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# Fast B-Spline 2D Curve Fitting for unorganized Noisy Datasets

Authors: N El-Hayek<sup>1,2</sup>, O Gibaru<sup>1</sup>, M Damak<sup>1,3</sup>, H Nouria<sup>2</sup>, N Anwer<sup>4</sup> and E. Nyiri<sup>1</sup>

- 1- Arts et Métiers ParisTech, Laboratory of Information Sciences and Systems (LSIS), 8 Blvd Louis XIV, 59046 Lille, France  
 2- Laboratoire Commun de Métrologie (LCM), 1 r. Gaston Boissier, 75015 Paris, France  
 3- GEOMNIA, 3D Metrology Engineering and Software Solutions, 165 Avenue de Bretagne, EuraTechnologies 59000 Lille, France  
 4- Ecole Normale Supérieure de Cachan, University Research Laboratory in Automated Production, 61 av. Président Wilson, 94235 Cachan, France

## Context

- Optical and Tactile Metrology for Absolute Form Characterization (EURAMET project IND10)
- Fast polynomial spline curve reconstruction from very large unstructured datasets

## Objective

Curve reconstruction of freeform shapes, specifically turbine blades, from data with unknown topology

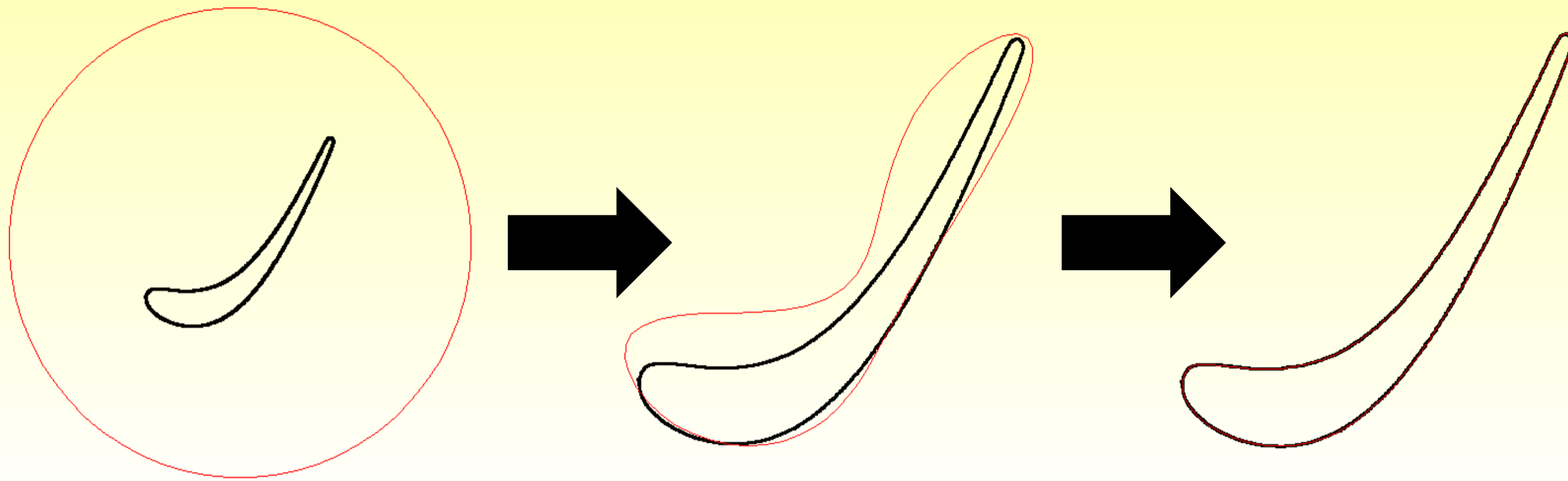
### Objective function

$$\min_{t_1, t_2, \dots, t_m} \sum_j \left( (MT_m)_j - \delta_j \right)^2$$

M is the subdivision matrix (1)

$T = \{t_1, t_2, \dots, t_m\}$  is the control points translations vector

### Discrete B-Spline Convection scheme



✓ NO initial parameterization

✓ NO differential calculations

✓ NO sampling requirements

✓ Invariance of final control polygon geometry to initial position and orientation

## Methodology

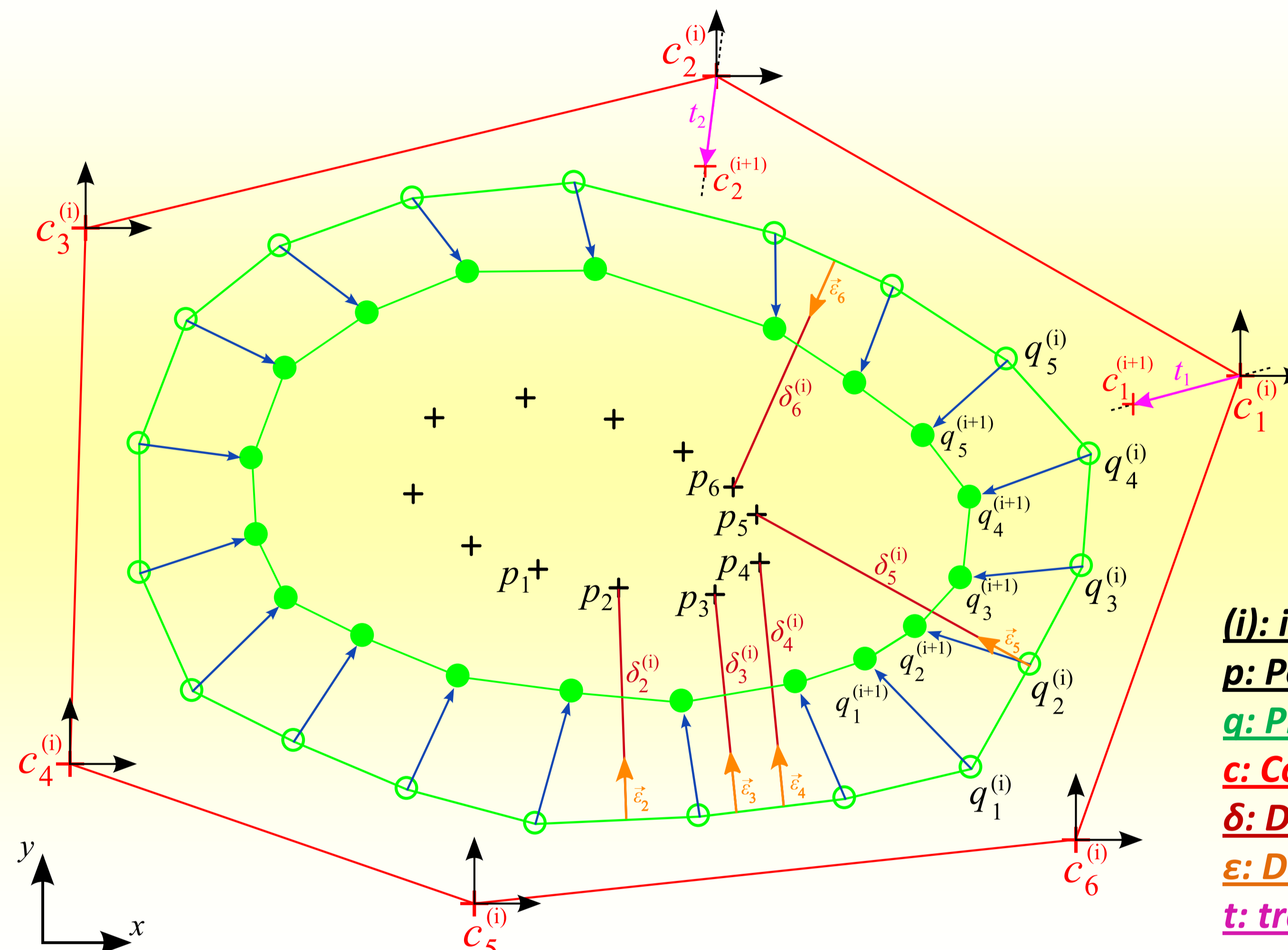
Coincide new B-Spline curve at iteration (i+1) with data points by minimizing the distances

The B-Spline (green) is initialized by a few control points around the data.

Distances  $\delta_i$  are calculated with geometrical and topological considerations.

$L_2$  minimization  $\rightarrow$  translation vectors  $\{t_1, t_2, \dots, t_m\}$  by which control points must move.

If the minimization does not meet the error tolerance, point insertion is applied locally.



### (1) Subdivision relation

$$q_j^{(i)} = MC_m^{(i)}$$

### (2) Convection equation

$$C_m^{(i+1)} = C_m^{(i)} + T_m^{(i)}$$

### (3) Solution equation

$$q_j^{(i+1)} = M(C_m^{(i)} + T_m^{(i)})$$

(i): iteration i

p: Point set

q: Piecewise linear B-Spline curve

c: Control points

delta: Distances

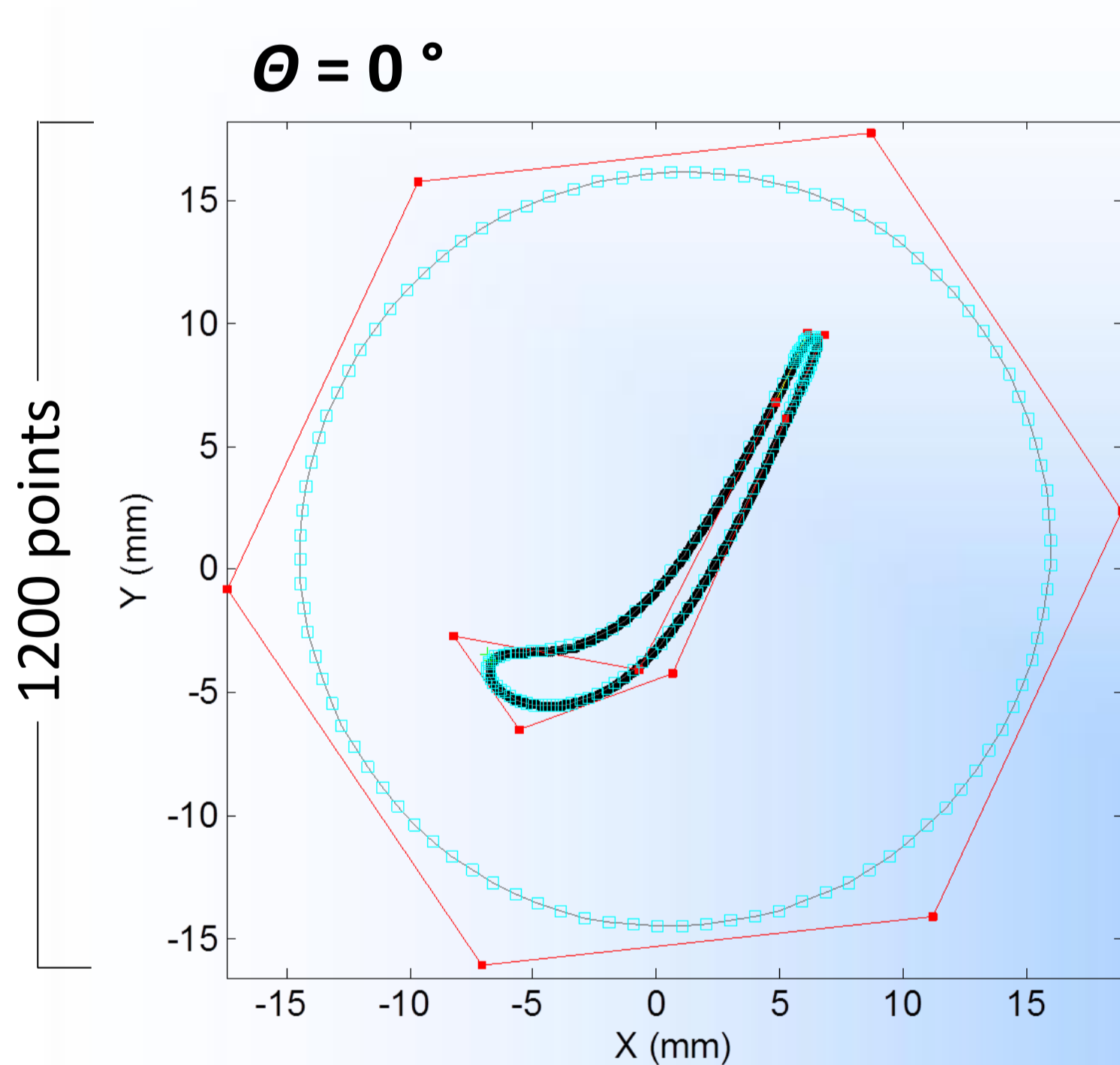
epsilon: Distance vectors

t: translation vectors

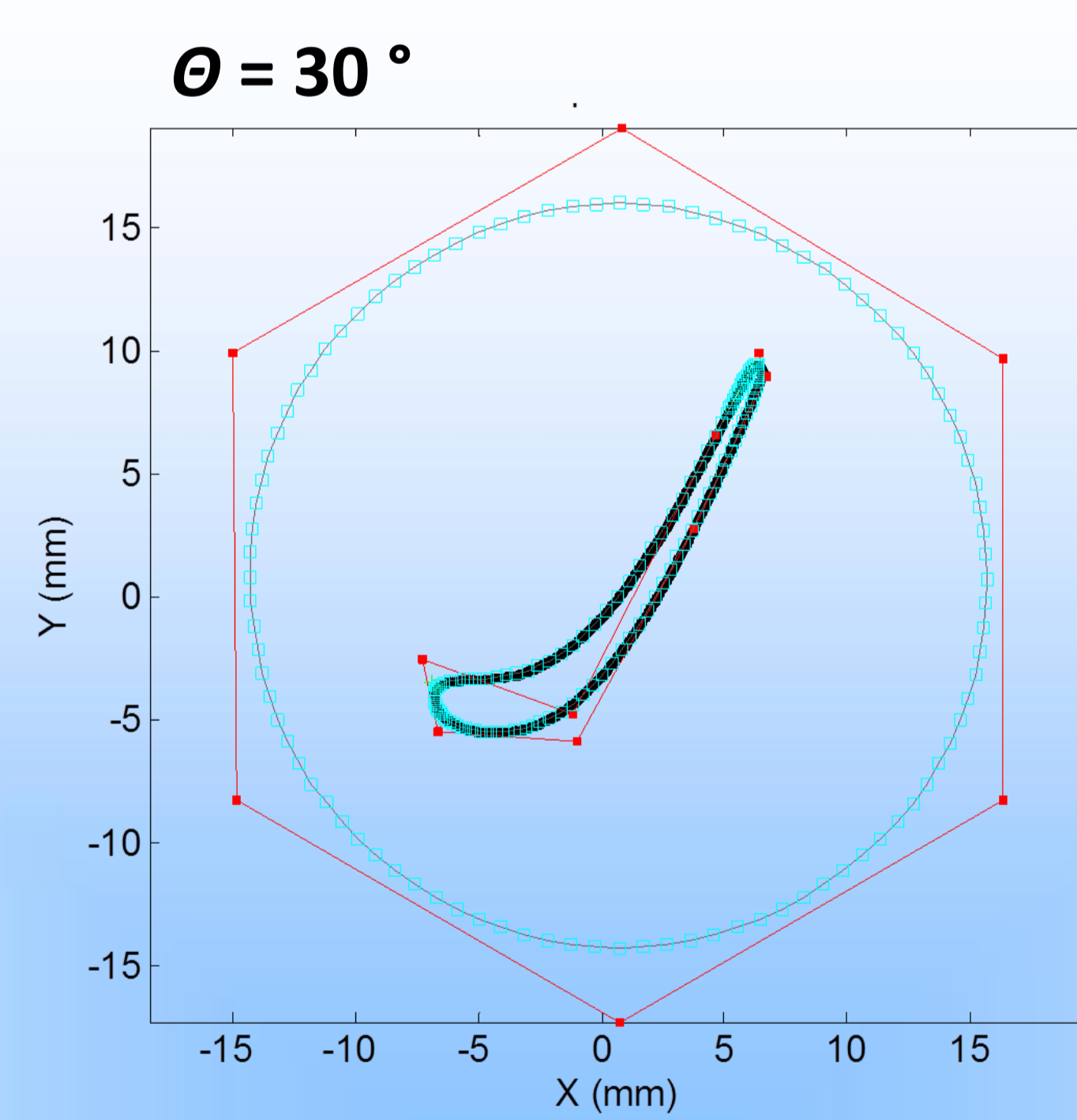
## Experimental results

### Invariance to point-set orientation

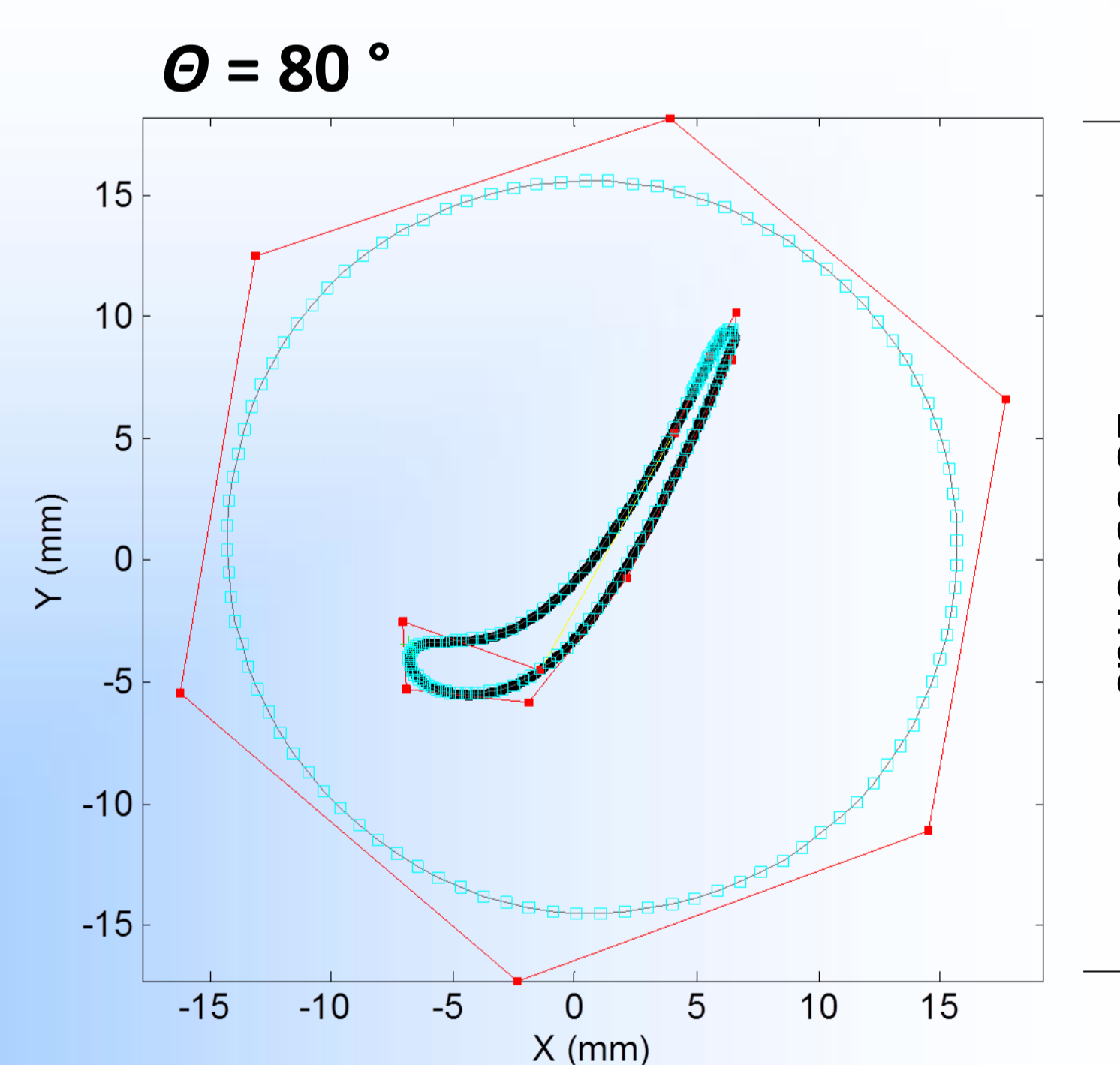
$\epsilon$  = mean of residual errors



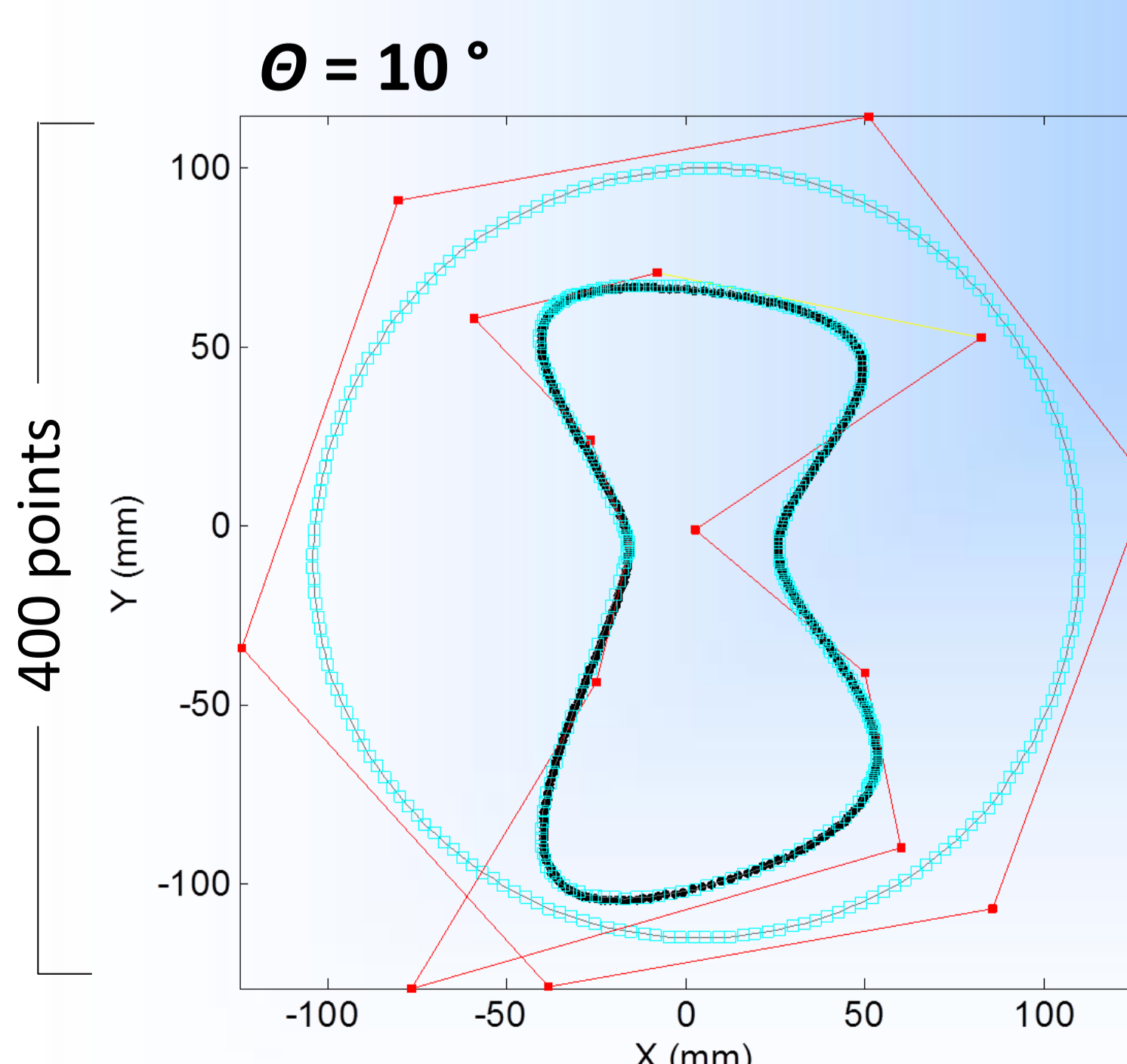
8 final control points:  
 $\epsilon \approx 0.0015$  mm, 140 iterations



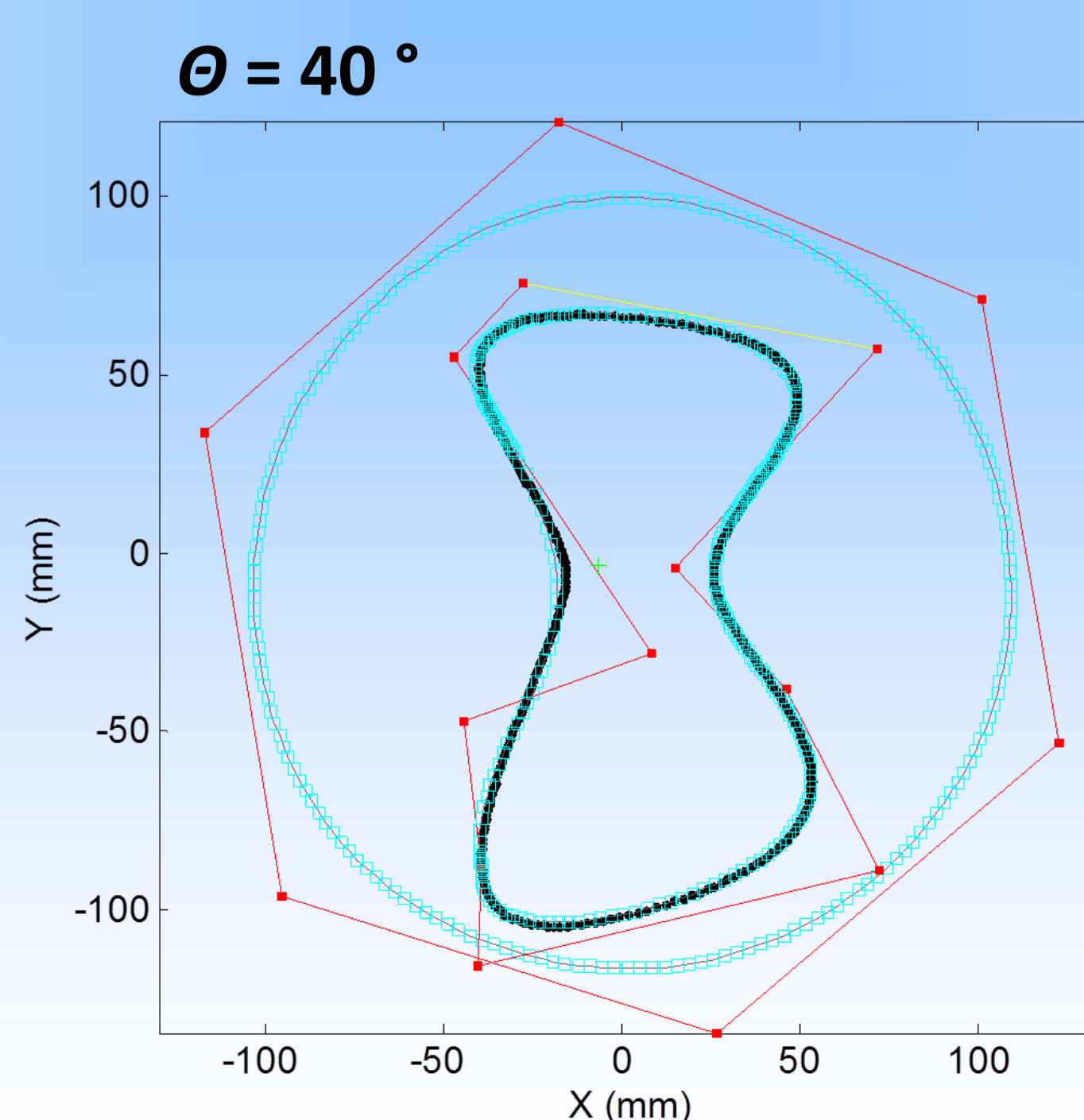
8 final control points:  
 $\epsilon \approx 0.00088$  mm, 140 iterations



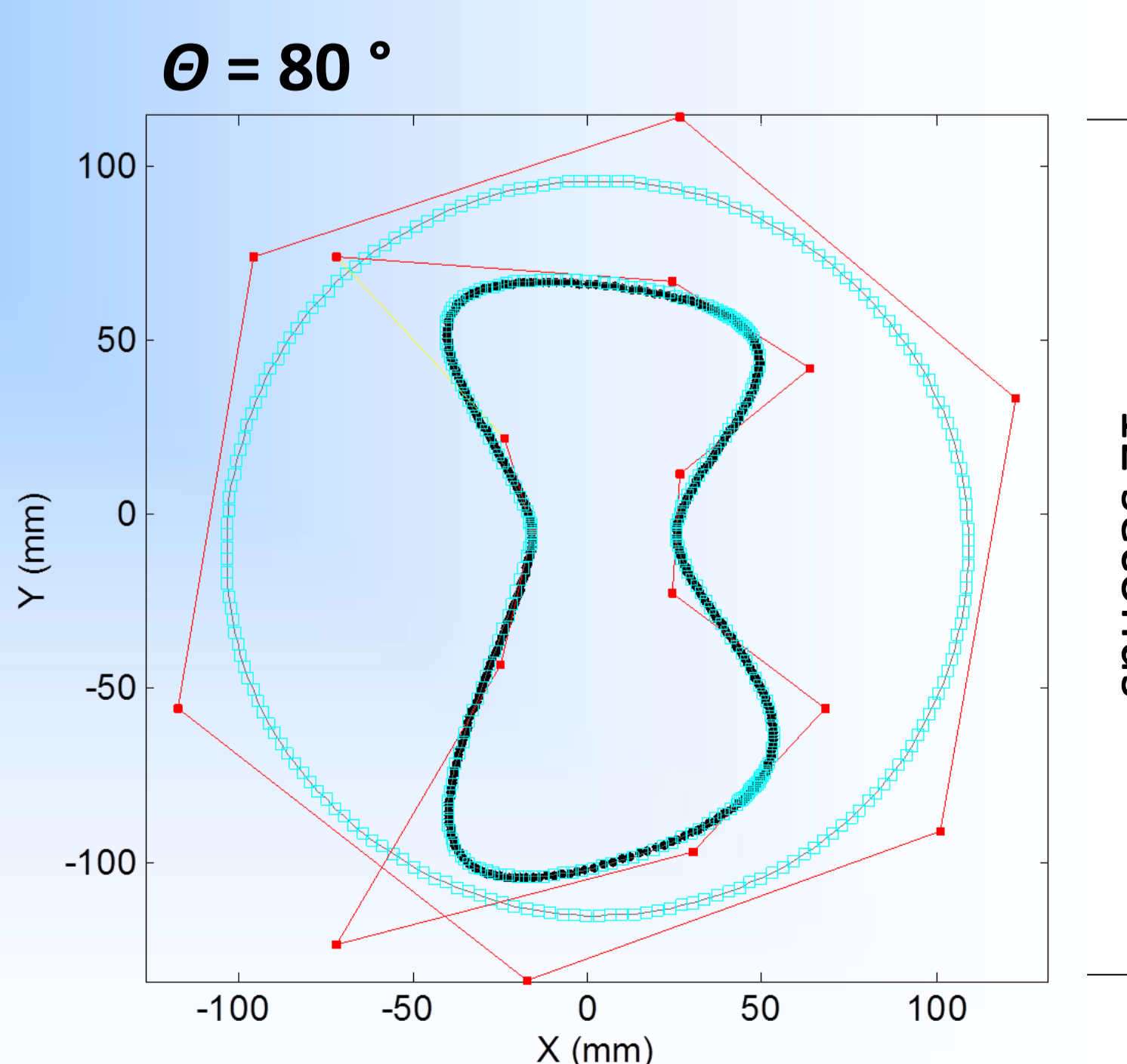
8 final control points:  
 $\epsilon \approx 0.0023$  mm, 140 iterations



10 final control points:  
 $\epsilon \approx 0.0024$  mm, 140 iterations



10 final control points:  
 $\epsilon \approx 0.0017$  mm, 140 iterations



11 final control points:  
 $\epsilon \approx 0.00091$  mm, 140 iterations

## Conclusions

- The B-Spline convection algorithm is founded on discrete computations.
- The algorithm is robust regarding the relative initial position of both the B-Spline and the data.
- The algorithm is tested on several shapes and returns residual errors below threshold if not too small.
- The initial number of control points must be minimal.
- The algorithm can be subject to time complexity improvement.
- Precision is not yet controllably achievable.

- [1] Speer T., M. Kuppe, and J. Hoschek. Global reparameterization for curve approximation, in Computer Aided Geometric Design, 1998.  
 [2] Wang W., H. Pottmann, and Y. Liu. Fitting B-Spline Curves to Point Clouds by Curvature-Based Squared Distance Minimization, in ACM Transactions on Graphics, 2006.  
 [3] Zheng W., P. Bo, Y. Liu, and W. Wang. Fast B-spline curve fitting by L-BFGS, in Computer Aided Geometric Design, 2012.

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