot, the terms of matrix $H = [H_{1RK}\ H_{2RK}\ H_{3RK}\ H_{4RK}\ H_{5RK}\ H_{6RK}\ H_{7RK}\ H_{8RK}\ H_{9RK}]^T$ are defined as follows.

$$
\begin{aligned}
H_{1RK} &= m_0\, s_x \left[\dot{q_1}^2 l_1' \cos\theta_1 + \dot{q_2}^2 l_2' \cos(q_0 - q_2) + \dot{\gamma_1}^2 (r_1 + r_2) \cos(q_0 - \gamma_1)\right] \\
&\quad + m_0\, sz \left[\dot{q_2}^2 l_2' \sin(q_0 - q_2) - \dot{q_1}^2 l_1' \sin\theta_1 + \dot{\gamma_1}^2 (r_1 + r_2) \sin(q_0 - \gamma_1)\right] \\
H_{2RK} &= m_0 s_x \left[l_2' \dot{q_3}^2 \cos(q_5 - q_3) + l_1' \dot{q_4}^2 \cos\theta_6 + (r_1 + r_2)\dot{\gamma_2}^2 \cos(q_5 - \gamma_2)\right] \\
&\quad + m_0 s_z \left[(r_1 + r_2)\dot{\gamma_2}^2 \sin(q_5 - \gamma_2) + l_2' \dot{q_3}^2 \sin(q_5 - q_3) - l_1' \dot{q_4}^2 \sin\theta_6\right] \\
H_{3RK} &= (m_0 l_1' + m_1(s_1 - r_1)) \left[r_2(\dot{q_2}^2 - (r_2/r_1 + 1)\,\dot{\alpha_2}^2) \sin\alpha_3 - l_2' \dot{q_2}^2 \sin\theta_2\right] \\
&\quad + m_0 r_1 \dot{q_0}^2 \left[s_z \sin(\gamma_1 - q_0) - s_x \cos(\gamma_1 - q_0)\right] - m_0 l_1' \dot{q_0}^2 \left[s_x \cos\theta_1 - s_z \sin\theta_1\right] \\
&\quad + m_0 l_2' r_1 \dot{q_2}^2 \sin\alpha_2 + m_1 l_2' r_1 \dot{q_2}^2 (\sin\alpha_2 + \sin\theta_2) \\
H_{4RK} &= (m_0 l_1' + m_1(s_1 - r_1))(l_2' \sin\theta_2 - r_2 \sin\alpha_3)\dot{q_1}^2 + (m_0 + m_1) l_2' r_1 \sin\alpha_2 \left[(r_1/r_2 + 1)\,\dot{\alpha_3}^2 - \dot{q_1}^2\right] \\
&\quad + m_0 \dot{q_0}^2 \left[r_2 \left(s_z \sin(\gamma_1 - q_0) - s_x \cos(\gamma_1 - q_0)\right) - l_2' \left(s_z \sin(q_0 - q_2) + s_x \cos(q_0 - q_2)\right)\right] \\
H_{5RK} &= (m_0 l_1' + m_1(s_1 - r_1))(l_2' \sin\theta_5 - r_2 \sin\alpha_4)\dot{q_4}^2 + (m_0 + m_1) r_1 l_2' \sin\alpha_5 \left[(r_1/r_2 + 1)\,\dot{\alpha_4}^2 - \dot{q_4}^2\right] \\
&\quad - m_0 \dot{q_5}^2 \left[l_2' (s_x \cos(q_5 - q_3) + s_z \sin(q_5 - q_3)) + r_2 (s_x \cos(q_5 - \gamma_2) + s_z \sin(q_5 - \gamma_2))\right] \\
H_{6RK} &= H_6 + m_0 s_x \left[(r_2 \dot{q_3} + r_1 \dot{q_4})^2 \cos(\alpha_5)/(r_1 + r_2) - \dot{q_3}^2 r_2 \cos(q_3 - q_5) - \dot{q_4}^2 r_1 \cos(\theta_6)\right] \\
&\quad + m_0 s_z \left[\dot{q_3}^2 r_2 \sin(q_3 - q_5) + \dot{q_4}^2 r_1 \sin(\theta_6) - (r_2 \dot{q_3} + r_1 \dot{q_4})^2 \sin(\alpha_5)/(r_1 + r_2)\right] \\
H_{7RK} &= 0 \\
H_{8RK} &= -m_0 (s_x \cos q_0 + s_z \sin q_0)\dot{q_0}^2 - m_0 (s_x \cos q_5 + s_z \sin q_5)\dot{q_5}^2 + m_3 s_3 \dot{q_6}^2 \sin q_6 \\
&\quad - (m_0 l_1' + m_1(s_1 - r_1))(\dot{q_1}^2 \sin q_1 + \dot{q_4}^2 \sin q_4) - ((m_0 + m_1) l_2' + m_2 s_2)(\dot{q_2}^2 \sin q_2 + \dot{q_3}^2 \sin q_3) \\
&\quad - (m_0 + m_1)(r_1 + r_2)(\dot{\gamma_1}^2 \sin\gamma_1 + \dot{\gamma_2}^2 \sin\gamma_2) \\
H_{9RK} &= m_0 (s_z \cos q_0 - s_x \sin q_0)\dot{q_0}^2 + m_0 (s_z \cos q_5 - s_x \sin q_5)\dot{q_5}^2 - m_3 s_3 \dot{q_6}^2 \cos q_6 \\
&\quad + (m_0 l_1' + m_1(s_1 - r_1))(\dot{q_1}^2 \cos q_1 + \dot{q_4}^2 \cos q_4) + ((m_0 + m_1) l_2' + m_2 s_2)(\dot{q_2}^2 \cos q_2 + \dot{q_3}^2 \cos q_3) \\
&\quad + (m_0 + m_1)(r_1 + r_2)(\dot{\gamma_1}^2 \cos\gamma_1 + \dot{\gamma_2}^2 \cos\gamma_2)
\end{aligned}
$$

The detail of the gravity matrix Q of the CK robot is :

$$\begin{aligned}
Q_1 &= m_0 g(s_z \sin(q_0) + s_x \cos(q_0)) \\
Q_2 &= (m_0 l_1 + m_1 s_1) g \sin(q_1) \\
Q_3 &= (m_0 l_2 + m_1 l_2 + m_2 s_2) g \sin(q_2) \\
Q_4 &= (m_0 l_2 + m_1 l_2 + m_2 s_2) g \sin(q_3) \\
Q_5 &= g \sin(q_4)(m_0 l_1 + m_1 s_1) \\
Q_6 &= m_0 g(s_x \cos(q_5) + sz \sin(q_5)) \\
Q_7 &= \quad m_3 s_3 g \sin(q_6) \\
Q_8 &= 0 \\
Q_9 &= (2m_0 + 2m_1 + 2m_2 + m_3) g
\end{aligned}$$

The detail of the gravity matrix Q of the RK robot is expressed by :

$$\begin{aligned}
Q_1 &= m_0\, g\, (s_z \sin(q_0) + s_x \cos(q_0)) \\
Q_2 &= (m_0 l_1 + m_1 s_1)\, g \sin(q_1) \quad (m_1 + m_0) r_1\, g\, (\sin q_1 \quad \sin \gamma_1) \\
Q_3 &= (m_0 l_2 + m_1 l_2 + m_2 s_2)\, g \sin(q_2) \quad (m_1 + m_0) r_2\, g\, (\sin q_2 \quad \sin \gamma_1) \\
Q_4 &= (m_0 l_2 + m_1 l_2 + m_2 s_2)\, g \sin(q_3) \quad (m_1 + m_0) r_2\, g\, (\sin q_3 \quad \sin \gamma_2) \\
Q_5 &= (m_0 l_1 + m_1 s_1)\, g \sin(q_4) \quad (m_1 + m_0) r_1\, g\, (\sin q_4 \quad \sin \gamma_2) \\
Q_6 &= m_0\, g\, (s_x \cos(q_5) + sz \sin(q_5)) \\
Q_7 &= \quad m_3 s_3\, g\, \sin(q_6) \\
Q_8 &= 0 \\
Q_9 &= (2m_0 + 2m_1 + 2m_2 + m_3)\, g
\end{aligned}$$

The matrix B of the CK robot is :

$$B = \begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix} \tag{3}$$

The matrix B of the RK robot is expressed by :

$$B = \begin{bmatrix}
1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{r_1}{r_1+r_2} & \frac{r_1}{r_1+r_2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{r_1}{r_1+r_2} & \frac{r_1}{r_1+r_2} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}$$

The detail of the Jacobian matrix A_{cL} of the left leg for the CK robot is :

$$A_{cL} = \begin{bmatrix}
0 & l_1 \cos(q_1) & l_2 \cos(q_2) & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & l_1 \sin(q_1) & l_2 \sin(q_2) & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

The detail of the Jacobian matrix A_{cL} of the left leg for the RK robot is :

$$A_{cL} = \begin{bmatrix} 0 & r_1\cos(\gamma_1)+l'_1\cos(q_1) & r_2\cos(\gamma_1)+l'_2\cos(q_2) & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & r_1\sin(\gamma_1)+l'_1\sin(q_1) & r_2\sin(\gamma_1)+l'_2\sin(q_2) & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The computation of the Jacobian matrix A_{cL} supposes that we have a contact with the entire sole of the foot and the ground is horizontal. In the other cases, we can have a contact with only the heel or the toes. In these two last conditions, the foot can freely rotate around the contact line between the foot and the ground. The matrix A_{cL} is then different. The notation $l_{p1} = L_p \quad l_p$ is used in the following equation.
For the contact with the heel, we obtain for the CK robot :

$$A_{cL} = \begin{bmatrix} h_p\cos(q_0)+l_p\sin(q_0) & l_1\cos(q_1) & l_2\cos(q_2) & 0 & 0 & 0 & 0 & 1 & 0 \\ h_p\sin(q_0) \quad l_p\cos(q_0) & l_1\sin(q_1) & l_2\sin(q_2) & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the contact with the toes, we obtain for the CK robot :

$$A_{cL} = \begin{bmatrix} h_p\cos(q_0) \quad l_{p1}\sin(q_0) & l_1\cos(q_1) & l_2\cos(q_2) & 0 & 0 & 0 & 0 & 1 & 0 \\ h_p\sin(q_0)+l_{p1}\cos(q_0) & l_1\sin(q_1) & l_2\sin(q_2) & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the contact with the heel, we obtain for the RK robot :

$$A_{cL} = \begin{bmatrix} h_p\cos(q_0)+l_p\sin(q_0) & r_1\cos(\gamma_1)+l'_1\cos(q_1) & r_2\cos(\gamma_1)+l'_2\cos(q_2) & 0 & 0 & 0 & 0 & 1 & 0 \\ h_p\sin(q_0) \quad l_p\cos(q_0) & r_1\sin(\gamma_1)+l'_1\sin(q_1) & r_2\sin(\gamma_1)+l'_2\sin(q_2) & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

For the contact with the toes, we obtain for the RK robot :

$$A_{cL} = \begin{bmatrix} h_p\cos(q_0) \quad l_{p1}\sin(q_0) & r_1\cos(\gamma_1)+l'_1\cos(q_1) & r_2\cos(\gamma_1)+l'_2\cos(q_2) & 0 & 0 & 0 & 0 & 1 & 0 \\ h_p\sin(q_0)+l_{p1}\cos(q_0) & r_1\sin(\gamma_1)+l'_1\sin(q_1) & r_2\sin(\gamma_1)+l'_2\sin(q_2) & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In the case of double support, the second leg is also in contact with the ground. We define then the corresponding Jacobian matrix A_{cR} with the same condition of contact as for the left foot.
For the contact with the toes, we obtain for the CK robot :

$$A_{cR} = \begin{bmatrix} 0 & 0 & 0 & l_2\cos(q_3) & l_1\cos(q_4) & h_p\cos(q_5) \quad l_{p1}\sin(q_5) & 0 & 1 & 0 \\ 0 & 0 & 0 & l_2\sin(q_3) & l_1\sin(q_4) & h_p\sin(q_5)+l_{p1}\cos(q_5) & 0 & 0 & 1 \end{bmatrix}$$

For the contact with the heel, we obtain for the CK robot :

$$A_{cR} = \begin{bmatrix} 0 & 0 & 0 & l_2\cos(q_3) & l_1\cos(q_4) & h_p\cos(q_5)+l_p\sin(q_5) & 0 & 1 & 0 \\ 0 & 0 & 0 & l_2\sin(q_3) & l_1\sin(q_4) & h_p\sin(q_5) \quad l_p\cos(q_5) & 0 & 0 & 1 \end{bmatrix}$$

For the contact with the toes, we obtain for the CK robot :

$$A_{cR} = \begin{bmatrix} 0 & 0 & 0 & r_2\cos(\gamma_2)+l'_2\cos(q_3) & r_1\cos(\gamma_2)+l'_1\cos(q_4) & h_p\cos(q_5) \quad l_{p1}\sin(q_5) & 0 & 1 & 0 \\ 0 & 0 & 0 & r_2\sin(\gamma_2)+l'_2\sin(q_3) & r_1\sin(\gamma_2)+l'_1\sin(q_4) & h_p\sin(q_5)+l_{p1}\cos(q_5) & 0 & 0 & 1 \end{bmatrix}$$

For the contact with the heel, we obtain for the CK robot :

$$A_{cR} = \begin{bmatrix} 0 & 0 & 0 & r_2\cos(\gamma_2)+l'_2\cos(q_3) & r_1\cos(\gamma_2)+l'_1\cos(q_4) & h_p\cos(q_5)+l_p\sin(q_5) & 0 & 1 & 0 \\ 0 & 0 & 0 & r_2\sin(\gamma_2)+l'_2\sin(q_3) & r_1\sin(\gamma_2)+l'_1\sin(q_4) & h_p\sin(q_5) \quad l_p\cos(q_5) & 0 & 0 & 1 \end{bmatrix}$$

4 Conclusion

This report gives the complete modeling of biped robot with two knee structures. These models are used in all software develop by Dr. Hobon during his PhD study. The models allow to compute the minimum torque required for the optimized walking of the biped robot that is used in the control law for the stabilization of the robot.