Optimal Sensors Placement to Enhance Damage Detection in Composite Plates

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Abstract

This paper examines an important challenge in ultrasonic structural health monitoring (SHM), which is the problem of the optimal placement of sensors in order to accurately detect and localize damages. The goal of this study is to enhance damage detection through an optimal sensor placement (OSP) algorithm. The problem is formulated as a global optimization problem, where the objective function to be maximized is evaluated by a ray tracing approach, which approximately models Lamb waves propagation. A genetic algorithm (GA) is then used to solve this optimization problem. Simulations and experiments were conducted to validate the proposed method on a carbon epoxy composite plate. Results show the effectiveness and the advantages of the proposed method as a tool for OSP with reasonable computation time.

Introduction

Structural Health Monitoring (SHM) is an emerging technology that aims at designing systems able to continuously monitor structures [1]. One of the most useful SHM implementation technique is that of "active sensing" whereby permanently attached actuators excite the structure under inspection and a set of sensors record the structural responses in order to extract some damage related information [2]. A wide range of theoretical, numerical, and experimental studies have been performed to assess the effectiveness and applicability of damage detection [3] and identification using guided-waves structural health monitoring techniques such as Lamb waves [4]. Given an active sensing Lamb waves-based approach, there exist primary constraints: the size of the structure under inspection, the total numbers of sensors available and their placement on the structure. Prior work in SHM has focused on damage detection methods[3]. The topic of optimal sensor placement (OSP) is still an open problem and a challenging task in Lamb waves-based SHM. Optimal sensor placement (OSP) strategies seek to maximize the probability of detection (PoD) of a damage given a sensor-actuator network configuration.

While there have been numerous studies on optimal sensor placement[5, 6, 7], there have only been few studies on sensor placement for Lamb-wave based SHM applications. Worden et al. [5] considered the case of passive monitoring with the objective of detecting impact induced-waves on a cantilever beam. Gao et al. [6] discussed active sensing sensor/actuator placement but limit their studies to the pulse echo actuation scheme. Todd et al. [7] discussed the use of evolutionary algorithms to tailor active sensing strategies for enhanced damage detection using a detection theory. Their method was tested on an aluminum bar.

In this work, we present a new approach for OSP designed to enhance damage detection in composite plates using Lamb waves propagation characteristics (group velocity, spatial attenuation). The novelty in this approach is the use of Lamb waves propagation characteristics to design the sensor placement, thus taking into account the physical phenomenon of the wave propagation in the OSP problem. A ray tracing approach is used for this purpose, modeling Lamb wave fronts as optical rays. Once the wave fronts are modeled, the OSP is presented as an optimization problem, where the objective function represents the probability that the damage is detected given a set of sensor location and a damage zone. Such optimization problems are often difficult to solve because of the complexity of the objective function and the large number of variables involved. There exist a numerous methodologies to solve this kind of problem. In this study, we use a Genetic Algorithm (GA) method with time varying mutation rate.

The paper is structured as follow. In the first part we present the mathematical formalism of the proposed method. In the second part we present the background of GA. The third part is devoted to the experimental studies allowing to extract Lamb waves propagation characteristics (group velocity, spatial attenuation). The fourth part presents the results and the last part deals with discussions and conclusions.

1 Description of the proposed method

1.1 Mathematical formalism

Mathematically, the sensor placement problem can be formulated as a constrained optimization problem:

$$\max_{s_l \in \mathcal{W}} \sum_{j=1}^{N_d} \sum_{i=1}^{N} \mathrm{PoD}_{ij}\{\boldsymbol{\theta}, s_l\} \qquad \text{subject to} \qquad G(s_l) \geq 0, \tag{1}$$

where $\boldsymbol{\theta} = [V_g, \alpha]^T$ is the Lamb waves propagation characteristics (group velocity, spatial attenuation), s_l : the set of sensor location. $\mathcal{W} \subseteq \mathbb{R}^{2\times N}$, N is the total number of sensors, N_d is the number of damaged zone which has to be monitored. $\mathrm{PoD}_{ij} : \mathcal{W} \to \mathbb{R}$ is the probability that the sensor i detect the damage in the zone j, which is the criterion to be maximized, and $G(\cdot) \geq 0$ represents physical constraints associated to the problem. $G(\cdot) \geq 0$ ensures that the sensors locations found by the algorithm remain on the surface of the plate.
Considering a set of sensor/actuator, one seeks the optimal position of these elements with respect to a damage zone, which maximize the probability that the damage is detected. Details of this function will be presented in section 1.3.

1.2 Geometric Dilution Of Precision (GDOP)

Since most of damage detection methods are developed in order to locate damage once detected, the objective function to be maximized is here modified in order to facilitate the localization step. This modification takes into account the spatial distribution of the sensor location which is an important issue for the localization. For this purpose, we introduce in our model a metric, termed as Geometric Dilution Of Precision (GDOP)[8]. The GDOP is used in satellite navigation and geomatics engineering to specify the positional measurement precision. In satellite navigation, such system employs the triangulation principle using several satellites to determine the position of an object on the ground [8], which is also a widely used principle in Lamb-wave based damage localization. A good GDOP (value around 1) is obtained for satellites distant rather than satellites near from each other. This metric is inversely proportional to the distance between satellites.

In damage detection framework, assuming that each point (x, y) in the plate is a possible damage zone, the GDOP can be computed for a sensors configuration as follow:

i. Compute the distance between each sensor and the damage :

$$R(i) = \sqrt{(x_s(i) - x)^2 + (y_s(i) - y)^2}, \qquad i = 1 : N$$

$(x_s(i), y_s(i))$ is the coordinates of sensor i.

ii. Compute the jacobian matrix of the distances:

$$\boldsymbol{H} = \begin{bmatrix} \frac{\partial R(1)}{\partial x} & \frac{\partial R(1)}{\partial y} \\ \vdots & \vdots \\ \frac{\partial R(N)}{\partial x} & \frac{\partial R(N)}{\partial y} \end{bmatrix}$$

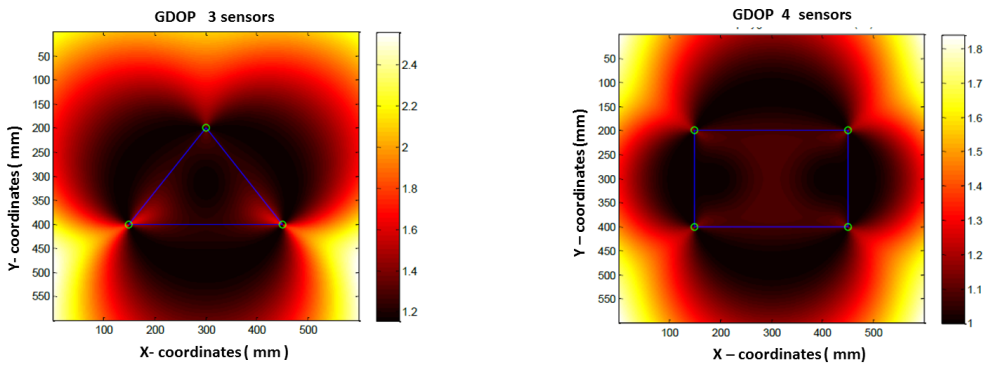
iii. Compute the matrix $\boldsymbol{U}$:

$$\boldsymbol{U} = (\boldsymbol{H}^T \boldsymbol{H})^{-1}$$

iv. The GDOP is obtained by :

$$GDOP = \sqrt{trace(\boldsymbol{U})}$$

To illustrate the importance of this quantity in the localization process, figure 1 shows the result of a GDOP map computed for two configurations of sensors. In the first case the number of sensors is N = 3, and N = 4 in the second case. The domain is a squared plate with dimensions (600×600) mm^2. Black regions represent low values of GDOP while yellow regions represents high values of GDOP. Since the GDOP should be as low as possible, it can be seen from these figures that there are some regions where the localization will be more accurate than others. It follows therefore that the GDOP in the damage zone has to be minimized in order to get more precision while detecting damage.



(a) (b)

Figure 1: GDOP Map for two configurations: 3 sensors (a) , 4 sensors (b)

Once defined, the GDOP is introduced in our objective function. The optimization problem can then be reformulated as :

$$\max_{w \in \mathcal{W}} \sum_{j=1}^{N_d} \sum_{i=1}^{N} \frac{\mathrm{PoD}_i\{\boldsymbol{\theta}, w\}}{GDOP(j)} \qquad \text{subject to} \qquad G(w) \geq 0, \tag{2}$$

1.3 PoD computation through a ray tracing approach

In Lamb wave-based damage detection approach, the PoD function depends on the scattering of the wave by the damage. Since this function depends on damage location, considered as punctual with coordinates (x,y), it can be seen as the amount of energy scattered by the damage at each point (x,y) in the plate. If we consider two states of the structure: S_0 structure without damage and S_1 structure with damage, the energy scattered by the damage can be expressed as :

$$E^{scatter} = \sum_{i=1}^{N} \int_{T_i}^{T_f} |(X^i_{S_1}(t)) - (X^i_{S_0}(t))|^2 dt \tag{3}$$

where $X^i_{S_1}$ and $X^i_{S_0}$ represent the measured time response of the sensor i with and without damage respectively. T_i and T_f are the starting and ending time for the integration. The main problem with this formulation is to compute this scattered energy. It can be achieved through a finite element analysis (FEA). The procedure consists of generating a narrow band excitation by an actuator bonded into the structure and then measure the responses of other sensors. This procedure is performed for both healthy and damaged structure, and then the energy of each recorded signal can be evaluated. For an optimization problem, this procedure has to be repeated a very large number of time until the solution converge to it's global optimum. Doing this by a finite element simulation would be very time consuming.
For example, if we consider a simple case study consisting of 1 plate with dimensions $400 \times 300 \times 1.2$ mm modeled with squared mesh elements of dimensions $1mm \times 1mm$, the resulting finite element model has 720.000 degrees of freedom (DOFs). The CPU time of the transient simulation is about two hours for 3 sensors bonded into the plate, and 8 hours for four sensors (RAM 8 Go, processor 2.5 GHz). This computation is performed at each iteration of the global maximum search algorithm. If the global maximum is obtained after 100 iterations, the total time of simulation will be 200 hours for a configuration with 3 sensors and 800 hours for the one at 4 sensors, which is very time consuming.

Instead of solving the optimization problem from a full transient finite element simulation which is time consuming, we propose a ray tracing approach which uses an analogy between Lamb waves and optical rays[9]. Lamb waves are discretized as rays that can sparse the full 2- dimensional domain of propagation (the direction of propagation is radial). Lamb waves group velocity then determines how fast the rays travel with respect to time while attenuation determines how the amplitude decreases while traveling. Reflection from discontinuities in the structure (boundaries or damages) are assumed to be governed by equation (4) :

$$\theta_i = -\theta_r \tag{4}$$

where θ_i is the direction of propagation of the incident ray and θ_r is the direction of propagation of the reflected ray. To build the ray tracing model, one needs two important parameters: the velocity of the traveling rays (Lamb wave group velocity) and the spatial attenuation of the rays (amplitude decrease of the Lamb waves while traveling). The major difficulty of the ray tracing approach in this case is the angle dependence of the group velocity and spatial attenuation, due to the anisotropy properties of the composite plate. These parameters are determined experimentally in §3.1.

As described earlier the proposed approach enhance detection of damage by maximizing the probability that a set of sensors detect a damage. Considering Lamb wave front as rays, this probability is nothing else than the numbers of rays that are reflected on the damage and go through a given sensor. If we discretize the Lamb waves with a fixed N_R rays, the optimal sensor placement is that for which the number of rays that are reflected from damage and then going through sensors is maximal. The objective function can then be defined as the number of these reflections.

2 Genetic algorithm : Background

Genetic algorithms (GA) were formally introduced in the 1970s by John Holland. GA is a class of evolutionary algorithms (EA) that mimics the process of natural evolution. This process is routinely used to generate useful solutions to optimization and stochastic search problems, which generate solutions using techniques inspired by natural evolution, such as inheritance, mutation, selection, and crossover[10].
The evolution usually starts from a population of randomly generated individuals, and is an iterative process. The population in each iteration is called a generation. In each generation, the fitness (which is the value of the objective function in the optimization problem) of every individual in the population is evaluated. The individuals with higher fitness are stochastically selected from the current population, and each individual's genome is modified (recombined and possibly randomly mutated) to form a new generation. The new generation of candidate solutions is then used in the next iteration of the algorithm. Commonly, the algorithm terminates when either a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the current population.

3 Experimental results

3.1 Experimental characterization of Lamb waves propagation in composite plates

The structure under consideration is a carbon epoxy monolithic composite plate (four layers) with ply orientation $(0^0/-45^0/45^0/0^0)$ and dimensions $400 \times 300 \times 1.2$ (mm). The mechanical properties of the lamina are listed in table 1.

Table 1: Mechanical properties of the lamina

Density (g/m^3)	ply thickness (mm)	$E_{11} = E_{22}$ (GPa)	E_{33} (GPa)	$G_{12} = G_{13} = G_{23}$ (GPa)	ν_{12}
1554	0.28	69	8.1	4.8	0.03

In the center of the plate (figure 2), was bonded a transmitter (source) serving as the excitation. In order to figure out the angle dependence of the group velocity and spatial attenuation (due to the anisotropy of the composite plate), reflective patches were bonded onto the plate with a specific configuration. They are disposed to form a quarter of circle with respect to the source. The step angle between two consecutive patches in the orthoradial direction is $\Delta\theta = 10^0$ and the distance between two consecutive patches in the radial direction is $\Delta d = 20$ mm. The distance between the source and the first patches is d = 40 mm.

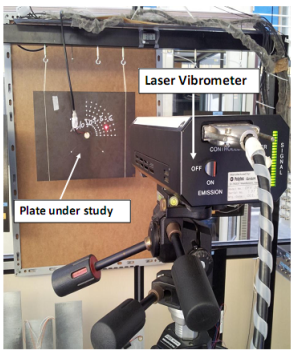
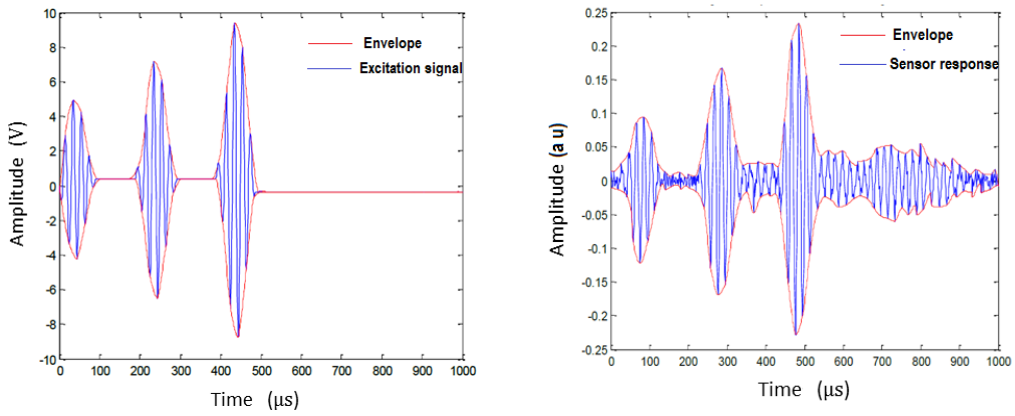


Figure 2: Experimental device for Lamb waves Propagation analysis

The signal sent to the emitting transducer, was a 5 cycles sinusoidal shaped signal, modulated by a Hanning window. We sent three pulses with different amplitudes as show in figure 3. The frequency of the excitation signal is selected at $f_0 = 50$ kHz to dominantly excite fundamental A_0 mode. A laser vibrometer (POLYTEC) was used as the receiver to measure out-of-plane displacements at each patches location. Figure 3 shows the excitation signal and the response measured at the first patches in 0^0 direction.



(a) (b)

Figure 3: Excitation signal (a), out-of-plane displacement 1st patch in 0^0 direction (b)

Given the distance between two consecutive patches in the same direction, the group velocity and the spatial attenuation can be computed respectively as :

$$Vg(\theta) = \frac{\Delta d}{t_{i+1} - t_i} \tag{5}$$

$$\alpha(\theta) = \frac{1}{\Delta d} \log\left(\frac{\tilde{Y}_{i+1}}{\tilde{Y}_i}\right) \tag{6}$$

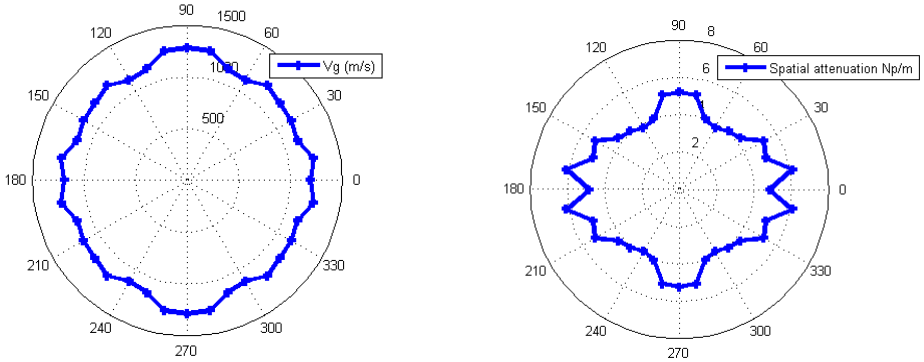
where t_{i+1} and t_i are the time of arrivals of the first wave packet at consecutive patches $i+1$ and i in the same direction respectively. $\tilde{Y}_{i+1}$ and $\tilde{Y}_i$ are the Fourier transform at $f_0 = 50$ kHz of the out-of-plane displacements measured at patches $i+1$ and i in the same direction respectively. Figure 4 shows the angle dependence of the group velocity and of the spatial attenuation of the Lamb waves. Once computed, these parameters are then used in the ray tracing model. Lamb waves are modeled as ray which travel with a velocity equal to $Vg(\theta)$. The amplitude of the rays decreases like Lamb waves signal with a spatial attenuation equal to $\alpha(\theta)$.

3.2 Optimal sensor placement

To illustrate the effectiveness of the proposed sensor placement method, a GA was used to solve the optimization problem (2) where the objective function is evaluated through the ray tracing model as shown in §1.3. The parameters of the algorithms are listed in table 2.

Table 2: Parameters of the GA

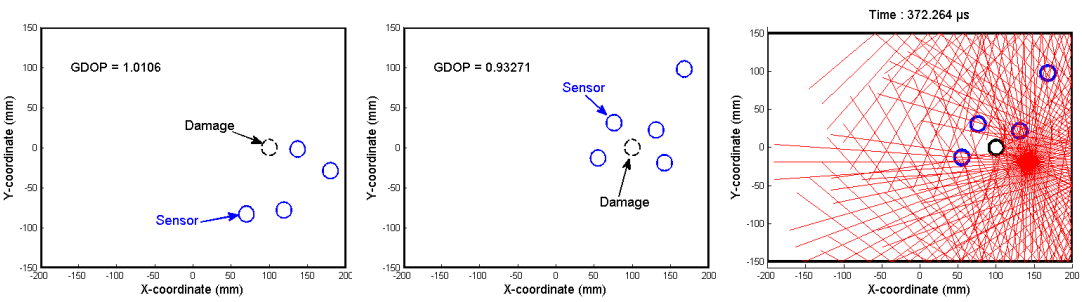
Nb of rays	Nb of generations	Nb of individuals	Nb of elites (crossover)
90	30	50	8



(a) (b)

Figure 4: Angle dependence of the Lamb wave group velocity (a) and spatial attenuation (b)

Figure 5 shows the optimal placement obtained by the genetic algorithm for a configuration of 4 and 5 sensors respectively. Each sensor has circular shape with 20 mm diameter. The center of the damage (modeled as a hole) is located at $(100, 0)$ mm, with 20 mm diameter.



(a) (b) (c)

Figure 5: OSP configuration for 4 sensors (a) and 5 sensors (b). (c)- ray tracing snapshot of (b)

Damage is marked by a black circle while sensors are marked by blue circles. Each transducer acts both as actuator and sensors in a permutation process. The figure 5c shows the snapshot of the ray of configuration 5b at time instant 327.264 μs when one of the transducers acts as actuator and then excites the plate and allows rays to propagate like Lamb waves. The simulated wave fronts based on ray tracing were plotted using Lamb waves group velocity and spatial attenuation obtained experimentally. 90 rays were emitted at 4 degree increments to form reasonable resolution in the wave front. Boundaries reflections were considered by the the solver, to accurately model the propagation phenomenon. A ray is supposed to be totally attenuated if its amplitude is less than 20% of its initial value. The decreasing law is exponential with respect to time. Prior verification were made to validate the rays tracing wave front from a FEA simulation, and a close match between both wave front has been found.
From figures 5a and 5b it is obvious that the optimization algorithm tends to find better solutions (sensors location) for transducers that are near of the damage zone. On the other hand, the effect of GDOP tends to spatially extend the transducers position to form a kind of a polygon, leading to values of 1.0106 and 0.93271 for 4 and 5 sensors respectively.
The results presented here do not replace currently available methods of observing Lamb

wave propagation. However this method is clearly faster (computational time around 12 hours and 16 hours for configurations with 4 and 5 sensors respectively, and for parameters of the GA listed in table 2) than these traditional methods. This enables a user to quickly model Lamb wave front in order to design a robust damage detection strategy in the SHM framework.

Conclusions

This paper describes the implementation of a new strategy for the optimal sensor placement problem using an analogy between optical rays and Lamb waves in order to model Lamb wave propagation. The optimization problem is solved by a GA, and the proposed method applied to experimental specimens is particularly effective in the case where the damaged zone to be monitored is prior known. Moreover, good results are obtained with lower computational time. Further taking into account the GDOP while maximizing the PoD would improve the localization of the damage, which is one of the fundamental challenge in SHM.

Acknowledgments

This study was supported by the CORALIE project. The authors would like to acknowledge the project head.

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